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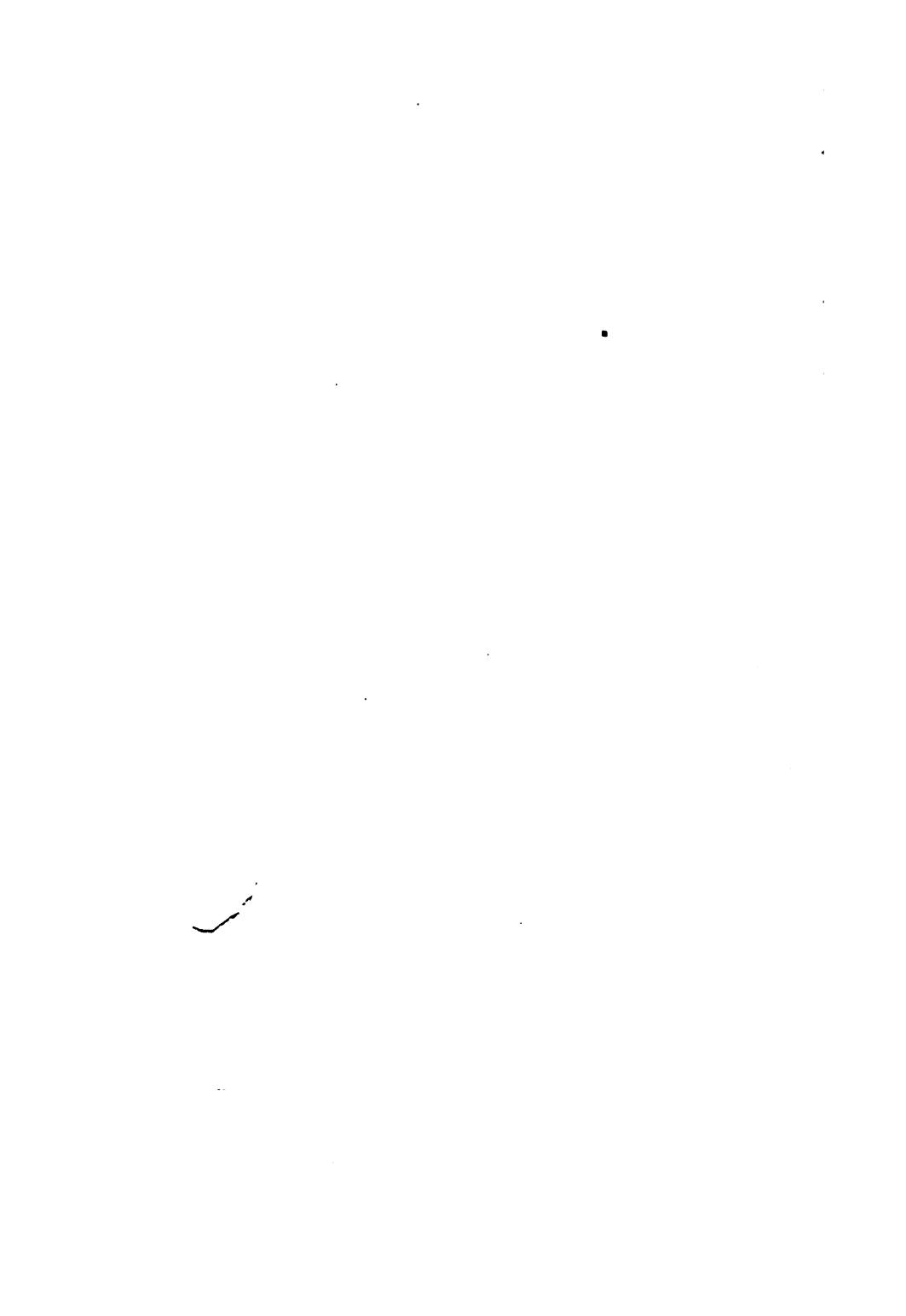
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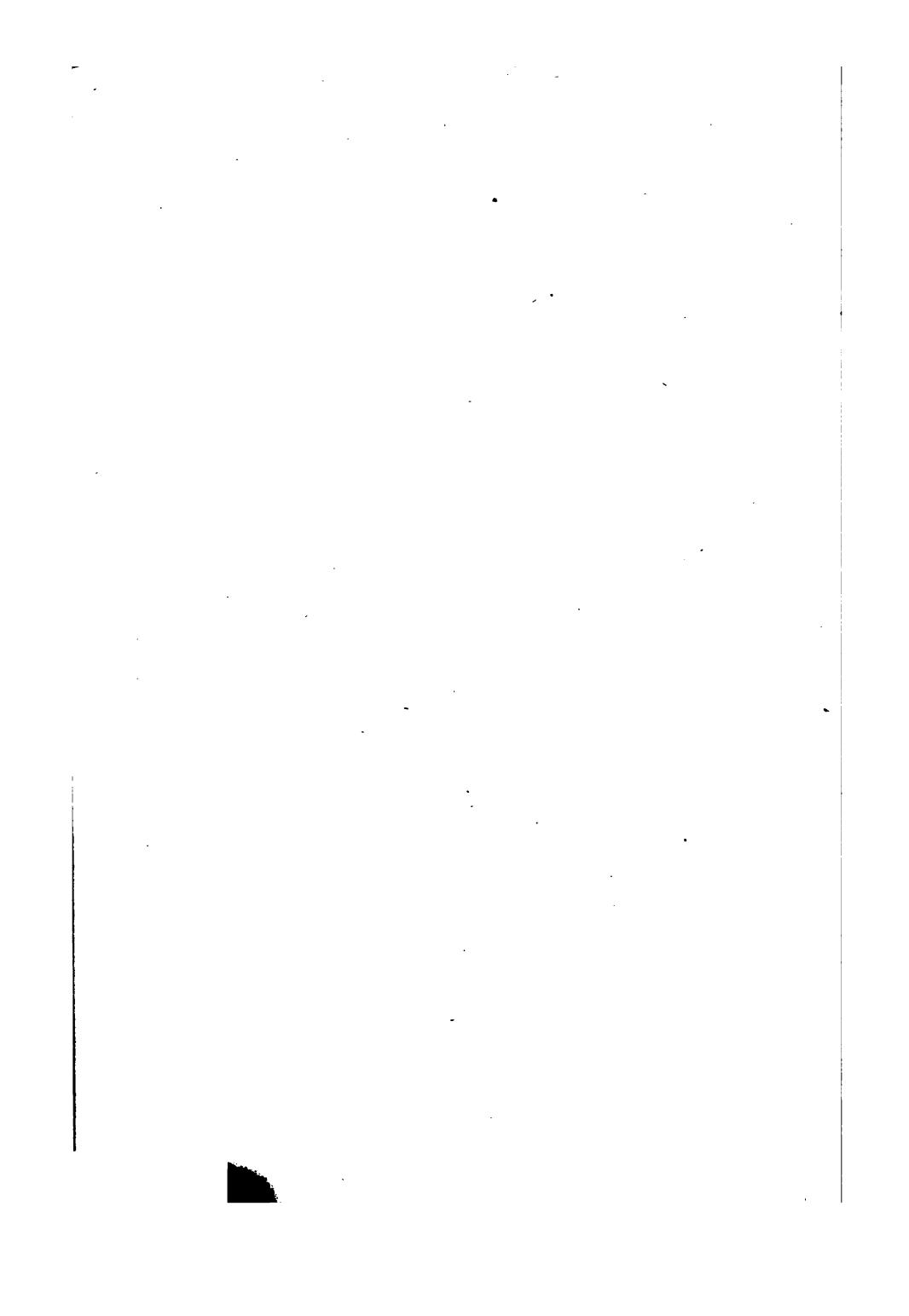
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# **LOGIC**



The University Series

LOGIC

BY

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LONDON

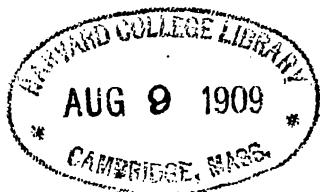
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## PREFACE

THE aim of the present treatise is to present the leading principles of Deductive Logic with as much lucidity and point as possible; and while preserving throughout an elementary character, yet with such a regard to critical discussions, as that the student who afterwards desires to take up any of the more discursive treatises will have nothing to unlearn, and will find in the numerous references throughout the work a useful guide to his further reading.

Being a volume of "The University Series," so far as its compass has permitted, the existing requirements of the universities, more especially of Great Britain and Ireland, have been steadily kept in view; and to the several books are appended copious sets of questions selected from the examination papers of Oxford, Cambridge, Dublin, the Royal, Edinburgh, and other universities. Some questions selected from papers set in the Queen's University by Professor Park of Belfast, and

Professor Moffett of Galway,<sup>1</sup> are marked with the initials of the examiners ; but as to the rest, with perhaps a single exception, the letters subjoined indicate the respective universities at whose examinations the corresponding questions were set. The papers of the higher Civil Service Examinations have also been occasionally laid under contribution ; and, whether specially indicated or not, questions from this source will almost surely be recognized for a peculiar educational and philosophical suggestiveness. In Logic, such questions, sometimes requiring considerable thought and exercise of the reasoning powers, tend to relieve the monotony of an unbroken mass of purely expository matter ; and conscientiously worked out, they are an invaluable aid to a thorough grasp of the principles of the science. It is hoped that this feature, while commanding itself to the private student, will render the work the more useful for class purposes. The fourth book, on Applied Logic, deals with Induction ; indeed, very briefly, but yet so, it is hoped, as to give the work a certain completeness for students who may find themselves obliged to confine themselves to it as their sole text-book.

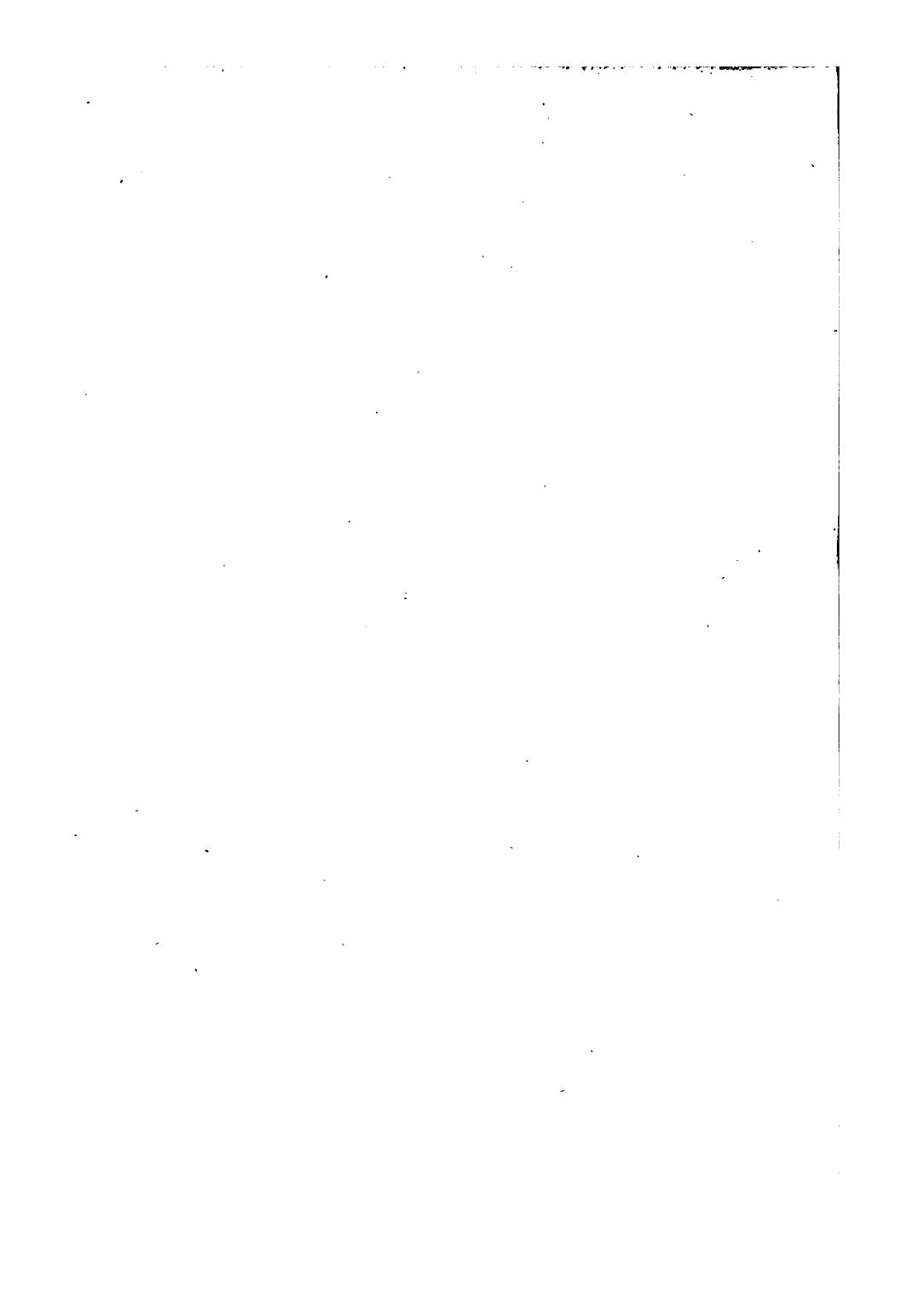
Perhaps, the most suitable acknowledgment the author can make to living writers who have left foot-prints on

<sup>1</sup> Sir Thomas W. Moffett ; a venerable name no longer, except in honoured memory, associated with Galway College, but still adorning the roll of the Senate of the Royal University of Ireland.

this path, and to whom he may feel indebted for a gracious concession on the part of any, or helpful guidance, and to the various educational bodies that have so willingly accorded him permission to transfer some of their questions to his pages, is the treatise itself now before the reader; one which, it is sincerely hoped, will to some extent assist them in their high aims, by, in some additional measure, popularizing the subject it is conversant with, and rendering this ancient study the more acceptable at the seats of higher learning in this country.

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*July, 1906.*



CORRIGENDA.

P. 3, line 13, *for* "Applied" *read* "Pure."

P. 6, line 2 from bottom, *for* "so" *read* "as."

P. 23, line 7 from bottom, insert "the" before "science."

P. 25, line 22, *read* "Conception."

P. 29, line 5, delete "All."

P. 44, line 7 from bottom, *read* "it is known."

P. 89, line 12, *real* "violated" *for* "isolated."

P. 100, line 11 from bottom, *read* "descriptio."

P. 162, interchange lines 10 and 11.

P. 184, line 24, insert "a" before "man's."

P. 220, line 6, *for* "methods" *read* "truths."

P. 231, line 5 from bottom, delete "and."

P. 232, lines 21-23, *for* "Definition. It must . . . Division" *read* "Definition and Division."

P. 233, line 5 from bottom, *for* "science" *read* "sciences."

P. 236, line 2 from bottom, *for* "conceivable" *read* "inconceivable."

P. 237, line 7 from bottom, *for* "Bouffier" *read* "Buffier."

P. 269, line 6 from bottom, insert after the period—"By a *fixed* error is meant one due to the instrument we use."

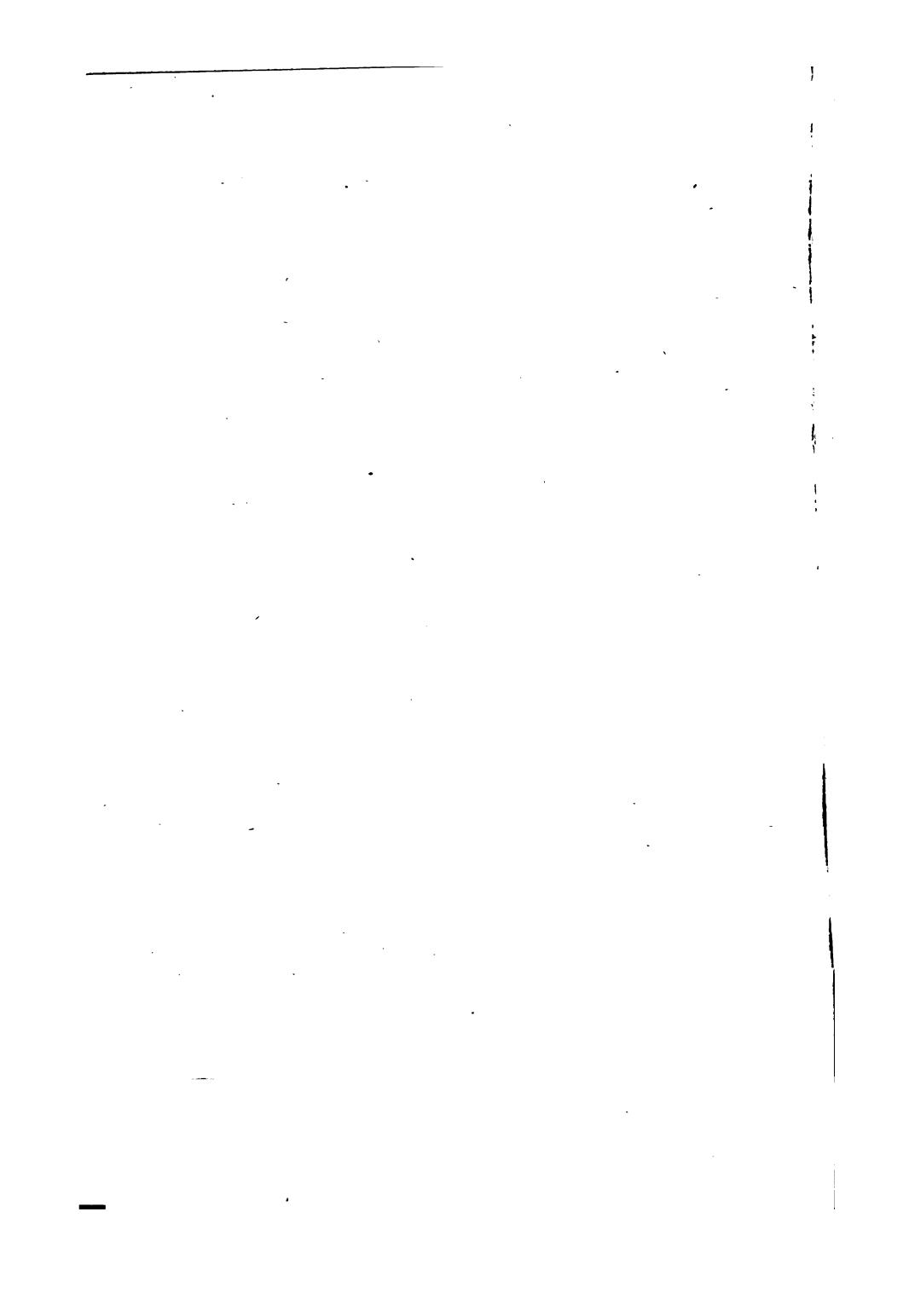
P. 271, line 8 from bottom, delete "if."

P. 279, line 15, *for* "Induction" *read* "Inductions."

P. 283, line 21, insert "yellow" before "double."

P. 288, line 16, *for* "simplicity's" *read* "simplicity'."

P. 312, line 15 from bottom, insert "not" before "respiration,"



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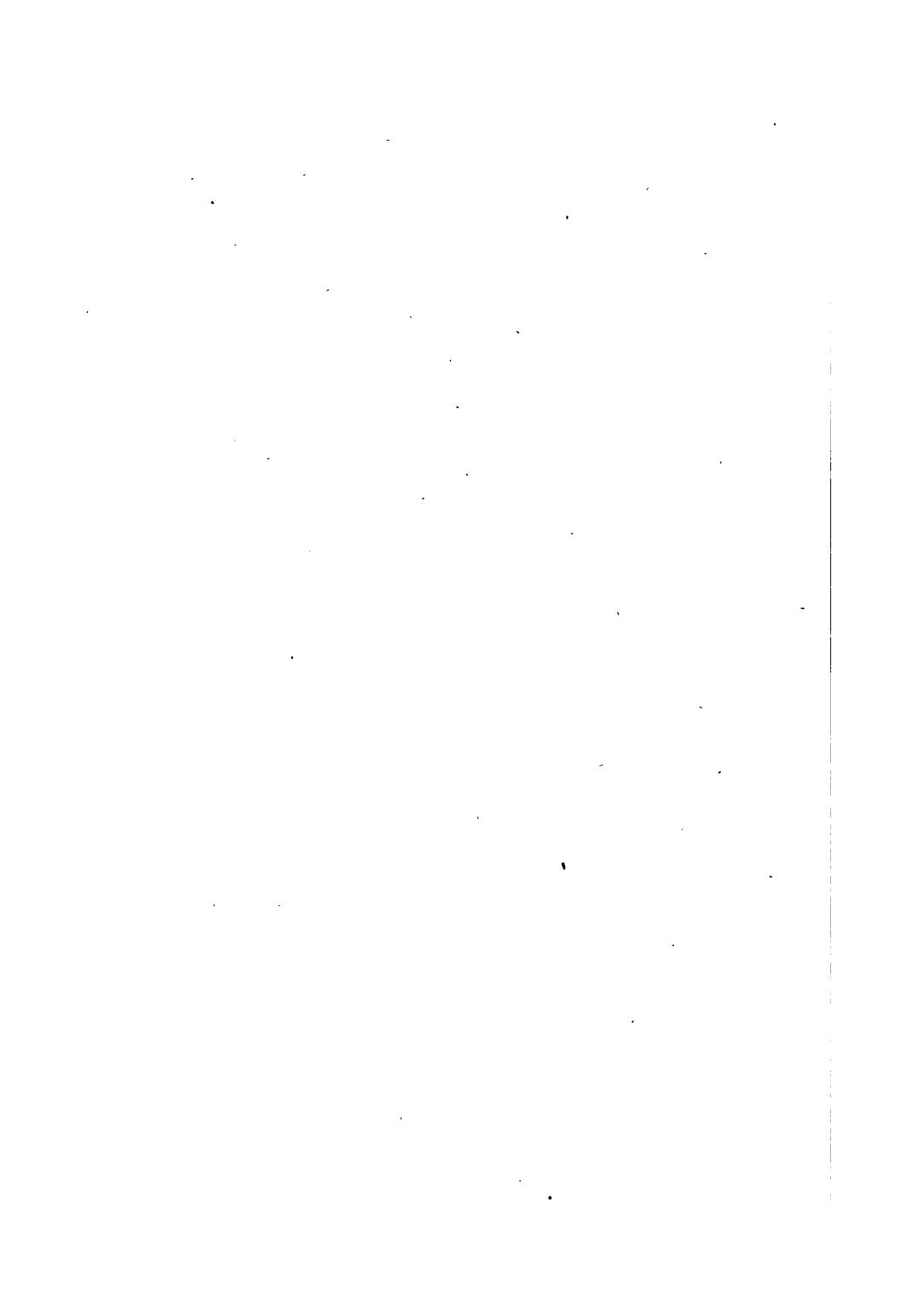
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# LOGIC

## INTRODUCTION

### CHAPTER I

#### PROVINCE OF LOGIC

**Scope of Logic.** § 1.—Logic, having regard to its scope in the widest sense, may be defined as the Science of the Necessary Laws of Thought, (1) in their own nature, and (2), as employed in attaining Truth or Knowledge.<sup>1</sup> Thought,

<sup>1</sup> Knowledge has been defined as consisting of “the sum total of what can be realised respecting the world of external and internal experience.” But, it is held by some philosophers that portion of our knowledge is *à priori*, that is, belongs to the mind independently of experience, that which is derived through the acquaintance of the mind with external facts being called *à posteriori*. The most perfect form of knowledge is science, which consists of a series of determinations of as great generality and certainty as possible, grouped into distinct departments, each embracing subjects of a kindred nature, with a definite order or arrangement suitable to their end in the collection, verification, and communication of knowledge. According to Leibnitz (1684), any element of knowledge, to be perfect, must be clear, distinct, adequate, and intuitive. He regards knowledge as intuitive when it consists of a clear insight into, if not all the attributes, at least the essential attributes of the object. This will generally be the case, however, only with the simpler elements of knowledge: the more complex are seldom fully realised; seldom, but only in name, and symbolically. Locke (1689) holds that our knowledge is conversant only with our ideas; and, in any individual instance, is but the perception of the agreement or disagreement of any of them. It is *intuitive* when the mind perceives the agreement or disagreement of two ideas by themselves, without the intervention of any other; *demonstrative*, when the mind perceives such agreement or disagreement through the intervention of one or more others.—*Essay concerning Human Understanding*, Bk. IV., chaps. i. and ii.

or Thinking, is that active function of the mind by which impressions received from within, or from without, are described, classified, and compared. The term, Thought, is sometimes applied to the process itself, sometimes to the results of the process. Impressions, generally, on which thought may be exercised—excluding acts of the Will—are called Presentations. If the Presentation be from within, it is called a Sensation ; if from without, a Perception. A Notion or Cognition is a clear, as distinguished from an obscure Presentation ; and an Intuition is a Notion formed in the mind from an immediate, and individual, or single object. Notions, or Cognitions, are the elementary data of Thought.<sup>1</sup> The necessary laws of thought are certain general principles that regulate the process of thinking. They are called *necessary*, because they *must* invariably be observed in thinking ; and *laws*, because all persons who think and reason correctly must observe them, and uniformly reason in accordance with them. Again, Logic is a *science*, in so far as it restricts itself to one special kind of subject-matter, namely, the necessary laws of thought, and investigates the theory of correct thinking as founded upon them, and, as distinguished from an *art*, which would have occupied itself in laying down rules to assist in the detection of false reasoning. As the science of the necessary laws of thought, Logic shows what principles, for example, must be put in practice in forming any general notion whatever, and on what principles one must proceed in combining one's impressions and drawing inferences from them. Further, Logic, as a science, may be regarded in a two-fold light, namely, as the science of the laws of thought in their own nature, or, as they are employed in attaining truth or knowledge. As the science of the laws of thought in their own nature, it is called *Pure*, or *Formal* Logic ; as employed in attaining truth or knowledge, it is

<sup>1</sup> Notions, or cognitions, are sometimes, though not with due precision, referred to as *Ideas*. "An Idea," according to Locke, is "whatsoever is the object of the understanding when a man thinks" ; or "whatever the mind perceives in itself, or is the immediate object of perception, thought, or understanding."

called *Material*, or *Applied Logic*. In its origin, Pure Logic may be regarded as a body of doctrine abstracted from that wider Logic called Applied: it is an abstract science, in relation to Applied which may be called concrete. Truth, in the sense of knowledge, means so much, including all that men can ever know, the science that explains how the mind deals with it must—the object-matter being infinite in extent—be necessarily loose and indefinite; and, consequently, Applied Logic can never attain completeness and precision, because it can never affirm that it has shown how the mind deals with every part of truth and knowledge. With Logic considered as the science of the necessary laws of thought in their own nature, otherwise called *Applied Logic*, we shall be mainly occupied. It has to recommend it in respect of scope and definition that it can be completely presented within comparatively moderate limits, and that, while embracing for its domain the most fundamental principles of all the sciences, it stands in no need of extraneous aid, and reposes on nothing more fundamental than itself.

*Pure or Formal Logic.* § 2.—Pure or Formal Logic is the science of the necessary laws of thought. It has *thought* rather than *language* for its adequate object-matter; for though it must express itself in language, and is very much concerned with it, language comes in only as the minister of thought. It is a *science*;—a science rather than an *art*.<sup>1</sup> In what precedes we have been considering Logic solely as a science; that is, as a body of principles with deductions. In the popular meaning of the term, however, the notion of an *art* is more prominent; the notion, that is to say, of a body of precepts intended to produce skill in reasoning; and, as such, we may regard Logic as the art of thinking consistently,—

<sup>1</sup> A Science may be briefly defined as a systematised body of principles and deductions to explain some object-matter; an Art is a body of precepts teaching how something must be produced, with the practical skill requisite for the completion or production of the work. A Science teaches us to know; an Art, to do. Art, according to Aristotle, is conversant with production; Science is conversant with that which is: *τερπ γένεσιν, τέχνην; ηερπ τὸ δό, επιστήμην*. *An. Post.* II., xix., 4.

that is, in accordance with the necessary laws of thought, or, as the art of attaining to correct, and exposing and avoiding incorrect thoughts. If, however, the subject were to be treated as an Art, it should be as the widest of all arts, containing specific rules for thinking in every region of thought, and on all sorts of matter, suitable at the same time to men of the most various capacities and under the most various circumstances, and admitting no principles of selection other than suitability for the performance of its work as an art. The result would be an expansion of the heterogeneous materials beyond all reasonable limits. And yet, withal, there would be a sense of incompleteness ; and, at the same time, such a treatment would necessarily fail in communicating the *art* in any eminent measure, because that unconscious skill which is the crown and completion of an art cannot be attained from what is taught in books. Not so, however, with a science : the whole of a science can be made the subject of teaching. So that Logic, as a body of principles with deductions,—teaching us to know, rather than a body of precepts with practical skill,—teaching us to do, may be regarded as a science rather than an art. The art of Logic, however, that is to say, the art of thinking, in so far as it is teachable, rests upon a moderate number of principles ; and these are so much bound up with the exposition of the subject as a science, and so naturally suggest the fitting application of them to practice, as to render the making of them into a distinct section on Logic as an art unnecessary.

Every act of thought is a thought about something. What it is about is the *matter* ; the mode in which the mind views the matter is the *form*. Pure Logic is a science of the form, or of the formal laws of thought or thinking, not of the matter : that is, it treats of the laws of thought in their own nature, and apart from the matter, or what we think about. Such laws have been called the forms of thought, or the formal laws of thought ; the meaning of the term *form*, like that of its correlative, *matter*, having been extended by analogy from material to immaterial things. For, *matter*, in the

original use of the term, meant the material or stuff, *form*, the shape the stuff assumed in the hands of the craftsman—the sculptor, or other manual artist. The terms, form and matter, then took successively other meanings, but still each with a certain relation to the original signification. Thus, *form* came to be understood, ( $\alpha$ ) as a *law*, or an *idea*, which mean the same thing, but looked at from two different standpoints, ( $\beta$ ) as that part of an object by which it comes under a given law, ( $\gamma$ ) as that class of cases brought together and united by a law, and, ( $\delta$ ) as the mode of viewing, or of knowing, objects presented to the mind—the *matter* being the perception, or the objects themselves that we have to know. The *idea*, or *essential* thing, in a statue, for example, which an artist designs to hew out of wood, or marble, or other material, might from his point of view, as a manual craftsman, or that of his pupil, be in a figurative sense, instead of a *law*<sup>1</sup> proper, to be strictly worked out by him, or followed in every touch of his chisel, and as nearly as possible expressed by his hand. By the artist, again, the *idea* or *design*, might, in a finished statue or other such work of art, be regarded as the distinctive and essential thing; and in a group of statues, to be the expression of it would be his *mode of knowing* that which he was in search of. And, from such uses, these terms, form and matter, passed on quite naturally to be employed in relation to immaterial things.

<sup>1</sup> *Law*, in relation to the order of nature, whether in mind or matter, is purely figurative, and expresses merely *uniformity of sequence*; not, as in Ethics or Politics, a rule set by intelligent superiors to intelligent inferiors, implying a consciousness of obligation, and entailing the infliction of pain on neglect.—See Bain's *Logic*, Part First, p. 245. But an important analogical feature in the application of the word to a principle regulative of the processes of thought is easily perceived. For, presupposing our minds capable of perception, and actually receiving impressions from the surrounding world, the Laws of Thought are the principles on which, in all cases without exception, we *must* proceed in combining such impressions and drawing inferences from them.—See the late Archbishop Thomson's *Outlines of the Necessary Laws of Thought*, § 10. The expression of any law of thought in general or indeterminied symbols, as, for example, when we say, “Everything is either S or not –S, whatever S may be,” gives perhaps a clearer insight into the nature of the term, law, as employed in Logic, than a long explanation in words.

Thus, they at length acquired in the language of philosophy the several cognate imports just given of them. To say, therefore, that Pure, or Formal Logic, is a science of the form, or the formal laws, of thinking, is to say that the science is concerned only with that which is essential to and distinctive of the thinking process.

Every act of thought is, as we have said, a thought about something ; and we have observed that what it is about is the *matter* of the thought : the mode in which the mind views the matter is the *form*. The matter may be varied indefinitely, and yet continue to be viewed by the mind in the same unaltered form ; and, conversely, we may, in one unaltered form, express thoughts about the most diverse kinds of matter. In conception, the matter consists of certain distinct notions—if not even distinct conceptions of a lower order, viewed by the mind under one common form ; and which being but *one*, and existing *only in the mind*, may be taken to be everywhere and absolutely the same—a characteristic of form as distinguished from matter. In judgment the proximate matter consists of two notions, viewed by the mind as subject and predicate, and so as to determine in the judgment a certain *quantity*—universal or particular, a certain *quality*—affirmative or negative, and if affirmative, a certain *relation*—attributive or substitutive. In syllogism, the proximate matter consists of two judgments, or premisses, viewed by the mind as necessitating a new and distinct judgment, not a mere repetition of the premisses or antecedents, the truth of which cannot be denied without impugning those laid down as premisses, and accepted for true. The *formal* or *logical* value of any thought is the view taken of it as a product of the form or law of thinking exhibited in connection with the particular kind of matter in the thought ; as concept, judgment, syllogism, and so on. Pure, or Formal Logic, is so far indifferent about the matter of a concept, or the matter or material elements of a judgment or of a syllogism, so that a purely symbolical expression, in which the particular nature of the material elements is kept

altogether out of sight, may be made to serve its purpose ; thus restricting its attention to the mere form, with which alone, in each case, it is concerned—a restriction that dominates its entire field. Formal Logic is, in fact, the theory of the form, or forms of thought ; or, in the terms of Dr. Thomson's definitions—(1) the science of the necessary laws of thought ; or, (2) the science of the form, or the formal laws of thought ; or, of Sir William Hamilton's—(1) the science of the laws of thought as thought ; (2) the science of the formal laws of thought ; (3) the science of the necessary forms of thought ; or (4) the science of the laws of the form of thought. Where in these definitions we find the plural, *forms*, employed, we may understand the term to mean *laws* ; and where we find the singular, *form*, we may understand it as signifying *mode of viewing*, or the *distinctive* or *essential* part. By *thought as thought*, Hamilton means, *the form of the thought*, to the exclusion of the matter.<sup>1</sup> It may be defined, in somewhat different terms, as the science that investigates the regulative principles of thought, that is to say, that investigates the principles that regulate the operation of thought or thinking, in itself, as a mental process, invariable in character, whatever the thinking may contingently be about.

Formal Logic, as thus defined, enounces the laws which we must observe in thinking. For explanations, however, of certain subsidiary processes, some or all of which must take place to allow us to think, it refers us to Psychology, or the science of the human mind. It takes no account of the modes in which we collect the materials of thought : it presupposes that we possess them, and but refers them to their proper heads or principles. It assumes that the thinking process, and the processes of the mind subsidiary thereto, are in their complete and perfect state. And it treats only of arguments that are certain and irrefutable. It has, for its end, logical truth,<sup>2</sup>

<sup>1</sup> See Mansel's *Aldrich*, Introduction ; Thomson's *Outlines of the Necessary Laws of Thought*, §§ 1-15 ; Hamilton's *Lectures*, vol. iii., pp. 24, 25, and 26.

<sup>2</sup> We shall, in a subsequent chapter, devote some space to a discussion of *Truth*. Here, perhaps, it will suffice to distinguish between *Truth*,

—that is, self-consistency in thought. Deductive Logic, so-called from the mode of reasoning employed in it, is regarded, generally speaking, as identical with Formal. It employs, throughout, but deductive reasoning, the general characteristic of which is that the conclusion is but a development of what is contained in, or implied by, the premiss or premisses—that is to say, is nothing more general. It contents itself with correctly tracing what follows when certain things are given as true, without inquiring whether its data are objectively, or, as we may say, *really* true or not.

<sup>The</sup> <sup>§ 3.—Before passing away from the Definition</sup> <sup>Definitions</sup> <sup>of Logic, something is to be said on the difficulties</sup> <sup>of Logic.</sup> inherent in it, the position of the question among the ancients, and the contributions to the literature of it by recent logicians, chiefly English—contributions which could not conveniently have been considered in the preceding part of our discussion.

“To define,” says Mr. Mill, “is to select from among all the properties of a thing those which shall be understood to be designated and declared by its name; and the properties must be well known to us before we can be competent to determine which of them are fittest to be chosen for the purpose.”<sup>1</sup> A satisfactory definition of Logic is difficult to frame, partly because of the necessity of deciding between certain attributes of the subject as to their relative claims to be designated by its name; partly because of the inadequacies and imperfections of language; partly because of certain traditional limitations on the one hand, and of general scientific and intellectual progress on the other: partly because of the elusive and intangible character of certain operations of the

considered as a quality, and Truth, considered as the equivalent of Knowledge. Truth, as a quality, is distinguished into *formal* or *logical*—which is self-consistency in thought, *objective*, *scientific*, *metaphysical*, or *real*—which is a correspondence between thoughts and things, and *moral*—which is a correspondence between thought and expression. When the products of thought are logically true, they are said to be *formally valid*; when they are objectively true, or agree with the reality, they are said to be *objectively valid*.

<sup>1</sup> Mill's *Logic*, Introduction, § 1.

human mind ; and partly because of the mysterious, inexpressible, and, in some respects, incomprehensible relation of the human mind to the universe around us ;—for Logic has something to say to each and all of these things. Accordingly, no definition of Logic has yet been offered that has not some element in it for one reason or another making against its general acceptance and finality ; and we find in works of authority considerable discordances as to (1) what Logic had best be called in a definition, whether a science, or an art, and (2) as to whether it is most properly to be described, and held to be, of subjective,<sup>1</sup> or of objective, of materialistic, or, of linguistic contents, —with all implied besides in the attitude taken respecting these questions by any particular logician with regard to the scope and province and general treatment of the subject ; and this, though Definition itself—how to define beyond reach of objection—is usually regarded as falling specially within the domain of Logic.

By Aristotle himself, Logic is not defined. He has no single term circumscribing those portions of his writings in which he lays the foundation of it. The name, *λογικὴ* (with some such word as *επιστήμη*, a science, understood), as applied to his logical writings, is of later origin, having been introduced by his followers, the Peripatetics.<sup>2</sup> The nearest equivalent for it in Aristotle's time, and a hundred years before, with Socrates, was *διαλεκτικὴ*, or Dialectic. But in its earliest uses, the province of Dialectic did not exclude Metaphysics ; and with Aristotle it includes merely the Logic of probable matter. The term, Logic, is, from its derivation, ambiguous, the Greek word, *λόγος*, meaning reason, or speech.

<sup>1</sup> The *subject*, in modern philosophical works, means the thinking mind ; the *object* is what it thinks about. Hence the terms, *subjective* and *objective*, as employed respectively with reference to what relates to the mind, and what the mind is occupied with other than its own nature and laws. Mental states as contemplated by the mind, might be called *subject-objects* ; whilst purely external things might be called *object-objects*, or simply *objects*, in a limited sense of the term.

<sup>2</sup> The word *λογικός*, however, is used by Aristotle in the *Organon*, Topics, i. 14, 4. He distinguishes three kinds of premisses, ethical, physical, and logical ; and he observes of the latter kind : *λογικαὶ δὲ οἷον πότερον τῶν ἐναντίων ἡ αὐτὴ ἐπιστήμη ἡ οὖτις*.

For this reason, Sir William Hamilton would call the subject Dianoetic,<sup>1</sup> to signify that it is chiefly conversant with thought.

Among the earlier philosophers, the Peripatetics regarded Logic as an instrument; the Stoics, as a science, in which they were followed by the schoolmen. Among modern logicians, Watts defines Logic as "the art of using reason aright in our inquiries after truth, and the communication of it to others"; Aldrich, as "the art of reasoning"; Whately, as "the science and the art of reasoning." To Watts' definition it is objected that the sole purpose of Logic is the guidance of our own thoughts; the communication of those thoughts to others falling within the province of Rhetoric, or of Education. Mill approves of the emendation made by Whately in Aldrich's definition;—for art necessarily presupposes knowledge, and, in any but its infant state, scientific knowledge; and, if we are to have an art of reasoning, that is, a set of rules for conducting the mental processes correctly when we reason, we require for its foundation a science of reasoning, that is, an analysis of that process; a right understanding of the process itself, of the conditions it depends upon, and the steps of which it consists, being the only basis on which a system of rules fitted for the direction of the process can possibly be founded. At the same time, the term "reasoning," as employed by Aldrich and Whately, is held by Mill, and also by Bain, to be insufficient. It is susceptible of more than one interpretation, since it may mean Deduction solely, or, Deduction together with Induction<sup>2</sup> or Applied Logic. Again, seeing that such subjects as Classification, Definition, and Division, if but subsidiary, yet touch so closely on Logic as to be popularly associated with it, it is held by both that the term "reasoning" is here too narrow,

<sup>1</sup> From *διάνοια*, the faculty (and also the exercise of the faculty) of reasoning.—*Lectures*, vol. iii., p. 65.

<sup>2</sup> Induction is not, strictly speaking, synonymous with Applied Logic. It is the main process of Applied Logic. Applied Logic teaches the application of the forms of thinking to those objects about which men do think—Man, the Universe, and Absolute Being. Induction is the process of discovering laws from facts, and causes from effects.

and that a definition is demanded, if for no other reason, more consonant than Whately's in this respect to the ordinary idea of the extent of the field that Logic covers.

Definitions of Logic, embodying the title, "Laws of Thought,"—such as Hamilton's and Thomson's, Bain regards as containing an element of ambiguity and uncertainty very difficult to mend by the help of a qualifying adjective, or limiting phrase; for the word, thought, covers much more than the mental processes—abstraction and reasoning—admitted into Logic, and thought, as the product of the process, is, besides, not always uniform in quality. When we narrow the signification of the word, as for example in Dr. McCosh's definition of Logic, as "the science of the laws of discursive thought," or, by prefixing "formal," we fail to relieve the perplexity; for the word, thought, obstinately points to Psychology, and to the laws of the rise and succession of our thoughts, that is, to the laws of Association of Ideas. And there remains the further question, Is it thought as it is, or thought as it ought to be? If the former, the subject is pure psychology; if the latter, the standard needs to be explicitly stated, and if so stated it can be nothing else but the standard of what is true.

Hamilton's definition of Logic is, it need hardly be observed, closely bound up with his psychological system. Logic is defined by Kant as, "the science of the necessary laws of the understanding and the reason"—a definition that rests upon the Kantian philosophy. Spalding defines Logic as, "the theory of inference"; Jeremy Bentham, as, "the art which has for its object, or end-in-view, the giving, to the best advantage, direction to the human mind in every branch of knowledge"; the Port Royalists, as, "the science of the operations of the understanding in the pursuit of truth."

Now, some truths relating to the present, such as our sensations and emotions, are known by direct consciousness, and admitted by all to be *intuitive* or immediate; some relating to the present, and all relating to the past and future, are mediate, and properly inferences; and some things lie between

intuitions and inferences on which philosophic opinion is undecided,—such, for example, as the controverted questions respecting the Origin of our Knowledge. It follows, that if Logic be concerned solely with inferences and not with intuitions, the Port Royalists' definition needs emendation,—since the operations of the understanding<sup>1</sup> include both ; and by way of such emendation, originated Mill's definition of Logic, as “the science of the operations of the understanding which are subservient to the estimation of evidence.”<sup>2</sup>

To Dean Mansel this definition of Mill's appears as much too wide as Whately's—“the science and the art of reasoning”—is too narrow : the latter excludes from the province of Logic processes of thought dependent upon precisely the same laws, and subject to the same method of discovery and criticism as that of reasoning ; the former includes within it processes governed by different laws, involving fundamentally different methods, and implying essentially distinct conceptions, united and confused by the ambiguities of a common language. It need hardly be added that Mill read this criticism, and yet, being unable to perceive any serious confusion in the structure he had raised, neither this nor any other similar attack induced him to alter or remodel his *System of Logic*.

Ueberweg's definition of Logic, as “the science of the regulative principles of human knowledge,” is, having regard to his

<sup>1</sup> “The power of perception is that which we call the understanding. Perception, which we make the act of the understanding, is of three sorts : 1. The perception of ideas in our mind. 2. The perception of the signification of signs. 3. The perception of the connexion or repugnancy, agreement or disagreement that there is between any of our ideas.”—Locke on *Human Understanding*.

<sup>2</sup> Mill, in his *Examination of Sir William Hamilton's Philosophy*, p. 448, defines Logic as “the art of thinking, which means correct thinking, and the science of the conditions of correct thinking.” In his *Logic*, footnote to Introduction, § 7, he marks the opposition between the view of Logic expressed by his definition, as given in the text, and that adopted by Sir William Hamilton and his pupils. The latter he characterises as framed for the express purpose of excluding, as irrelevant to Logic, whatever relates to Belief and Disbelief, or to the pursuit of truth as such, and as restricting the science to that very limited portion of its total province which has reference to the conditions not of Truth but of Consistency.

view of the scope of the subject, open to a like criticism with that of the Port Royalists. His definition is framed so as to include intuitive truths. In interesting contrast with this definition may be added the view of Logic held by the late Mr. Bain. He regards Logic, first, as a theoretical abstract science, dealing with all affirmation, and embodying the results in suitable formulæ,—the *logica docens*, as distinguished from the *logica utens*, of the older logicians; secondly, as a practical science, the object being proof or evidence, and the laws of evidence being regarded in their widest compass so as to include deduction and induction; and, thirdly, as a body of method auxiliary to the search for truth, and giving an account of all known processes—provided they are of a general kind—that aid the understanding in proving or evolving it.

**Classification of the Definitions of Logic.** § 4.—It will, of course, have been noted, in passing on, that the definitions of Logic admit of classification on certain principles. One of the most common grounds of distinction is that of regarding the subject as a Science, or an Art, or both a Science and an Art. Sir William Hamilton has taken much trouble with the writers on Logic on the ground of this distinction; but the principle is so obvious as that it is unnecessary to dwell further on it.

If we extend the scope of Logic, however, classification assumes a more complex character. But, even with the most extensive scope, the great bulk of the definitions of Logic in modern treatises may be divided into two classes, namely, *Subjective*, and *Objective* or *Material*, on a principle of division indicated by the terms employed. As, however, a third kind is occasionally met with having some claim to be formed into a distinct group, we may for inclusiveness' sake, divide Definitions of Logic into three classes, each of which has a distinct end in view:—

1. Definitions of which the contents are *Subjective* elements, and which take into account but the subjective aspect of Logic; thereby, limiting its province to the

pure forms of thinking, and its *end* to *self-consistency*, and making it dependent on mental science.

2. Definitions of which the contents contain *Material*, or *Objective* elements, and which extend the province of Logic to all objects of thought ; thereby, determining its *end* to be *objective truth*, and placing it among objective sciences.

Such, for example, is the definition of Logic by Mr. Herbert Spencer, namely, as the science that "formulates the most general laws of correlation among existences considered as objective."

3. Definitions of which the contents are *Linguistic* in character, and which are mostly equivalent to the statement that Logic is the science which has for its object the prevention of fallacy and self-contradiction in the use of Terms, Propositions, and Arguments ; thereby, making Logic chiefly dependent on Grammar and Language. The *end* aimed at, according to this view, is to *secure such forms of language* and modes of expression as must obviate the occurrence of self-contradiction, or fallacy.

**Pure Logic.**      § 5.—Pure, or Formal Logic, has been called *à priori*, an *à priori* science. How far any truths or ideas are possible *à priori*, that is to say, are possessed by the mind *prior* to all that it acquires from the world around, is the great controverted question of mental philosophy. It is not denied, however, that the mind always takes in the impression of an external object subject to certain conditions, as of transmuting it into its elements, and grouping, classifying, and drawing inferences in relation to these elements. Writers who characterise Logic as an *à priori* science do not necessarily involve themselves in the controversy beyond holding, that these conditions, which, in fact, are the conditions of all knowledge, are *à priori*,<sup>1</sup> that is to say, are laws or con-

<sup>1</sup> According to the theory of Evolution, there is both an *à priori* and an *à posteriori* element in our knowledge, or to speak more accurately,

ditions of the mind itself, with an existence prior to the impression of the external object ; and that, as Pure Logic treats only of such laws or conditions, namely, those laws or conditions to which objects of sense are subjected in the mind, or, in other words, as they form the subject-matter of Pure Logic, the epithet is justifiable. By way of contrast, sciences of which the subject-matter consists of external facts, such for example, as Astronomy, or Physics, or Chemistry, are called *à posteriori* sciences.<sup>1</sup>

**Relations of Logie to Psychology, Meta-physics, Grammar and Rhetoric.** § 6.—In defining Logic as the science of the necessary laws of thought, we found ourselves in contact with Psychology, or the science of the Human Mind. This science investigates, analyses and classifies, the various phenomena of the mind intellectual and emotional ; including thought, or thinking, and the processes subsidiary thereto, such as memory, and association of ideas, some or all of which must take place to allow us to think. It investigates the genesis of thought as a mental phenomenon ; but it regards the laws of thought as laws in the sense of observed uniformities in the modes in which men are found by experience to think and reason, not as determinative of the mode in which they *ought* to think

there are both *à priori* and *à posteriori* conditions of our knowing ; the *à posteriori* being, as in all systems, individual experience ; the *à priori* conditions being inherited mental aptitudes which facilitate the formation of certain general conceptions, concurrently, or almost concurrently with the presentation of individual experience.

<sup>1</sup> Aristotle distinguishes two uses of the epithet *à priori*, namely as applied to (1) what in its own nature is prior to some other thing, and (2) what is prior *in respect of us*. The general truth is, in its own nature, prior to the particular, and the cause to the effect. To us, the particular truth, since our knowledge of it precedes, is prior to the general, and the effect to the cause. In accordance with the first of these senses, the Schoolmen call an argument from cause to effect an *à priori* demonstration. The epithet, *à priori*, appears to have been first employed as in the text in the sense of prior to, or independent of experience, by Hume ;—a sense which Kant has fixed as the usual one in modern philosophy. See Trendelenburg, *Elementa Logices Aristoteleæ*, Editio septima, pp. 6, 29, and 86-88. But truths, originally of an admittedly *à posteriori* origin, are now, sometimes, nevertheless, if long in possession, and of the highest certainty, said to afford *à priori* grounds of deduction, and to give *à priori* knowledge as the result of such deduction.

and reason, or of valid as distinguished from invalid reasoning ; with the modes in which men actually reach conclusions, not of the logical value of the process employed in arriving at them. Logic is, thus, in one sense narrower than Psychology as concerning itself with one, not with all mental processes ; and, yet, in another sense, wider,—as it is wider than every other science, inasmuch as it furnishes to Psychology, as to all other sciences, a body of method, with the logical principles of scientific investigation.<sup>1</sup>

In alluding to the distinction between *à priori* and *à posteriori* truths, we found ourselves in contact with another related science, Metaphysics,—the science which treats of Being, or Real Existence, and of the attributes that belong to it as such, including such questions as Cause, the Finite and the Infinite, the nature of Substance, Space, Time, the Good, the Beautiful, and the True. Metaphysical speculations, have, some of them, a special importance in connection with Applied Logic. Locke lays down a typical proposition in Metaphysics when he says : "We have the knowledge of our own existence by intuition ; of the existence of God by demonstration ; and of other things by sensation." Metaphysics investigates the validity of the primary or fundamental assumptions on which the various other sciences are based, towards the unification of all in a single fundamental principle. Formal Logic furnishes the instrument for correctly proceeding in such investigations, some of which relate to the metaphysical assumptions of even Logic itself.<sup>2</sup>

<sup>1</sup> Psychology touches Logic on one border, Physiology on another. Even for Logic itself some philosophers claim a physiological basis, regarding thinking as synonymous with cerebration, and subject as such to the general laws of nervous action.—See *American Journal of Mathematics*, vol. iii., p. 15.

<sup>2</sup> It should be observed that the science of Being is by many writers denominated Ontology, the term Metaphysics being applied to a whole consisting of Ontology and Psychology. Leibnitz, in his *Monadologie* (1714), attempts to solve the various problems of Metaphysics and Psychology by means of his curious and interesting hypothesis of Monads. His Monads are immaterial, intelligent beings, infinite in number, each attached to an ultimate particle of matter, capable in various ways of uniting into corporations, or aggregates, of an intelligence increasing in volume, so to speak, with the multiplication of

Certain other sciences, for their points of contact with Logic, may be mentioned, in passing, namely, Grammar and Rhetoric. Logic, Grammar, and Rhetoric, are all three conversant with language. Grammar, however, is concerned primarily with language, towards making it a perfect interpreter of human thought. Logic,—primarily concerned with thought, is secondarily concerned with language, towards employing it in expounding the laws of correct thinking, with truth as the ultimate object. It sometimes employs analysis for the purpose of bringing out the contents of language with the necessary distinctness ; and it must always have in Grammar its most useful ally.

Rhetoric, too, like Logic, is conversant with language ; but it employs the resources of language to produce persuasion—not necessarily conviction, in using special means to work on the emotions ; and Logic is sometimes called upon to expose the artifices this science thinks it not inconsistent with its scope and end to have recourse to, namely, in the adroit utilization of fallacies—often most captivating in their ingenuity, in order to entrap the unwary.

**Utility of the Study of Logic.** § 7.—Duns Scotus, writing in the thirteenth century, characterises Logic as that “*ars artium et scientia scientiarum, qua aperta, omnes aliae aperiuntur, et qua clausa aliae clauduntur.*” He was satisfied that, to whomsoever Logic stood open and familiarised, to him also the other sciences of that day were open, or at least, through its assistance, could present but little difficulty : and to whomsoever Logic was closed—unattained, or unattainable—they were closed. The enthusiastic tone of our British philosopher should not raise undue expectations respecting the

aggregations. Each human mind is one monad. It knows the sensible world by intuition, is capable of reflection, and so, of referring particular phenomena to general laws ; and, in fine, of overpassing the limits of the sensible universe, raising itself to a knowledge of the *Absolute*, and founding Metaphysics. The Absolute is that which is not relative to the Ego or Consciousness, but stands in relation to Being in general. The problem concerning it constitutes the *Higher* or *Transcendental* Logic, with which Kant's name is associated. Hence, Metaphysics is sometimes called, also, the Science of the Absolute.

worth of the science ; but, at the same time, his words will often still be repeated, for the considerable amount of truth they contain. Logic is in itself, indeed, no golden key to unlock the treasure-house of the knowledge of the universe, as some ancient writers would take it to be ; but the science of Formal Logic constitutes the basis of an art of correct reasoning, the rules of which are employed in every scientific inquiry, and direct the procedure in all other sciences : and since reasoning, to be correct, must conform to these rules, it is plain that a person ignorant of them is less likely to reason correctly than a person acquainted with them.

We have seen that Logic may be regarded both as a science and as an art. Not, however, until we have completed our survey of it can its enlightening and disciplinary effects on the mind as a science, or its utility as an art, be fully understood. A few thinkers, not undistinguished in the annals of philosophy, have urged, that so careful and systematic a cultivation of correct habits of thought as a course of logical studies implies, is not at all essential in the affairs of life, and can be dispensed with by men, without their, in any appreciable degree, feeling the want of it. Even Locke, though at a time when the subject was in some disrepute in this country, perhaps owing to too classical a dress, wrote :—“ God has not been so sparing to men as to make them barely two-legged creatures, leaving it to Aristotle to make them rational. . . . He has given them a mind that can reason without being instructed in methods of syllogising ; the understanding is not taught to reason by these rules ; it has a native faculty to perceive the coherence or incoherence of its ideas, and can range them right without such perplexing repetitions.”<sup>1</sup> But, the aim of Logic is not, to supply the want of this “ native faculty,” or to ignore it : it is, instead, to recognise its existence, and to cultivate it, on whatever subject-matter employed, to more easy, speedy, and sure perceptions of thought that is right, and thought that is wrong,—the clear understanding of the former serving as a light for the avoidance of the latter.

<sup>1</sup> See Locke on the *Human Understanding*, Bk. IV., ch. xvii., § 4.

It is true that, in the use of any instrument, repeated exercise, and a continuous endeavour to obtain the best results will sometimes, without any previous acquaintance with theory, or with certain general principles that underlie excellence of performance, lead, as of nature, to a degree of skill, not, perhaps, rare or remarkable, but sufficient for ordinary requirements ; and, just as many speak correctly who never studied the principles of grammar, so it may be admitted that many of a naturally clear understanding, and a certain power of imitation, reason correctly who never learned the principles of Logic. No one will deny, however, that a preliminary initiation into the principles, whether of instrumental or grammatical art, must always, other things being the same, lead up to a given standard of excellence with greater certainty, and generally in a much shorter time. So it is with the reasoning mind. Habits of reflection, natural keenness of faculty, and continuous exercise, may be quite enough to secure men from erroneous conclusions affecting matters of every-day life ; but, when complexity is introduced into the data set to natural Logic to work upon, the conclusion is reached and accepted by the untrained intellect with tardiness and perplexity ; and the desirability of a preliminary initiation towards greater familiarity and expertness, namely, in the rational considerations and principles that, apart from any given kind of subject-matter, determine the course of a sound argument, cannot fail to impress itself on even such a mind as a useful, if not rather as a necessary, preparation for other cases of a similar nature.

It is not at all to be regarded as intended by the science of Logic, in insisting on a complete analysis and a thorough knowledge of certain typical forms of reasoning, to bring forward such forms that they might be substituted for any others ; they are intended, merely, to be understood as the forms to which all valid reasoning may be ultimately reduced, and which—when Logic is employed as an art—will enable the reasoner to test the validity of an argument and to detect any possible source of vitiation. Thus, the

systematic, or scientific study of the process of reasoning itself, so as to familiarise oneself with the legitimate modes of procedure in the most complex as in the simplest cases, till accuracy be a habit, and certainty in the use of the mere instrument be achieved to a degree of unconscious ease, is placed beyond question as a preparation the absence of which will always be more or less seriously felt when speculations of any considerable subtlety have to be weighed and discussed.

It is not, however, solely to persons who are much engaged in abstruse speculations that a preliminary training in Logic should be of great and lasting advantage. To all, in whatever business or profession, it is of the highest importance to know, and to be conversant in, the method of reasoning justly. The study of Logic furnishes, too, in strengthening and developing the faculties, a mental discipline the value of which, considered as an end in itself, cannot be overestimated.

The chief practical rules of the subject become an acquisition not readily forgotten; and, when undertaken in the proper season, and in some measure under the influence of those *mirabiles amores* with which, an eminent writer reminds us, science can inspire her followers, it always repays the pains taken in the study and mastery of a not too burthensome terminology and collection of principles.<sup>1</sup>

<sup>1</sup> Of the high value attached to Logic by the older writers, the names of distinction bestowed upon it by them afford sufficient evidence. It is called the *Architectonic Art*, owing to the supremacy of its rules in the building up of the arts in general. For a similar reason it was called by many of the followers of Aristotle, the *Organon* or Instrument and the Instrument of Instruments; by Duns Scotus, as observed above, the Art of Arts, and the Science of Sciences; by others, *Zetetic*, or the Art of Seeking, namely, Truth, and *Heuristic*, or the Art of Discovering Truth; by others, *Medicina Mentis*, and Cathartic of the Mind, owing to its curative and cleansing effects on minds subject to prejudice and error; by others, Dialectic, as teaching the art of Discussion, not necessarily with Truth as an object; and by Epicurus and his followers, the Canon, as being a gauge or standard for the measure of thoughts, towards securing their correctness. See Hamilton's *Lectures*, vol. iii., p. 36;—noting the limitation to the utility of Logic in his view, consequent on the restriction of its scope in accordance with his definition of the subject.

## CHAPTER II

### LOGIC AND LANGUAGE

**Language and its Functions.** § 8.—We have observed that Pure or Formal Logic has thought rather than language for its adequate object-matter. Language may, in a general sense, be described as any mode, whether natural or artificial, of expressing our thoughts. For the purposes of Logic, however, it may be regarded as a system of articulate words, corresponding, by convention, with the internal thought to be expressed. We are about to consider briefly how far it is justifiable thus to make thought rather than language the primary object-matter of Logic. But, first, it is to be understood that as to the great importance of the functions of language in relation to Logic there can be no difference of opinion. For language is not simply an interpreter of thought; and, again, as an interpreter of thought, is not simply a bare interpreter of thought. It serves as a means (1) of communication between mind and mind respecting the whole field of thought, and (2) of recording mental impressions; and, further, the language of words (3) abbreviates thinking by enabling us to substitute a short word or symbol for a highly complex object of thought, without being compelled on each occasion to pause to realise how much it implies. When thus employed in thought for a notion the contents of which we do not immediately realise, a word or name is called a *symbolical* cognition or conception, as distinguished from an *intuitive* or *notative* conception which implies a clear insight into, at least, the essential attributes of the object. In recording an impression, whether

external or internal, the language of words (4) always in some measure analyses such impression into its parts. Language, in its record of an objective or a subjective impression, can reproduce, not only the objects observed and their qualities, but also their mutual relations, the relation of the observer to them, and the order and relation of the observer's thoughts themselves due to such impression. The language of words is thus analytic, and is to be contrasted with the language of painting or of sculpture which some would call synthetic, as throwing together the elements or qualities of the object and representing them at one view. And, derivatively associated with the functions of recording and analysing complex impressions is another function of language, namely (5) that of recalling the parts of a decomposed impression of sense by labelling them with general names, even though these parts may have no independent existence. Such general names, or common terms, as expressing concepts or general notions, are the means of fixing and recording trains of thought which, without them, must be repeated frequently, and on each repetition, with all the pain of first effort.

**Logic, secondarily concerned with Language.** § 9.—Logic thus, in these several ways, exercises a powerful influence on the thinking process; but it is concerned primarily with thought, and only secondarily with language. There are, as we have seen, logicians who hold a different view, and who would place Logic among the linguistic sciences, defining it as "the science of the principles and rules to which we must conform that we may be free from fallacy and self-contradiction in the use of names, propositions, and arguments," or in some such terms. Whately, one of the most influential of the modern exponents of the subject, lays great stress upon the importance of language in relation to Logic. He would have it clearly understood, he observes, that Logic is entirely conversant about language, and considered as an art, he defines it to be, "the art of employing language properly for the purpose of reasoning, and of distinguishing what is

properly and truly an argument from spurious imitations of it." The view, however, that Logic treats primarily of language is one that most logicians pronounce to be untenable. And Whately, notwithstanding the terms in which he expresses himself, cannot be said to have worked it out. He, in fact, appears to have meant no more than this, that, considering Logic as "the science and also as the art, of reasoning," in so far as it is an art of reasoning,—seeing that reasoning is conducted but by language, and that many of the chief impediments to the correct performance of the process lie in the defects of expression—it is concerned primarily with speech, not with thought. It is not the art, however, but the scientific foundation of the art, that goes to the root of the matter; and its scientific foundation consists of laws of thought; so that Formal Logic, even on foot of Whately's own definition, is primarily concerned with thought, and consequently, only secondarily, with language. Whether it is possible to think without the aid of any signs, or any language, is a question which has been a constant source of dispute among logicians and psychologists. It is, however, possible to think without the aid of any given form of articulate speech—though, when this takes place, we find thought proceeding by help of some other class of signs. It appears, therefore, to be most philosophical to conclude that Logic should expound the laws of thinking, just as universal grammar should those of speech, apart from their special modification in any given language. No confusion can arise, however, in introducing principles of language into Logic, or in taking account of the way in which language performs its functions, so long as thinking is made the adequate object-matter of science, and language comes in only as the minister of thought; in other words, so long as it is regarded as concerned primarily with thought and only secondarily with language.

**Nominalistic and Conceptualistic Modes of Treatment.** § 10.—A treatment of Logic, such, for example, as Whately's, that gives special prominence to language, may be called *nominalistic*, as contrasted with a treatment in which, like Sir William

Hamilton's, an attempt is made to denote the mental impressions, operations, or products of thought by words which do not imply their expression in language, and which, from the prominence given to the objective subject-matter, may be called *conceptualistic*. These terms, to some, will naturally bring to mind the philosophical doctrines of Nominalism and Conceptualism ; but, as thus used, they need not be regarded as implying on the part of the authors of logical treatises any necessary adhesion, one way or the other, to those doctrines. A conceptualist, as well as a nominalist, in philosophy, might, in Logic, if desirous of laying stress upon the importance of the consideration of language, as to its functions generally, and in respect, more particularly, of terms, propositions, and reasoning, equally adopt a treatment of the subject which might be designated as nominalistic. A nominalistic treatment has the advantage of enabling the logician to dispense with a considerable amount of psychological preliminaries rendered necessary by the adoption of the opposite method. Greater compactness is thus secured, and also the avoidance of conflict with disputed theories in a science such as psychology, in which so much is still, among philosophers, matter of disagreement and speculation. Of course, when the tendency of philosophy is nominalistic, as it has been in England since the days of Locke, the treatment of Logic may be expected to exhibit at the hands of many writers a correspondence and conformity with the nominalistic doctrine.

**Nominalism, Conceptualism, and Realism.** § 11.—We have just alluded to the philosophical doctrines of Nominalism and Conceptualism. A third philosophical doctrine called Realism is usually contrasted with them. All three owe their origin to the different views that have been held by philosophers respecting what are called Concepts, Conceptions, or General Notions. A Notion of anything is a clear presentation or impression of it. Notions formed in the mind are either of single objects, or of many gathered into one. Notions of single objects are called Intuitions ; notions formed from

many objects are called Concepts, Conceptions, or General Notions, as being produced by the power which the mind has of taking, or grasping, many things together. - The process of Conception, or the formation of a Concept consists of five steps, namely, Comparison, Reflection, Abstraction, Generalisation, and Denomination—sometimes, however, described by the most important of them, namely Abstraction, or by the two most important, namely Abstraction and Generalisation. We have, for example, an intuition, to begin with, say of a *rose*, and also of a *lily*. The mind lays these two, as it were, side by side, or regards them rapidly in succession,—that is to say, *compares* them ; it *reflects* upon them, noting their points of agreement and of difference ; it *abstracts*, that is to say, draws away, or separates the points of agreement from those of difference that they may constitute a new or common nature different from, yet including that of *rose*, and of *lily* ; it *generalises*, or recognises a class of objects, each, of this new or common nature ; and lastly, it *denominates*, or imposes a name on them that shall serve to recall the common nature,—the name, *flower*. Such a common nature we call a Concept, or General Notion, and the several steps just indicated as necessary for its formation we call Conceptions. We thus, with the materials assumed, arrive at the concept, *flower* ; and, in a similar way, we may arrive at other concepts, starting from the intuitions in which they inherit.

Under the name of Universals, Concepts, at one time, formed the subject of keen philosophical controversy. Are they *real existences*, apart from the mind that forms them by the several steps, and in the manner just indicated ? Or, have they *only a subjective existence* ? Or, seeing that we represent them in language by General Terms, are they *mere names* ? Hence arose the three celebrated philosophical doctrines of Realism, Conceptualism, and Nominalism, according to the views held by philosophers on these points.

Nominalism and Conceptualism may be regarded as still holding their ground on the debatable line between Psychology and Logic. At the same time it is urged against the

opinion of the Nominalists, who hold General Terms to be mere names, that there must be an act of mind corresponding, or such names would be valueless. The best received view is that of the Conceptualists, who hold that the General Notion is the knowledge in the mind, or the intellectual synthesis, of the common properties of the things included under the Notion. The Realists contended that Universals have a real existence, apart from the mind itself, and from the particular objects distinguished by the common properties. This view has ceased to have any adherents.<sup>1</sup>

<sup>1</sup> William of Champeaux, who died in 1121,—the founder of scholastic Realism, maintained that Universals are the only real entities or existences. Abelard (1079–1142), his pupil, ascribed a reality both to Individuals and Universals, subject to the distinction that Individuals have an essential existence, and Universals an existence ideally real. The latter are extracted from particulars, and exist in the mind in the form of *Concepts*, and are held together by words called *General Terms*. Hence his theory was called Conceptualism, or Conceptual Nominalism. In their views respecting the nature of General Notions the majority of modern writers, Mansel and Thomson included, are Conceptualists. Nominalists contend that General Terms are mere names. The foundation of Nominalism is commonly assigned to Roscelin, or Roscelinus, Canon of Compiègne, who died some time after 1121 ; but its real founder appears to have been John, called the Sophist, supposed by Du Boulay to have been chief physician to Henry I. (1031–1060) of France. General Terms, according to Roscelin, are mere words (*status vocis*), employed to designate qualities common to different individuals. Aristotle maintained the doctrine of *universalia in re* ; the Realists, and probably Plato among the ancients, *universalia ante rem* ; the Nominalists, *universalia post rem*. Hobbes, and, after him, Hume, have gone to the utmost limits of Nominalism, holding the Universal Term to be merely a convenient abbreviation whereby a number of objects with features in common can be at once denoted. Mr. Bain, among recent writers, may be regarded as an adherent of Nominalism ; also, John Stuart Mill, though in a modified form,—touching on Conceptualism. The theological aspects of Nominalism it would be out of place to dwell upon here.

## CHAPTER III

### DIVISIONS OF LOGIC

Terms ;      § 12.—Formal Logic may be conveniently divided  
Simple      into three parts. The first treats of Terms, the  
Apprehen-      second of Propositions, the third of Inferences.  
sion.     

A Term has been briefly defined as an idea objectified,<sup>1</sup> or as one or more words denoting an idea or notion : William, house, field, Pilgrim Fathers, Locksley Hall, are instances of Terms. Some Terms may be applied to several objects in the same sense when such objects possess certain features in common : these are called General or Common Terms, as man, voyagers, chase, villa, lotus-eaters, and with such Logic is chiefly concerned. The formation of the subjective counterparts of terms is due to Simple Apprehension or Perception. This may be defined as that operation of the mind by which we mentally perceive or form a notion of some object. Of the General Term the subjective counterpart is the Concept. We have already traced the steps by which the Concept or General Notion is formed. These, as we have observed, are sometimes represented by the names of the two principal of them, Abstraction and Generalisation. Abstraction may be defined as the operation of the mind by

<sup>1</sup> Objectivity may relate either to Things in themselves, or to the Marks, Types, or Symbols of Things ; and, as some writers regard Logic as conversant with the relations of Things in themselves rather than with either their subjective counterparts, or linguistic or other signs or symbols, we may conveniently distinguish these two points of view, respectively, as the Real-Objective and the Verbal-Objective, or Linguistic. Thus, the verbal-objective counterpart of an idea is a term.

which we draw off in thought, and attend to separately, some particulars of an object of perception ; and Generalisation as the act of comprehending under a common name several objects agreeing in such particulars, and which that common name serves to indicate. Thus by drawing off in thought the features in which Windermere, Loch Lomond, Lough Neagh, agree, disregarding the features in which they differ, we arrive at a general notion which will be in agreement with any of the three ; and the name of this general notion or concept, which name, lake, is in like manner applicable to any of them, we speak of as a General Term. The various kinds of Terms we shall consider very fully and apart in a later chapter.

Proposi-      § 13.—A Proposition is a Judgment objecti-  
tions ;      fied, or the expression of a Judgment in words.  
Judgments.      A Judgment is our mental pronouncement on the  
agreement or disagreement of two of the notions<sup>1</sup> obtained  
by simple apprehension. “A hyacinth is fragrant,”  
“Rhetoric is the science of expression,” “John is not tall,”  
“Place is the relation which any object bears to two or  
more fixed points,” are examples of propositions.

Propositions may be divided, as sentences, into two classes : Simple (or Categorical<sup>2</sup>) and Complex (or Hypothetical). The Simple Proposition is resolvable into three elements : the *Subject* and the *Predicate*, called the terms of the proposition (from *termini*, boundaries), and the *Copula*, which lies between them. The Subject is that which is spoken of ; the Predicate is that which is said of it ; and the Copula is the verbal connecting link. The latter, according

<sup>1</sup> On this point, Mill holds that judgments express agreement or disagreement of things, not of our notions of things. (See Mill's *System of Logic*, vol. i. p. 96.) Dr. Fowler's definition can be regarded only as evading the difficulty by concealing it. “A proposition,” he states (*Deductive Logic*, p. 24), “asserts or denies, as the result of comparison, some word or combination of words of some other word or combination of words.” But, as the result of comparison between what ? The answer necessarily brings out the controverted point.

<sup>2</sup> Categorical, from its derivation, *κατηγορικός*, properly means affirmative, and is so employed by Aristotle. It seems, therefore, inappropriate to extend it to the whole class of simple propositions.

to the Logic of the schools, must be always either *is* or *is not*, *are* or *are not*; and, as expressing merely the agreement or disagreement of subject and predicate, has no relation to time. Thus, in the proposition, "All the flowers are blowing," "All the flowers" is the subject, "blowing" is the predicate, and "are" is the copula; so in the proposition, "He is not at home," "He" is the subject, "(a person) at home" is the predicate, and "is not" is the copula. Other verbs, when they are used, must be resolved into the substantive verb and a participle, or adjective, or their equivalent; or words must be supplied so as to enable us to express the meaning in the normal form. For instance, "James struck the ball" is for purposes of logical analysis to be read, "James is a person that struck the ball." As thus expanded, "James" is the subject, "a person that struck the ball" is the predicate, and "is" is the copula. The Simple Proposition, as we see from the preceding examples, asserts merely that the predicate does, or does not, apply to the subject. The Complex Proposition makes its assertion under a condition, or with an alternative. "If the book be in the library I shall lend it to you," "Either the one clock is slow or the other is fast," are examples of Complex Propositions.

Other sentences, such as those expressing a *question* or a *command*, differ from propositions in wanting their essential feature, namely, predication, or the expression of affirmation or denial. And this is so, even though the sentence be substantially equivalent to an affirmation or denial, as, "Can Honour's voice provoke the silent dust?" With Complex Propositions and their classification we shall deal at length hereafter.

Simple Propositions are divided into two classes, based upon the character of the Copula, namely, *Affirmative* and *Negative*. This is called a division of propositions according to their *Quality*, that is to say, their characteristic quality, namely, *assertion*. "Logic is the science of reason expressed," "The veeries are American birds of song," are affirmative propositions. "Twas no hypocrisy in him to

flatter," "He cannot be said to have fallen prematurely," "She is not dead," are negative propositions. A further division of propositions is into *Universal* and *Particular*, according as we affirm or deny the predicate of the *whole* subject, or of *part* of it only. "All was peace," "No birds fly over it," "All the apple trees are in bloom," "Here all things always seem the same," are universal propositions. "Some islands are not fertile," "Some quadrupeds lay eggs," "Some bees have no sting," "All are not wise," are particular propositions. This is called a division of propositions according to their *Quantity*.<sup>1</sup> Such propositions as "Britain is an island," in which the subject is a proper name, or applies but to one individual thing or object—being what is called a singular term—are called *Singular* propositions, and are treated in Logic as universal. Such propositions as, "Travellers tell us so," "Its springing violets blossom," "Birds have wings," are called *Indefinite*, and are treated in Logic as universal or particular, according to the meaning to be inferred from the rest of the discourse in which they occur, or their proper context. We shall return to the consideration of Indefinite Propositions in a later chapter.

Having regard to their quality and quantity combined, propositions are classed as Universal Affirmative and Particular Affirmative, briefly denoted, respectively, by A and I, and Universal Negative and Particular Negative, denoted respectively by E and O.<sup>2</sup> Moreover, it is often convenient to denote the subject of a proposition by a single letter and the predicate by another, in discussing purely formal relations; subjects and predicates relating to any given subject-matter, being, of course, introducible at will into such *propositions* in blank. Our four forms, therefore, stand thus:—

Universal Affirmative. All S is P. (A)

Universal Negative. No S is P. (E)

com.

comb. what *quality* and *Quantity* were called by the Schoolmen, the *affections*

<sup>2</sup> *Catēgories.*

affirmative, from the two first vowels in the Latin *affirmo*, and inappropriative from those in *nego*.

Particular Affirmative. Some S is P. (I)

Particular Negative. Some S is not P. (O)

sometimes written, also, *SaP*, *SeP*, *SiP*, *SoP*, respectively : and to one or other of these four forms it is maintained by logicians that all categorical propositions can be reduced.

In each of the four propositions just written down, S and P are the terms, and stand respectively for the subject and the predicate. In A and E, the subject is taken in its whole extent, the assertion being made with regard to all the individuals included under the term, S ; and this is technically expressed by saying that in A and E the subject is *distributed*. In I and O, the subject is taken in but part of its extent ; in other words, the assertion is made only with regard to a portion of the individuals included under the term S, and this is technically expressed by saying that, in I and O, the subject is *undistributed*. The meaning of the *distribution* of a term, then, is the taking of it universally ; and the mark of universality is the word *All* in the sense of *Every*. On the other hand, the explicit mark of particularity, and therefore of non-distribution, is the word, *Some*. We thus have the following *Rule of Distribution* affecting the *subject* of a proposition :—All universal propositions, and no particulars, distribute the subject. With regard to indefinites, the distribution of the subject in such propositions depends on the knowledge, or intended meaning (or both) of the speaker or writer. As to the predicate, in A and I it is undistributed ; in other words, neither A nor I necessitates its distribution towards their being true, since it suffices that, in either case, a part of P should agree with S ; but, in E and O, it is distributed, for if any part of P were applicable to S it could be affirmed, and, of course, could not be denied of S ; hence, to deny P of S must imply that no part of P is applicable to S ; in other words, that the whole of P is denied of S. We therefore, have the following *Rule of Distribution* affecting the *predicate* of a proposition :—All negative propositions, and no affirmatives, distribute the predicate. Thus, the distribution of the predicate depends on the quality of the proposition.

**Inferences;** § 14.—We are now in a position to explain **Reasoning**; some of the principal matters that present themselves for treatment in the third part of Formal Logic. These may all be included under the general head of Inferences. A formal inference may be defined, in an objective sense, as a *proposition* deducible from one or more antecedent propositions, but at the same time expressing a judgment distinct from any that has preceded it. It is employed, however, to mean also, a *combination of propositions* consisting of one or more antecedent propositions leading to a consequent proposition distinct from any that has preceded it. The subjective counterpart of the latter definition is *Reasoning*, or *Discourse*, which may be defined as the process of proceeding from one or more judgments to another as a consequence. Reasoning has been otherwise defined as “the process of the mind, whereby from facts or truths, known or assumed, we come to the knowledge or belief of other facts or truths not directly known.” This explicitly raises questions as to the nature of *Truth*, *Knowledge*, and *Belief*, the discussion of which properly falls within the province of Applied Logic.

Inferences are divided into immediate and mediate. An immediate inference is a combination of two propositions of which one is founded on the other, being virtually included in it. Sometimes the word *inference* is employed to denote the *process itself* of deriving, in accordance with the fundamental principles of Logic, one proposition from one or more. This consequent proposition is then said to be *inferred* from its antecedent or antecedents. When from “All the leaves had fallen,” we derive, as virtually implied therein, “Some of the leaves had fallen,” we may (1) call both propositions taken together an immediate inference, or (2) call the consequent proposition or conclusion an immediate inference from the other, or (3) call the *process* of deriving the conclusion or consequent proposition from the other immediate inference. As additional examples of immediate inference, we have from No A is B, some A is not B; from All A is B,

Some A is B ; from some S is P, some P is S ; from All men are animals, Some animals are men ; and so on.

Of mediate inferences the fundamental kind is the Syllogism. A Syllogism may be defined objectively as a combination of two propositions necessitating a third as a consequence of their mutual relation. The two propositions from which the third is derived or inferred are called the *premisses* of the syllogism, and the inferred or consequent proposition is called the *conclusion*.

As an example of the syllogism, we may consider the following :—

All A is B ; (A)

All B is C. (A)

∴ All A is C. (A)

All A is B, and All B is C, are the premisses. All A is C, is the conclusion. And while any two propositions at random may be said to contain four terms, the two premisses of this syllogism contain in reality but three, since one, namely the term B, appearing in both, does service for two distinct terms. This is called the *middle term* of the syllogism. It is repeated in the premisses, and does not appear in the conclusion. Of the other two terms, that which appears as the *subject of the conclusion* is called the *minor term* and that which appears as the *predicate* the *major term*. Also, as one or other of them appears in each premiss, that premiss which contains the major term, namely, the premiss, All B is C, is called the *major premiss*, and that which contains the minor term, namely, All A is B, the *minor premiss*.

In every syllogism, the middle term must, as in that under consideration, appear twice in the premisses. It may, however, be the subject or the predicate in either. Hence with A and C as the other terms, and disregarding the copula and the word or mark of quantity in each premiss, we have the four following possible arrangements of the terms in the premisses of a syllogism consequent upon the positions the middle term may take in them combined :—

$$\left. \begin{matrix} A, B \\ B, C \end{matrix} \right\} (1); \quad \left. \begin{matrix} A, B \\ C, B \end{matrix} \right\} (2); \quad \left. \begin{matrix} B, A \\ B, C \end{matrix} \right\} (3); \quad \left. \begin{matrix} B, A \\ C, B \end{matrix} \right\} (4).$$

Not every group of three propositions fulfilling the conditions as to relation and form that we have so far laid down is, as we shall hereafter show, a valid syllogism; but all valid syllogisms can be written in one or other of the four following forms, in which, with an object to be explained immediately, the terms of the major premiss are placed first in each group:—

$$\left. \begin{matrix} B, C \\ A, B \\ A, C \end{matrix} \right\} (1); \quad \left. \begin{matrix} C, B \\ A, B \\ A, C \end{matrix} \right\} (2); \quad \left. \begin{matrix} B, C \\ B, A \\ A, C \end{matrix} \right\} (3); \quad \left. \begin{matrix} C, B \\ B, A \\ A, C \end{matrix} \right\} (4).$$

These four forms represent respectively what are called the First, Second, Third, and Fourth *Figures* of the Syllogism. A division of syllogisms in this way is, therefore, a division according to Figure: it is, as we see, based upon the positions of the middle term in the premisses.

If, again, we write the letters in order that denote respectively, the major premiss, the minor premiss, and the conclusion, in quantity and quality—the priority of the major premiss being purely *conventional*—we thus denote what is called the Mood of the syllogism. Thus:—

$$\begin{aligned} \text{All } B \text{ is } C; & \quad (A) \\ \text{All } A \text{ is } B. & \quad (A) \\ \therefore \text{All } A \text{ is } C. & \quad (A) \end{aligned}$$

is said to be in the mood AAA. Mood, then, is a distinction of syllogisms according to the quantity and quality of the premisses and the conclusion, and it is expressed by the letters that denote them, written in the conventional order. Take another example:—

$$\begin{aligned} \text{No } A \text{ is } B; & \quad (E) \\ \text{Some } C \text{ is } B. & \quad (I) \\ \therefore \text{Some } C \text{ is not } A. & \quad (O) \end{aligned}$$

The mood here is EIO, and the syllogism, as may be easily seen, is of the second figure.

The following matters are noteworthy in these syllogisms :—

The middle term is distributed in at least one of the premisses.

There are but three terms in each syllogism.

In the syllogism containing a negative premiss there is but one such negative premiss.

In the syllogism containing a particular premiss there is but one such particular premiss.

The conclusion is negative with a negative premiss, particular with a particular premiss (or, as it is said, follows the inferior or weaker part).

No term is distributed in the conclusion unless it is distributed in the premisses.<sup>1</sup>

And the same hold true with regard to every valid syllogism. Where, in what have been called quasi-syllogisms, they do not hold, and no conclusion can be drawn, we find involved what are termed formal *Fallacies*. Of these, some of the most common are *Undistributed Middle*, when the middle term is undistributed in the premisses, and *Illicit Process*, when a term is distributed in the conclusion which is not distributed in the premisses. For example: Every orang-outang is a vertebrate animal; every monkey is a vertebrate animal. Therefore, every monkey is an orang-outang. Here, vertebrate animal, the middle term, is undistributed. As another example, take the following :—A man who has enough is rich; no miser has enough. Therefore,

<sup>1</sup> These points are embodied in the hexameters :—

Distribuas medium, nec quartus terminus adsit,  
Utraque nec praemissa negans [nec particularis]  
Sectetur partem conclusio deteriorem,  
Et non distribuat, nisi cum praemissa, negetve.

Thus translated :—

Thou shalt distribute the middle, nor let four terms be present;  
And neither may both premisses be negative or particular;  
The conclusion must always follow the weaker part;  
And it shall not distribute or deny unless when one of the premisses  
does so.

no miser is rich. Here, we have *illicit process of the major term*: it is distributed in the conclusion, but not in the major premiss.

**Method.** § 15.—Method may be regarded, (1) as a well-defined progress of thought to attain its end in the advancement of knowledge, or, (2) as a body of rules and principles to which it ought to conform towards the attainment of such end. Many of the older treatises, accordingly, give the subject a place as a fourth part of Logic. But it more appropriately comes under the head of Applied Logic, and except in a very restricted sense will not occupy us except under that title. Method, as thus understood, is to be distinguished from what is known as Logical Methodology, which, according to Sir William Hamilton, embraces the treatment of the regulated ways or methods in which the means of thinking are conducted to their ends of thinking well. Perfect thought should be clear, distinct, and connected; and these qualities are secured by illustration and definition, division, and proof or inference. Methodology, as so defined by Hamilton, embraces, however, merely topics that, though some of them are regarded by many writers as extra-logical, we shall for convenience treat in their usual places as part of the ordinary matter of Formal Logic.

#### QUESTIONS.

1. Discuss the causes that retard the establishment of a final definition of Logic. C. S.
2. State and classify the different definitions of Logic, pointing out any different conceptions of its province they involve. C. S.
3. Explain in your own words the meaning and use of Logic. Why has it been called the “science of sciences”?
4. Distinguish between the provinces of Psychology, Logic, and Metaphysics.<sup>1</sup>

<sup>1</sup> See *Mind*, vol. xiii., p. 527.

5. Distinguish between the provinces of Rhetoric, Grammar, and Logic.

6. Discuss the province of Logic in view of Mr. Adamson's definition of it as the "theory of the normal laws of knowledge."<sup>1</sup>

7. Describe generally the view of Logic in accordance with which it may be defined as "the science of the necessary Laws of Thought," pointing out particularly—

(1) The limitation of the science consequent upon this view, as compared with that of Mr. Mill.

(2) The technical phraseology appropriate to this view. C. S.

8. Is Logic concerned with Names, Thoughts, or Things ? Give reasons for your view. C. S.

9. Define Realism. In what parts of Logic, and with what modification, does it survive ?<sup>2</sup> C. S.

10. Distinguish between Art, Science, Knowledge. Explain Mr. Adamson's definition of Knowledge as, "the whole sum of the determinations in and through which the world of external and internal experience is realised for us."

11. Distinguish between Formal and Material Logic.<sup>3</sup>

12. Reply to objections commonly urged against the utility of Logic.<sup>4</sup>

13. "People can reason without the help of Logic." Why is this not a sufficient objection to the study ?

14. How far is it justifiable to call Logic an *a priori* science?<sup>5</sup>

15. Define Language, and point out the main functions it performs.

16. What is meant by the distribution of a term ? Why is the predicate always distributed in negative propositions ?

17. Designate by the technical symbols the quantity and quality of each of the following propositions :—

<sup>1</sup> *Encyclopaedia Britannica* (ninth edition).

<sup>2</sup> See Mill's *System of Logic*, Book I., chapters ii., v., vii., and viii., and §§ 35-42 of the present work.

<sup>3</sup> See *Mind*, vol. iv., p. 362.

<sup>4</sup> See INDEX, and Whately's *Logic*, Introduction, § 4.

<sup>5</sup> See Thomson's *Laws of Thought*, p. 49.

A thing of beauty is a joy for ever.  
Solomon built the Temple at Jerusalem.  
There is no happiness but in well-doing.  
This is the loved, the well-known spot.  
There is a pleasure in the pathless woods.  
Longfellow wrote "Evangeline."  
Any two sides of a triangle are together greater than  
the third.

18. Give an example of a syllogism, and enumerate all the  
logical parts of which it is made up.

19. What is meant, respectively, by Mood and by Figure?

20. Distinguish the quantity and quality of each of the  
following propositions:—

Virtue is its own reward.

Not to know me argues thyself unknown.

Nought to admire is all the art I know.

To make man happy and to keep him so.

The English can hardly be called humble-minded. L.

21. Define Concept, Judgment, Reasoning.

## BOOK I.—ON TERMS

### CHAPTER I

#### MODES OF GROUPING TERMS

Terms :      § 16.—It is usual to include in Formal Logic a discussion of the various kinds of Terms, especially when an objective rather than a subjective mode of treatment is adopted. A *Term* is, as we have seen, a word or combination of words, so-called from its being capable of forming by itself one of the extremes, the subject or the predicate, of a proposition. Hence, by the older logicians, the Term was called a *Categorematic* word (from *κατηγόρημα*, a predication), while words incapable of being so employed were called *Syncategorematic* words, as being capable of forming terms only in conjunction with other words. Adjectives can be employed as terms only by ellipsis. Adverbs, prepositions, and nouns in any other case than the nominative, are syncategorematic. All verbs other than the infinitive, the participle, and the substantive verb, used as the copula, are mixed words, being resolvable into the copula and the (participle, or filling up the ellipsis, the) predicate. For example, “The sun shines,” we resolve into, “The sun” (subject) “is” (copula) a “shining body,” or into what, of course, is equivalent to the same, though with an ellipsis, “The sun is shining.”<sup>1</sup> A mixed word is categorematic in

<sup>1</sup> Such a proposition as the former, namely, “The sun shines,” in which the verb stands unanalysed into the copula and the participle, is, in the language of the scholastic logicians, said to be “secundi

the sense that it includes a Term. A Term may consist of a single word or of a combination of words. In the former case it is said to be *Single-worded*, as "charm," in the latter, *Many-worded*, as "The charm of earliest birds."

Considered in its representative character, and apart from its mere place as the subject or the predicate of a proposition, a Term is synonymous with a Name. The latter is defined by Hobbes as "a word taken at pleasure to serve for a mark which may raise in our mind a thought like to some thought we had before, and which being pronounced to others may be to them a sign of what thought the speaker had before in his mind."<sup>1</sup> The definition just quoted naturally suggests the question, Should names, properly speaking, be regarded as names of things, or of our ideas of things? To put the matter more clearly, let A represent the name, B, our *idea* of the thing, when the latter is objective in character, and C, the *thing itself*, of which B is the subjective counterpart, the question is, Does A more properly stand for B or for C? To the consideration of this question we shall return in our next chapter.

To illustrate further the meaning of the Term, let us take the proposition, "The wise are thoughtful": in this, the terms are "wise" and "thoughtful," both put substantively and so standing for "wise persons" and "thoughtful persons" respectively. In the proposition "He was buried in the Vale of the White Horse," the predicate, *in full*, consists of a group of words, namely, "a person that was buried in the Vale of the White Horse," and this is all to be regarded as a single term. Again, in the proposition, "The art which we profess

adjacentis," while such a proposition as, "The sun is shining," in which the verb is so analysed, is said to be "tertii adjacentis."

<sup>1</sup> Molesworth's edition of the Works of Hobbes, vol. i., p. 16. *On Computation or Logic*. Professor Peirce distinguishes signs as *indexes*, *tokens*, or *icons*. They stand in conjoint relation to the things denoted by them and to the mind. The *index* asserts nothing, but, like a pointing finger, may be taken as saying, "There!" Demonstrative and relative pronouns are nearly pure indexes, since they denote things without describing them. Where mental association exists as regards the sign and its object, it may be called a *token*, and, in the case of a certain degree of similarity between them, an *icon*. (See *American Journal of Mathematics*, vol. vii, p. 165.)

has beauty for its object." "The art which we profess" is to be regarded all as one term, forming the subject; and the full predicate, namely, "an art that has beauty for its object"—supplying the words omitted by ellipsis, is also to be regarded but as one term. Moreover, "vale" and "art" are categorematic words, though not employed by themselves, that is to say as terms, in the examples before us.

A very usual division of terms is that into Univocal, Equivocal, and Analogous. This, however, is, strictly speaking, not so much a division of terms, as of the *modes of employing words as such*,<sup>1</sup> since the *same word* may be employed univocally, equivocally, or analogically. *Univocal Terms* are such as are employed in but one sense. *Equivocal Terms* are such as are employed in more than one sense. *Analogous Terms* are equivocal terms whose meanings have a relation of analogy to each other. The term, "cathedral," for example, as applied to St. Paul's, and to the principal churches of Hereford, Salisbury, and several other towns, may be regarded as univocal. The term, "box," is equivocal, since it may be used in different senses. The term, "post," also may be used in different senses, but senses that are analogous, as connected with the root-meaning by relation of analogy. But, for logical purposes, a word must be regarded as a distinct term for each distinct sense in which it is employed.

**General and Singular Terms.** § 17.—The first really important division of terms is that into General, and Singular, or Individual. A *General Term*,<sup>2</sup> applies to an indefinite number of

<sup>1</sup> And the like may be said of the division of terms by the older writers into those of the *First* and those of the *Second Intention*. According to Hobbes, "Names of the first intention are *names of things*, as, a man, a stone; and of the second, the *names of names and speeches*, as, universal, particular, and other logical words." Looking to the *notion*, that of the first intention, according to Mansel, is "a conception under which the mind views things whether facts of external or of internal perception," that of the second intention as "a conception under which the mind regards its first intentions as related to each other." Thomson reminds us that it is with the second, not with the first, notion of things the logician is chiefly concerned.

<sup>2</sup> We have already treated of the Concept as the subjective counterpart of the General Term. The Concept is defined by Hamilton as, "the

individuals, and may be truly affirmed in the same sense of each of them, as city, tree, statue. A *Singular Term* is a term that can be truly affirmed in the same sense, but of one thing;<sup>1</sup> as Plato, Iseult, the Peninsula. General Terms are also called *Common Terms*. A *Collective Term* can be truly affirmed of the individuals of a multitude only when all are taken together; not of each separately. A term may be both collective and general, or collective and singular or individual. Regiment,

cognition or idea of the general character or characters, point or points, in which a plurality of objects coincide." The following he distinguishes as the principal steps in conception, or the process of forming concepts:—(1) Through the senses, or the imagination, we become conscious of a confused multitude of objects; (2) a comparison of these objects shows them to be possessed of attributes, some similar throughout the series of objects, others dissimilar; (3) attention is concentrated upon the objects which partially agree in attributes, and in them upon those attributes in which they agree; (4) the unification, in consciousness, of these attributes gives a concept. On this unity the seal is set by giving it a name (see Baynes's *New Analytic*, p. 6). A concept involves no image. It is said to be *clear*, as opposed to *obscure*, when we are able to distinguish it from other concepts; and *distinct*, as opposed to *confused*, when we are able to distinguish and enumerate the attributes of which it is composed. The most important distinction of concepts, according to Hamilton, is that into *Congruent*, or those which can be united in thought, and *Conflictive*, which cannot be. This distinction serves as a basis for his theory of judgments. For other divisions of Concepts, see Veitch's *Institutes of Logic*, p. 182.

The following observations on Conception are repeated from § 2 of the present work:—"The matter of thought may be varied indefinitely, and yet continue to be viewed by the mind in the same unaltered form; and, conversely, we may, in one unaltered form, express thoughts about the most diverse kinds of matter. In Conception, the matter consists of certain distinct notions viewed by the mind under one common form, which being but *one*, and existing *only in the mind*, may be taken to be everywhere and absolutely the same; a characteristic of *form* as distinguished from *matter*." To harmonise with this view of the process, Conception may be briefly defined as "the bringing of a series of distinct notions of objects under a law of thought." The law of thought chiefly prominent in the process may, by a slight anticipation, be indicated as the Law of Identity.

<sup>1</sup> Mr. Bain holds that the names of materials—as earth, stone, water—are singular, since they each denote the entire collection of one species of material. So, with Space and Time, if not regarded as abstractions. Other logicians, however, maintain a different view, insisting that, in the use of such a Term as *water* as the name of a material, it is of *any* portion of water we predicate, not of water in the aggregate; and, therefore, that the term is general. In this view, Space is general, if we mean a particular portion of space; singular if we mean space as a universe.

library, crew, are terms at once collective and general ; the Light Brigade, the Black Watch, the Gallant Six Hundred, are collective and singular. A *Concrete Term* is a term that stands for a thing, or an individual, or for an attribute, or a group of attributes considered with reference thereto ; as man, Solomon, wise. An *Abstract Term* is one that stands for an attribute or group of attributes, considered apart from the individual ; as wisdom, humanity, fortitude. Concrete terms are some of them general, some singular. Bird, happy, bower, for example, are concrete and general ; while Edwin, Shottery, Glastonbury Thorn, are concrete and singular. Abstract terms, too, are distinguished as, some of them general, some singular. When considered as strictly the names of attributes, they are to be regarded as singular, since attributes are one and indivisible ; but when used as concrete terms they may become general. Thus, *heat* is an abstract term, according to Shedden and others ; and, while boiling-point-heat is abstract and singular, as expressing under given circumstances one precise degree of heat, the term *heat* may become general, as including the different heat-points expressed by the terms, summer-heat, blood-heat, and so on.

**Positive, Negative, and Privative Terms.** § 18.—Terms have been also divided into Positive, Negative, and Privative. If we were to regard all attributes as positive, we should define a *Positive Term* as one expressing the presence of one or more attributes in the object to which it is applied, a *Negative Term* as expressing the absence of an attribute or attributes that could not conceivably exist in the object, and a *Privative Term* as one expressing the absence of an attribute or attributes that might conceivably exist in it, but do not. "Speaking," "dumb," "silent," are examples of such terms from this point of view, as when we say, "a man speaking," "a dumb statue," "a man silent." Since, however, there is no agreement among logicians that all attributes should be regarded as positive, while, at the same time, there is no test by which to decide as between what are called positive and what might be called negative attributes, it would seem to be the better

way, as suggested by Hobbes, to define such terms on purely linguistic grounds, "not," or its equivalent in compounds indicating a negative term as not-man, not-philosopher, "ex—" and its equivalents a privative term, and the absence of negative and privative particles a positive term. Some logicians consider, however, that, inasmuch as position and negation properly belong only to propositions, to speak of a term when by itself as positive or negative, is under any circumstances misleading.

**Contradic-** § 19.—To be associated with the foregoing is **tory and** the distinction of terms into Contradictory and **Contrary** Terms.

**Contrary.** *Contradictory Terms* are such as differ in verbal form by the particle "not—" or its equivalent in sense, being such as, therefore, cannot both be predicated at once of the same subject, while one of the two must be predicable of any subject whatever. This is otherwise expressed by saying that they exhaust between them the *Universe of Discourse*—that is, the entire universe to which reference is made; whilst within that universe there is no individual of which they can be simultaneously affirmed. Of course the universe of discourse may, at the option of the speaker, be taken to mean the universe of all existence. A pair of such terms will obviously be, a positive term and the corresponding negative, as Hobbes would have such terms defined. *Contrary Terms* are terms which, coming under some one class, are the most opposed of all that belong to that class, as *black* and *white*, among colours. The phrase, "most opposed," is, however, very indefinite; and it has been suggested to define contrary terms as such as cannot at once be predicated of the same subjects, while there may be subjects of which neither can be predicated; or such that is known *à posteriori* by experience, and not *a priori* by the very form of expression, that they cannot be predicated of the same subject.

**Relative** § 20.—Another usual division of terms is that **and** into Relative and Non-Relative or Absolute, the **Absolute** word *Relative* being here understood in a very **Terms.** special sense; for any given term having regard to the

relativity of the idea it expresses to other ideas in the universe of knowledge, may be considered a *Relative Term*, if the two-sidedness of its meaning be the main subject of thought. In this general sense, positive and negative terms, as just treated, are relative terms, each pair dividing a certain universe between them. Considered, however, in the special sense intended, *Relative Terms*, while they denote certain objects to which thought is in the first place directed, necessarily bring also before our minds some other object, or objects, not denoted by them, but the names of which, called *Correlative Terms*, are usually mentioned in any explanation of their meaning; both terms being generally expressive of some fact of complicated relationship, while *Non-Relative Terms* denote objects without reference to any other such necessarily associated objects. The fact, or facts, constituting the basis of relation between the objects represented by both correlative names logicians call the *fundamentum relationis*. The characteristic property of relative terms, as thus understood, is that they are always associated in pairs, as, father, son; ruler, subject; cause, effect; while non-relative terms, as, man, being, do not pair in this way, and are, in their ordinary use, definable alone. The term *Absolute*, as Mill points out, is of too considerable importance in Metaphysics to be used as a definite name for such terms in Formal Logic.

**Connegative and Non-Connegative Terms.** § 21.—Finally, we have to consider Terms as divided into Connegative and Non-Connegative, a division that Mill regards as the most important of all, and one of those that go deepest into the nature of language. A *Connegative*, or *Attributive Term*,<sup>1</sup> is one that denotes a subject, and implies an attribute; as, horse, white, strong, holy. A *Non-Connegative Term* is one that denotes a subject only, as Victoria, England, or an attribute only, as

<sup>1</sup> The word *Attributive* is employed by Dr. Fowler to mark a distinct class of terms, those included under Adjective names by Shedd, and grammatically represented by adjectives and participles. Such terms may form the predicate, but cannot, unless joined with a noun, form the subject of a proposition (*Deductive Logic*, pp. 13, 16).

whiteness, length, virtue. A subject, here, means anything that possesses attributes. All concrete general terms are connotative ; as, man, city. They signify subjects directly, attributes indirectly.<sup>1</sup> Connotative terms are also called *denominative*, because the subjects they denote receive a name from the attribute they connote ; for example, snow is called *white*, as possessing the attribute, *whiteness*. Even abstract names, though usually non-connotative, may, in some instances, according to Mill, be considered as connotative ; for attributes themselves may have attributes assigned to them. For example, the word *fault* in such an expression as " slowness in a horse is a fault," connotes *hurtfulness*, or an undesirable peculiarity. This view, however, is disputed. As to concrete singular terms, proper names, according to Mill, are not connotative. Though, in their first application to individuals they may have had connotation, the name once given is independent of the reason, and, in fact, frequently remains attached to the object after the reason has ceased to exist. Hence such names are to be considered as mere meaningless marks. Some singular terms, however, are really connotative, namely, such of them as express the individualising fact ; for example, "the *first* Emperor of Rome," "the author of the *Iliad*," and the term "God," when used by a monotheist. The view held by Mill with regard to the non-connotative character of proper names is opposed by Jevons, Sheddern, and others, who contend that a proper name has not merely connotation, but

<sup>1</sup> Mr. James Mill appears to have been the first comparatively recent writer to adopt from the Schoolmen the word *to connote* : he speaks of a term, however, as *noting* the attribute, and *connote*ing the things possessing the attribute. Mr. J. S. Mill, in defending his own mode of employing the word (*System of Logic*, vol. i., p. 43), points out that the Schoolmen clearly explained that nothing was said to be connoted except *forms* ; forms, in their writings, being synonymous with attributes. This appears to be in accordance with the meaning attached to the connotative term by Dr. Narcissus Marsh (*Institutiones Logicae*, ed. 1697, p. 6) :—" Does a connotative term express its matter and form in the same way ? " " A.—No ; its matter is expressed *in recto* ; its form *in obliquo* : that is said to be expressed *in recto*, which in the meaning of the term, is expressed in the nominative case, and *in obliquo*, which is in an oblique case ; as when we explain a *white* thing as 'a thing affected with *whiteness*.' "

is the most connotative of names; the group of attributes increasing from the more general name to the more particular included under it, till in the proper name we find their number culminate.<sup>1</sup> Dr. Fowler employs the term *connotative* in a sense somewhat different from that attached to it by Mill. According to the latter a connotative term must possess both connotation and denotation. According to the former "a term may be said to *denote* individuals, or groups of individuals; to *connote* attributes or groups of attributes." Thus regarded, general terms are both connotative and denotative; and abstract terms which, according to Mill, are denotative, but not connotative, are connotative, but not denotative.

**Extension and Intension.** § 22.—By the *Extension* of a term is meant the number of its significants, that is to say, the number of individuals to which it is applicable; by *Comprehension*, the number of attributes that constitute the notion for which it stands. Hamilton calls Extension, also, the *sphere* of this notion, and Comprehension, the *Intension*.<sup>2</sup> In the same sense, that is to say, as applied, the one term to significants, the other to attributes, Extension and Comprehension are respectively synonymous with *Breadth* and *Depth*. Taking, for illustration, such a term as *man*, it includes certain marks, the more obvious of which, say, corporeality, animal life, rationality, and a certain external

<sup>1</sup> Though when the object denoted by a term is familiar to us, we can usually enumerate, without difficulty, its more prominent attributes; yet anything approaching an exhaustive determination of its connotation is always a matter of difficulty, unless, indeed, the meaning has been settled by definition. Again, when owing to a certain superficial similarity, the same name is applied to things in other respects wholly different, the difficulty of such a determination is much increased. Even such familiar terms as *man*, *stone*, are illustrations in point. Sometimes, owing to advances in science, it is found necessary to increase or decrease the connotation of a term. This, of course, narrows the unexplored field, but every further acquisition of connotation is attended with an advancing amount of difficulty. The term *metal*, in the light of recent progress in chemistry, affords a good illustration. (See Mill's *Logic*, vol. i., pp. 38-42.)

<sup>2</sup> Extension and Intension are sometimes called the *affections* of terms.

form, enable us to single out from the field of Being around us the individuals to which it is applicable. Thus, the comprehension of the term enables us to determine its extension. If, moreover, we were to increase the comprehension by adding to the number of attributes, as for example, by understanding the given term *man* to mean no other than a person *six feet high*, the result would be to diminish the extension of the term, as several individuals wanting in the attribute of specified height would be thus excluded ; and if, on the other hand, we were to diminish the comprehension, for example, by omitting the attributes of rationality and human form, the result would be to increase the potential extension of the term, as a greater number of animals would, or could, now be included under it. If, again, we take a series of general terms, such as, Being, Animal, Biped, Man, Englishman, we see that if they stand in the order of generality, Being includes the greatest number of individuals, and implies in each the smallest number of attributes, or, in other words, has the greatest extension and the least intension of these five terms ; and that the extension diminishes from term to term, to Englishman, the last in the series, while, at the same time, the intension increases from term to term, so that this last term implies the greatest number of attributes. And the same is true with regard to every such series of terms, similarly ordered or subordinated. This relation is sometimes expressed by saying that, *there is an inverse law<sup>1</sup> between extension and comprehension ; the one diminishes as the other increases.* The terms, denotation and connotation, as employed by Dr. Fowler, are equivalent to extension and comprehension as here explained. Denotation and connotation, however, are not employed with reference to *concepts*, while extension and comprehension apply more properly to concepts than to terms.

<sup>1</sup> Not an *inverse ratio*. This mathematical term is very improperly employed in statements such as that in the text, by Hamilton and after him by Baynes. See Mill's *Examination of Sir William Hamilton's Philosophy*, p. 612.

## CHAPTER II

### THE THINGS DENOTED BY TERMS

**Import of Terms.** § 23.—Owing to the fact that the Doctrine of Terms is concerned mainly with the *matter* of the Term, involving considerations of a metaphysical and grammatical, rather than of a purely formal nature, some logicians hold that it cannot properly be said to fall within the province of Logic. This, however, depends on the limits we assign to the science; such writers as Hamilton and Thomson, who adopt a subjective standpoint, regarding the full treatment of the Term from their point of view as extralogical, while Mill and others, who take an objective view of the science, consider that a discussion of the various kinds of terms and their import forms a necessary part of it, as being the first step towards the analysis of propositions, on which the logical superstructure rests. We have already drawn attention to the inquiry, Does a term or name, such as the word *sun*, properly stand for a Thing, or for our idea of a Thing? It is obvious that where we desire to secure fixity of meaning the latter could not be accepted; especially when, owing to the want of final determination of the comprehension of the term, or from other causes, different persons might have different ideas of the same object-matter. Hence, where, as here, an objective substratum lies, or is conceded to lie beneath, we may take, with Mill, the Term to stand for the Thing itself, which, in the case of the example chosen, is the *noumenon*, sun, with its potentiality of phenomena. But an objective substratum, or

substance, is not the only kind of Thing that a name, or term, may denote. Attributes, Feelings, and the like, at once suggest themselves in addition. And it is obvious that if it be important to us to know the possible materials of propositions, each of which brings into conjunction for affirmation or denial two terms, we must ask ourselves what are the various kinds of Things that names or terms may denote ; or, what is the final analysis of Things to which all nameable Things are reducible. The analysis of Terms, as given in our preceding chapter, in some measure indicates the direction in which the answer to this inquiry lies, since it brings before us distinctions between the Things signified by such Terms generally recognized. Many logicians, however, regard much that is included in the analysis of Terms as superfluous, or overlapping, or as expressive of distinctions existing among the mere names rather than among Things in their own nature. Hence, an independent analysis of nameable Things appears to be necessary towards a complete treatment of the import of Terms.

**Aristotle's Categories.** § 24.—The Categories<sup>1</sup> or Predicaments of Aristotle are sometimes mentioned as an analysis of

<sup>1</sup> The term Category is derived from *κατηγορία*, which with Aristotle means assertion or predication, so that the plural, *αἱ κατηγορίαι*, may be translated, the kinds of assertion or predicates. The Categories form the subject-matter of the first of the six treatises that compose the *Οργάνον* of Aristotle, viz. :—

1. *Κατηγορίαι*, Praedicamenta ; on the Categories.

2. *Περὶ ἐρμηνείας*, De Interpretatione ; on the Expression of Thought.

3, 4. *Ἀναλυτικὰ πρότερα* and *ὕστερα*, the Prior Analytics, and the Posterior Analytics, each in two books ; on the Theory of Conclusions.

5. *Τοπικά*, De Locis, in eight books ; on the General Points of View (*τοποί*) from which Conclusions may be drawn.

6. *Περὶ σοφιστικῶν ελεγχῶν*, concerning Fallacies.

The passage in the Organon, in which the Categories are enumerated, is as follows :—

Τῶν κατὰ μηδεμίαν συμπλοκὴν λεγομένων ἔκαστον ήτοι οὐσίαν σημαίνει η ποσὸν η ποιὸν η πρός τι η ποῖ η ποτὸν η κείσθαι η ἔχειν η ποιεῖν η πάσχειν. (Magrath's *Selections*, p. 2.) “Of things expressed without syntax (i.e., single words, and so, of course, without reference to truth or falsehood), each signifies either substance, or quantity, or quality, or relation, or place, or time, or disposition (i.e. attitude or internal arrangement), or appurtenance, or action (doing), or suffering (being done to).”

From its more technical meaning of “that which more naturally

the very kind we require. These Categories are ten in number, and with corresponding Latin and English renderings are as follows:—

(1) Οὐσία, Substantia, Substance ; (2) Ποστὸν, Quantitas, Quantity ; (3) Ποιόν, Qualitas, Quality ; (4) Προς τι, Relatio, Relation ; (5) Ποῦ, Ubi, Place or Location ; (6) Πότε, Quando, Time or Period of Time ; (7) Κείσθαι, Positio, Attitude, Posture, or Disposition ; (8) Ἐχεῖν, Habitus, Equipment, Appurtenance, Property ; (9) Ποιεῖν, Actio, Action, Active Occupation ; (10) Πάσχεῖν, Passio, Passion, Passive Occupation.

From the earliest times considerable difference of opinion has prevailed among logicians as to what these Categories are an enumeration of, whether of names, or notions, or things, or of all three. They have been variously regarded as—

(1). An enumeration of all things capable of being named ; an enumeration by the *summa genera*, *i.e.*, the most extensive classes into which things could be distributed ; which, therefore, were so many highest predicates, one or other of which was supposed capable of being affirmed with truth of every nameable thing whatsoever. (*The Schoolmen.*)

(2). An enumeration of the different grammatical forms of the possible predicates of a proposition, viewed in relation to the first (namely, substance) as subject. (*Mansel* and *Trendelenburg.*)

(3). An enumeration of the different significations of simple terms, apart from their connection in the proposition.

(4). An enumeration of the several genera under which Aristotle's four heads of predicables fall.

(5). An enumeration of the different modes in which Being may be signified. (*Adamson*<sup>1</sup>)

(6). A generalisation of *predicates* ; an analysis of the final

takes the place of predicate," the term, Category, has now, in common language, come to signify a head or class to which any object may be referred.

<sup>1</sup> See *Encyclopaedia Britannica* (ninth edition), vol. v., p. 221.

import of predication, including verbal (see § 33) as well as real predication. (*Bain.*)

(7). The different classes to which are reducible all the objects of our thought comprising all *substances* under the first, and all *attributes* under the nine others. (*Port Royal Logic.*)

(8). The forms of the individual conception,<sup>1</sup> and of their verbal expression, in relation to the corresponding forms of existence (and these last themselves metaphorically). (*Ueberweg.*)

Considered, however, as an enumeration by the *summa genera*, of all things capable of being named, Aristotle's list of Categories appears to Mill to be at once redundant and defective. It is redundant in that some of the categories are not exclusive; for instance, the Category of Relation ( $\pi\tau\circ\tau\iota$ ) should be comprehensive enough to include at least the fifth, ninth, and tenth, while the difference between others (as  $\pi\tau\circ\tau\iota$  and  $\kappa\epsilon\iota\sigma\theta\alpha\iota$ ) is merely verbal. It is, also, defective, in that it takes no notice of feelings and states of mind,—as hope, joy, fear, and the like.

(9). Taken according to Bain's view, and not as an enumeration of Things, the categories do not seem to be open to the charge of being defective, as urged by Mill, since sensations, feelings, and states of mind can be considered only as in the import of propositions. Nor can the charge of non-exclusiveness or redundancy urged against them be in this light so strong, seeing that they thus acquire some distinctness of meaning not taken account of in Mill's criticism. Bain's opinion is that the original purpose of the categories "was simply to exhaust the possible predicates regarding an individual, and not either to exhibit a classification of nameable things or to analyse the import of propositions with a view to the arrangement of logical departments."<sup>2</sup>

<sup>1</sup> "The individual conception (representatio, or conceptus singularis) is the mental picture of the individual existence, which is, or at least is supposed to be, objective. (See Ueberweg's *Logic*, p. 110.)

<sup>2</sup> Bain's *Logic*, part i., p. 226.

(10). Sir William Hamilton proposes to amend the categories so as to obviate the charge of redundancy. They are, according to him, a metaphysical, not a logical reduction. They are all divisions of Being ( $\tauὸ \; ὄν$ ), *Ens*. Being is divided into *Ens per se*, and *Ens per accidens*. *Ens per se* corresponds to substance, the first of the categories. *Ens per accidens*, which denotes either something *absolute*, or something *relative*, comprises the other nine. As absolute, it flows either from the matter or from the form of things. If from the matter, it is Quantity, Aristotle's second category; if from the form, it is Quality, the third. If something relative, it corresponds to the fourth category, Relation; and to Relation the remaining six are reducible.

His arrangement will, therefore, stand thus:—

Being ( <i>Ens</i> )	Being, by itself ( <i>Ens per se</i> ).	Substance;
	Being, by accident ( <i>Ens per accidens</i> ).	Quantity, Quality, Relation;

Relation, as just observed, including the last six categories of the Aristotelian system.<sup>1</sup>

(11). A statement of the purpose of Aristotle's categories in harmony with Dr. Bain's opinion is that they constitute the heads of the verbal analysis into its elements of an

<sup>1</sup> Thomson's classification, based on Hamilton's, is:—Conceivable Things are, Substance, Attribute {Quantity, Quality, Relation [of Time, of Space, of Causation, of Composition, of Agreement and Repugnance, of Colour Opposition, of Finite to Infinite.]} The Stoicks reduced the Aristotelian Categories to four, which they called the Most Universal Kinds. These are (1) the Substratum, (2) the (Essential) Property, (3) the (Unessential) Quality, and (4) the Relation. (See Ueberweg's *Logic*, p. 119.) Kant's Categories may be regarded as a classification of what can be asserted about Things, arrived at through a classification of Judgments. His four leading Categories are Quantity, Quality, Relation, and Modality. Under the first of these he places the sub-categories of Unity, Plurality, Totality; under the second, Reality, Negation, Limitation; under the third, Substance, Causality, and Community or Reciprocal Action; and under the fourth, Possibility, Existence, Necessity and their opposites. Of these, Dean Mansel recognises none but Unity, Plurality, and Totality; while M. Cousin tries to reduce the whole to Substance and Causality. But Kant makes use of his Categories chiefly in Metaphysics, or Psychology, not in Logic.

objective complex impression. It was, in the schools, a usual task for the wits of a student to include in a single distich, or stanza, a concrete illustration of the entire ten categories. Of such exercises Shedden gives two examples:—

1. “Arbor, sex servos, fervore, refrigerat ustos,  
Kuri eras stabo, nec tunicatus ero.”
2. “A lady stout, too warmly clad,  
At Bow, one summer day,  
Walking a mile to see her son,  
Was melted quite away.”

**Mill's Classification of Things.** § 25.—As a substitute for the Categories of Aristotle, considered as a classification of Existences, Mill suggests the following:—

1. Feelings, or States of Consciousness, including Sensations, Thoughts, Emotions and Volitions; Actions being merely Volitions followed by an effect.
2. The Minds which experience those feelings.
3. The Bodies, or external objects which excite certain of those feelings, together with the properties whereby they are commonly regarded as exciting them. Bodies and Minds exhaust what are known as Substances.
4. The Successions and Co-Existences, the Likenesses and Unlikenesses, between feelings or states of consciousness. On these are grounded all Attributes.

These four classes comprise all Nameable Things; and these, or some of them, must, therefore, compose the import of all Terms or Names. They form, however, a psychological, not a logical enumeration; and they are, further, based upon a psychological theory that has not met with universal acceptance. Nor is their exclusiveness beyond objection. Why, it may be asked, should Likenesses and Unlikenesses between Feelings and States of Consciousness be regarded as other than States of Consciousness as well.

**Bain's Classification.** § 26.—Mr. Bain bases his Enumeration of Things, in the most general sense, upon the principle

of Universal Relativity, by which all objects of knowledge are two-sided, or go in couples. This gives a fundamental and independent position to Relation, which occupies so subordinate a place in Aristotle's list. He divides the totality of Existing Things into :—(1) The Subject, that is to say, Mind, or the Internal World ; (2) the Object, that is to say, Matter, or the External World ; (3) the Attributes common to Object and to Subject—namely, Quantity, Succession, and Co-Existence ; (4) the Attributes special to the Subject, namely, Feeling, Will, and Thought ; and (5) the Attributes special to the Object, of which the principal are, Extension, Resistance, Colour, Touch, Sound, Odour, Taste, Heat, and Cold. As to Substance, it is not the antithesis of all Attributes, but that Attribute, or group of Attributes, which in each case is the most fundamental, persisting, inerasible, and essential.

This enumeration has the advantage of clearness, and is framed with an important purpose in relation to Applied Logic, namely, towards a division of the field of knowledge in a manner corresponding to distinct methods of enquiry. A discussion of it at length, here, however, would be out of place.

#### QUESTIONS.

1. "Sir William Hamilton says that Logic deals with the Notion, Judgment, and Reasoning ; Mill that Logic has to do with Names, Propositions, and Arguments. Hamilton takes a subjective view of its matter, Mill an objective view."<sup>1</sup> Explain. M.

<sup>1</sup> It will prevent a possible misconception with regard to Hamilton's meaning, though scarcely likely to arise in connection with the question before us, to mention that he qualifies his definition of Logic thus :—"Logic considers Thought not as the operation of Thinking but as its product ; it does not treat of Conception, Judgment, and Reasoning, but of Concepts, Judgments, and Reasonings." Mill, in his *Examination of Hamilton's Philosophy*, p. 463, gives his entire adhesion to this distinction ; but he seems to shift his position in his *System of Logic*, in which, instead of Concepts or Notions, Judgments and Reasonings, he treats of Names as standing for Things, of Propositions (except when merely verbal, and, as such, assertions about Names) as assertions about Things, and of the Proof, or Disproof, of such assertions.

2. Explain the meaning of Term. Mention, with examples, the various kinds of Terms. 0.

3. Give the logical characteristics of each of the following Terms<sup>1</sup> :—Mortal, Mortality, Immortal, Being, Equality, Governor, Regiment. In what respect do Abstract Notions differ from Concepts?

4. Assign the logical characteristics of the terms :—Astronomy, World, Antidote, Independent, Investigate.

5. Thing, White Object, Metal State, Machine, Horse. In what respects do these Concepts differ from one another?

6. Give a classification of the various kinds of Terms in the following sentences :—

Jeremy Bentham is the author of Chrestomathia.

It bore the legend of St. George and the Dragon.

John Kyrle was the Man of Ross.

Liquids find their level.

A rose by any other name would smell as sweet.

What is the blessing of the sight?

Oh ! tell your poor blind boy.

She answered, We are seven.

7. Discuss the claims of the Doctrine of Terms to be included in a treatise on Logic.

8. What is meant by saying that a word is equivocal, and how do words become so?<sup>2</sup>

<sup>1</sup> That is to say, state of each, whether it is singular or general, collective, concrete, or abstract, positive, negative, or privative, relative or absolute, connotative or non-connotative. If a number of words were promiscuously given, it should be stated also, of each of them, whether it is categorematic, or syncategorematic. An Abstract Notion is the notion of a quality considered apart from the subject in which we should look to find it, as prudence, strength. A Concept (from Lat. *con*, and *capere*, to take or group together) is the notion of a quality or a number of qualities, considered as belonging to each of a number of individuals, and in virtue of which we group them all together as forming a distinct class.

<sup>2</sup> A word is said to be equivocal when it has two or more different meanings. Words become equivocal from the accidental confusion of different words derived from different languages, or from different roots of the same language; or, from the transfer of the meaning by the association of ideas from the thing originally denoted by the word to some other thing usually associated with it; or from the transfer of meaning by analogy, or owing to real or fancied resemblance.

9. Distinguish between the different kinds of Terms employed in the following propositions :—

His manner is lifeless.

He fell a lifeless corpse.

The viewless wind left no trace behind.

Dante describes the hopeless abyss.

When these symptoms appear, the case is hopeless.

The thoughtless man never regards it.

10. What do you understand by *clearness*, and what by *distinctness*, of Thought. By what logical processes are these to be obtained ?<sup>1</sup>

11. "Every Concept is, in fact, a judgment, or a fasciculus of judgments" (Hamilton). Explain and criticise this statement.<sup>2</sup> C. S.

12. Distinguish between Collective and General Names. Of what kind are the subject and predicate, respectively, of each of the following propositions :—

The really disinterested are scarce.

Few men can forgive.

13. Distinguish between a General and an Abstract Term. O.

14. Mr. Monck states that one of the main distinctions between General, Collective, and Singular Terms, for logical purposes, is "that Singulars and Collectives do not admit of being used as the subject of particular propositions, and are very seldom used as predicates." Examine this statement in connection with the example—"Some John Smith was born in Bristol." Discuss the use of Collectives and Singulars as predicates.

<sup>1</sup> See footnote, § 17. *Clearness* is obtained by Definition; *distinctness* by Division, and Classification.

<sup>2</sup> See Hamilton's *Lectures*, vol. iii., p. 117. By this is meant that a concept is nothing but the result of a foregone judgment, or series of judgments, fixed and recorded in a word. For instance, the Concept, Man, which is represented as equivalent to the combination, Rational-Animal, is, according to Hamilton, the result and record of a judgment in which we affirmed a relation between the Concepts, Rational, and Animal. But as this judgment may be any one of at least four different forms, the question arises, Which of these forms is the Concept the record of ?

15. "For every positive Concrete name a corresponding negative one might be framed."<sup>1</sup> Illustrate the meaning of this statement, and find the precise negatives of the positive terms—Man, Physician, Red, Thing. C. S.

16. Explain the distinction of Comprehension and Extension in Concept, and show the application of it in Judgments and Reasoning.

17. Distinguish between the Extension and Intension of the terms—Monarchy, Virtue. According to what law are Extension and Intension related?

18. Explain and illustrate what is meant by the Denotation and the Connotation of a Term. What Terms have both, and what have one of these only?

19. Discuss the question whether the following Terms are Connotative, or Non-Connotative:—Dryburgh Abbey, The Master of Ravenswood, The Hollow-blade Sword Company, Newcastle.

20. Is it accurate to say that, as the Denotation of a General Term increases its Connotation decreases, and *vice versa*? What extent of attributes may be regarded as connoted by a General Term?

21. State your view as to whether proper names are Connotative, or Non-Connotative, and give the grounds on which you support your opinion. C. S.

22. What are the principal difficulties by which we are beset in determining the Connotation of any given Name?

23. In what different senses have logicians spoken of Connotations and Connotative Names? O.

24. Show, by examples, that the division of Names into General and Singular does not coincide with the division into Abstract and Concrete; and give three instances of each of the following:—Relative, Privative, Negative, and (Singular) Connotative Names. L.

25. Write a note on—"The Concept *horse* cannot, if it remain a Concept, be represented in the imagination; but except it be represented in the imagination it cannot be

<sup>1</sup> Mill's *Logic*, vol i., p. 43.

applied to any object, and except it be so applied it cannot be realised in thought at all.”<sup>1</sup>

26. Show that if the Comprehension of one Term be part of the Comprehension of another, the Extension of the latter will be part of the Extension of the former. M.

27. Illustrate the difference of Connotation and Denotation with reference to the following Terms:—Cæsar, Roman, Virtue, Virtuous, Equality. L.

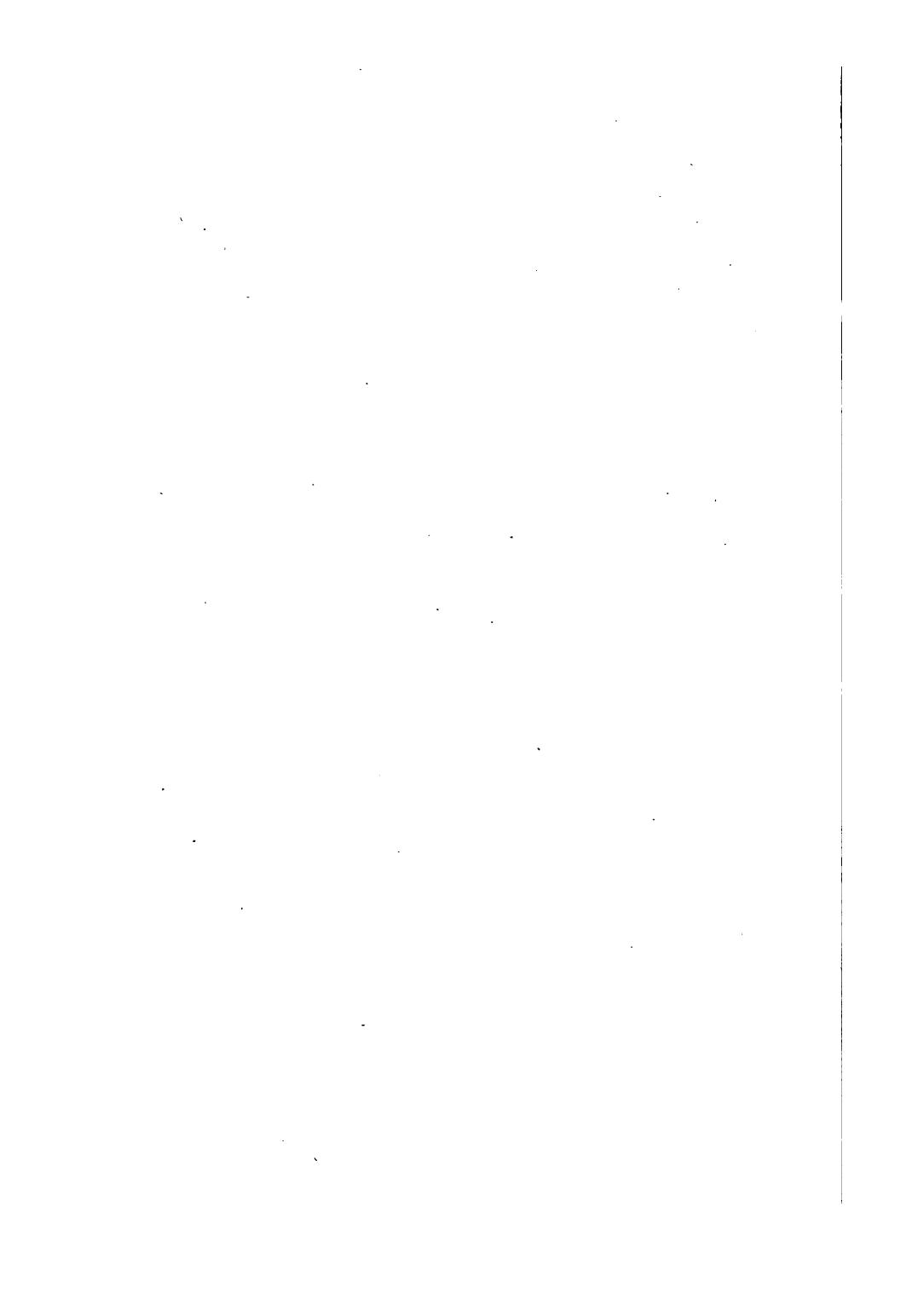
28. What is meant by the *material*, and what by the *formal*, relation of Terms to each other?

29. Distinguish between Perception and Conception. Give some account of the matters at issue between Nominalism and Realism.

30. State and compare the various arrangements of the Categories which have been proposed by logicians.

31. What is meant in Logic by the term “Categories”? Give Aristotle’s list of Categories. What faults have been found with this classification, and what emendations in it have been suggested? Criticise the view taken of them as, “the heads of the verbal circumscription of a concrete object.”

<sup>1</sup> See Hamilton’s *Lectures*, vol. iii., pp. 131–137, and Mill’s *Examination of Hamilton’s Philosophy*, p. 388.



## BOOK II.—ON PROPOSITIONS

### CHAPTER I

#### ANALYSIS AND CLASSIFICATION OF PROPOSITIONS

*Elements of a Proposition.* § 27.—We have already explained the nature of the Proposition. Briefly, a Proposition is a predicative sentence, or the verbal expression or enunciation of a judgment. Propositions, as to their substance, have been distinguished into Simple, Complex, and Compound, a distinction, as we shall have to observe again, only partially belonging to Logic. All propositions, however, are ultimately expressible in terms of a single simple proposition, or a number of such propositions united ; and every simple proposition is easily seen to be analysable into three distinct elements—the *Subject*, the *Predicate*, and the *Copula*. The Subject is that of which something is affirmed or denied. The Predicate is what is affirmed or denied of the subject. The Copula is the verb which expresses the relation of the Predicate to the subject, whether as of affirmation or denial. In English, a proposition must contain at least two words ; as, for example, “Water flows.” A two-worded proposition, however, must for logical purposes be reduced to the normal type, which will be effected by resolution of the verb into the substantive verb and the predicate ; thus, “Water is a flowing substance.” Here, “water” is the subject ; “is” is the copula ; and “flowing substance” is the predicate. And it will be apparent from the consideration of a complex or a compound proposition that it is in like manner so resolvable.

For example, "If a man is upright he need not fear accusation" is complex, but may be otherwise expressed as "*The case of a man's being upright is the case of a man's not fearing accusation*"—a simple proposition, and, of course, analysable, as before, into subject, predicate, and copula.

According to the more usual arrangement of words in an English sentence, the subject is the first term or extreme of the proposition, and the predicate the last. This, however, is not always so. For example, in the proposition, "Great is Diana of the Ephesians," "Diana of the Ephesians" is the subject, and "great," or, "great goddess," is the predicate. Although the order of the terms, therefore, generally indicates which is the subject and which is the predicate, in particular cases it will be often found that the term which conveys to us information about the other is the predicate, and that the other is the subject; in other words, the subject is that which more usually brings before the mind the object or objects, the predicate, the attribute or attributes pertaining thereto. Of course, when the subject and predicate are merely two names for the same thing, or are co-extensive, and so interchangeable at will, the order of the terms in the sentence may be taken as decisive of the point.

The terms, that is to say, the subject and predicate, represent, in a certain sense, the objective element in a proposition; the copula represents the subjective. The subject and predicate are said to constitute the *matter* of it; the copula and the sign of quantity the *form*. The copula is usually regarded as consisting of some part of the verb *to be*, in the present tense (Logic contemplating only what is present to the mind), and with, or without, the particle "not."<sup>1</sup> It is to be regarded merely as a sign of predication; and is not to be understood as implying time, or real existence. As to

<sup>1</sup> Hobbes and others, however, recognise only one form of Copula, the *affirmative* copula, *is*, *are*, &c.; and in all cases attach the negative particle to the predicate, and this with the idea of getting rid of the distinction between affirmation and denial by treating every case of denial as the affirmation of a negative name. Such a proposition as, "A is not B," would, on this understanding, be read as, "A is not—B."

the latter, for example, "A griffin is a fabulous animal," cannot, of course, imply that such a creature as a griffin actually exists. But when the verb *to be*, in its persons, is employed to stand for both copula and predicate, as in the proposition, "A *is*," that is, "A *is* an existing thing," then, existence is implied, though whether objective, or purely subjective, is left undetermined.

**Classification of Propositions.** § 28.—Propositions may be classified for logical purposes on various principles. But, first,

may be mentioned the distinction of propositions, according to substance or external form, into *Simple*, *Complex*, and *Compound*, a distinction, however, which only partially belongs to Logic. A simple proposition contains but one subject and one predicate. In a complex proposition, a subordinate clause, or complement, enters into the subject, or predicate, or both. In a compound proposition, there are more than one subject, or more than one predicate, or both. These three classes Bain reduces to two, namely, the Simple, and the Complex or Compound; subdividing the Compound, in which the separate propositions are conjoined in one meaning, into *Conditional* and *Disjunctive*, as the kinds of Complex propositions more especially entering into Logic.

Mansel, regarding chiefly the relation between the terms, or the nature of the affirmation or denial, divides propositions into three classes—(1) *Categorical*, (2) *Hypothetical* or *Conditional*, and (3) *Disjunctive*.

In the treatment of the class called by Bain, Complex or Compound, much independence in the feature of nomenclature appears in recent writers. Mill calls the entire group *Hypothetical*, subdividing it, as does Bain, into Conditional

But the affirmation of a negative name is simply an assertion that something is not; to express which no word seems so proper as the word denying. And, at the same time, the real distinction is between a fact and the non-existence of that fact. Or, rigidly holding the mind to A and B, the question is, Are these to be kept together or put asunder? "To put things together," Mill observes, "and to put or keep them asunder, will remain different operations, whatever tricks we play with language."

and Disjunctive; Hamilton and Thomson, *Conditional*, subdividing it into Hypothetical and Disjunctive; and Dr. Fowler, *Hypothetical*, subdividing it into Conjunctive and Disjunctive. The Disjunctive Proposition is of one or other of the forms, "A is either B or C," "Either A is B or C is D"; and the other kind of complex proposition, variously called Conjunctive, Conditional, and Hypothetical, is of the form, "If A is B, C is D."

A Categorical Proposition is an unconditional expression of the relation between the subject and predicate, and the notions, or, according to Mill, the *things* they represent; and to this form disjunctives and conditionals are both reducible. The reduction of the disjunctive, however, involves a question of interpretation. Taking for illustration, the disjunctive proposition, "Either A is B, or C is D," Hamilton, Thomson, Dr. Fowler, and some other logicians, hold that the truth of "A is B" depends on the falsity of "C is D," and *vice versa*—the disjunctive thus expressing the relation between two propositions which cannot both be true together, but one or other of which must be true. Whately, Mill, Jevons, and their followers, on the contrary, hold that the falsity of "C is D" implies the truth of "A is B," and not *vice versa*;—the disjunctive thus expressing the relation between two propositions which cannot both be false together, though they may be simultaneously true. Dr. Fowler (*Deductive Logic*, p. 115) analyses the above disjunctive into four conjunctive or conditional propositions. As to the conditional proposition, the formula, "The *case*, fact, or notion, of this existing, is a *case*, fact, or notion, of that existing," is sufficient, as above shown (§ 27), to reduce any given example to a categorical. "If A is B, C is D," is thus reduced to—"The *case* of A being B is a *case* of C being D." The second clause of the conditional, namely, "C is D," the truth of which depends on that of the first clause, is called the *Consequent*, and the first clause, "A is B," the *antecedent*.

Having regard to their *Quality*, as indicated in simple or

categorical propositions by the copula, and, in conditionals, by the copula of the consequent clause, propositions are divided into Affirmative and Negative. Having regard to their *Quantity*, that is to say, to the quantity of the subject, whether as consisting of the whole or only of part of the objects denoted by the term, propositions are divided into *Universal* and *Particular*. Having regard to the relation between the connotation of the predicate and that of the subject, propositions are divided into *Verbal* and *Real*, called by Kant, respectively, *Analytic* and *Synthetic*. Analytic propositions merely unfold the nature or meaning of the subject, without determining anything new concerning it. Synthetic propositions attribute to the subject something not directly implied in it, and so enlarge our knowledge.

*Exponibile Propositions* are propositions which, though not compound in form, need *exposition*. They are each resolvable into two or more independent and simpler propositions necessary to be set forth separately for logical purposes. Of these propositions there are four species, called, respectively, exclusive, exceptive, comparative, and inceptive or desitive propositions. Of exclusives, "Virtue alone is happiness below," may be taken as an example. This is equivalent to—(1) Virtue is happiness here on earth. (2) Nothing but virtue is happiness here on earth. "All except the wise man are mad" is an exceptive proposition; though turned thus—"The wise man alone is not mad," it becomes an exclusive. "To be deprived of a friend is the greatest of losses," and, "After the death of the Gracchi, Rome ceased to be free," are, respectively, comparative, and inceptive or desitive propositions.

Having regard to their *form*, Propositions are sometimes divided into *Pure* or *Absolute*, and *Modal*. A pure or absolute proposition is one that asserts something without any qualification, as "Henry is at home." A modal proposition is one that asserts something with some particular qualification, as "Henry is probably at home." We shall return to the consideration of modal propositions in § 34.

**Quality and Quantity.** § 29.—We have, in § 13, distinguished *propositions*, according to their *Quality* into Affirmative and Negative. The term, *Quantity*, as we have there explained, is employed in Logic in a very special sense. The quantity of a proposition is the same as the quantity of its subject ; and the quantity of its subject means the extent to which the predicate is affirmed or denied of the individual or individuals, thing or things, of which the subject is a name. If the affirmation or denial be made of the whole of the individuals included under the subject-term, the quantity is said to be *Universal* (or *Total*), as, "All the stars were twinkling," "Every man's house is his castle," "No minstrel's raptures swell"; if of part only, the quantity is said to be *Particular* (or *Partial*), as, "Some historians tell us so," "Some rare things are not precious."

In affirmative propositions the word *All*, in the sense of *Every*, denotes universality. In negatives, however, when the negative particle is regarded as part of the copula, these words, as well as the word *Some*, in both positives and negatives indicates particularity ; thus, "All were not saved," "Every river is not navigable," are equivalent to, "Some were not saved," "Some rivers are not navigable." But, if the negative particle be joined to the predicate so as to render the negative proposition constructively an affirmative, *All* prefixed to the subject-term denotes totality ; thus, "All were not-saved" is equivalent to "None were saved," which is a universal negative.

In the case of particular negatives, denoted by *Some*, prefixed to the subject or subject-term, the negative particle may be joined to either copula or predicate without altering the meaning. The force of *Some* in Logic is usually *Some at least*, not excluding the possibility of *All* ; so that "Some A is B" is compatible with "All A is B." Hamilton, however, having a distinct object in view, would understand by *Some*, *Some only* ; though the attachment of this meaning to the term leads to much inconvenience.

As to *Singular Propositions*,—in which the subject is

either a proper name, as "John Locke," or a common name with the demonstrative pronoun prefixed—such propositions are equivalent to universals of which they are regarded as forming a sub-class ; the subject-term being taken in its whole extent.

Besides *Some*, we often find other words prefixed to the subject in particular propositions, such as *Most*, *Few*, and the like, which require to be interpreted for quantity. Of these *Most* implies *More than half*, and *Few* is equivalent to *Most not* ; thus, "Few of the sailors were saved" is equivalent to the *Plurative*<sup>1</sup> proposition, "Most of the sailors were not saved."

When the quantity of a proposition is expressed by such words as *All*, *None*, or *Some*, the proposition is said to be *Definite*, or *Pre-designate*, as "All art worthy of the name is energy." When the subject is a general term, and no word is prefixed to it to indicate explicitly whether it is taken in the whole or in only part of its extension, the proposition is called *Indefinite*, *Indesignate*, or *Pre-indegnate* ; as, "Books are not absolutely dead things." As Formal Logic does not assume a knowledge of the *matter* of which the proposition treats, it is held by some logicians that propositions of this kind ought to be struck out of our list altogether. It is obvious, however, that in reality all such propositions must be either universal or particular ; and where, in any example, the quantity of the subject may be assumed to be sufficiently obvious the ellipsis may for logical purposes be filled up.

**Distribution of Terms.** § 30.—With regard to the distribution of the terms in propositions expressed in the ordinary way, very little need be added to what has been said in § 13. It was there shown that the *quantity of the predicate* depends on the *quality* of the proposition,—being *universal* for *negatives*, *particular* for *affirmatives* ; and that the quan-

<sup>1</sup> Propositions have been called *plurative* which give the distinct idea of the fraction or number of the subject about which assertion is made. Such propositions, however, are, in the view of Hamilton and others, not very important for philosophical purposes.

tity of the subject, and of the proposition, are of the same name, both universal, or both particular. From this it follows, that if, in a negative proposition, both terms be distributed, the proposition must be E ; if only one term, O ; if in an affirmative proposition a term be distributed the proposition must be A ; if neither, I ; the latter being the only one of the four forms in which no term is distributed.

**Quantification of the Predicate.** § 31.—Quantity, as explained in § 13, has reference to the subject ; and nothing has been said of the quantity of the predicate. But, according to Sir William Hamilton, Logic postulates, as a fundamental point, that we be allowed to state explicitly in language, all that is contained implicitly in thought ; and he and other logicians after him maintain that, in thought the predicate is always quantified, and therefore the quantity should, on demand, be expressed in language. Thus, *Predicates may be of any quantity in propositions of any quality.* The adoption of this principle, making the predicates explicitly universal or particular in each of the recognised four forms, A, E, I, O, gives us the following eight propositional forms denoted respectively by the letters annexed :—

- All X is all Y. (U)
- All X is some Y. (A)
- No X is any Y. (E)
- No X is some Y. ( $\eta$ )
- Some X is all Y. (Y)
- Some X is some Y. (I)
- Some X is no Y. ( $\circ$ )
- Some X is not some Y. ( $\omega$ )

If *some*, in these forms, be understood in the sense of "some, but not all," the meaning attached to it by Hamilton's pupil, Professor Baynes, the foregoing eight forms become reduced to five :—All X is all Y, Some X is some Y, All X is some Y, Some X is all Y, No X is any Y. But this use of *some* would be often found very inconvenient. "If, how-

ever, we employ the word in the strict logical sense of "some at least," that is, "some, it may be all," a number of these forms may be regarded as representing two propositions expressed in the ordinary way. Thus, All X is all Y, or, its equivalent, Every X is every Y, strictly speaking, means Every member of the class Y is a member of the class X; that is, Every X is Y, and every Y is X.<sup>1</sup>

Thomson adopts Hamilton's forms with the exception of  $\omega$  and  $\eta$ . His reason for excluding them is that, though admissible in a table including all conceivable cases of negative predication, they should have no place in a table of actual cases, as they have no practical utility. For example, "Some common salt is not *some* chloride of sodium" ( $\omega$  of the table), and "No birds are *some* animals" ( $\eta$ ), though they have the semblance, have not the force of negatives, since they do not prevent our making another judgment of the affirmative kind from the same terms. "Common salt is chloride of sodium," and "All birds are animals" are also true.

Mill rejects Hamilton's theory of the quantification of the predicate on two grounds:—

(1) Men do not quantify the predicate in thought as asserted; they think of its attributive meaning or comprehension, not of its extension.

(2) All reasoning is carried on in the ordinary forms of

<sup>1</sup> See Monck's *Introduction to Logic*, pp. 141-158. It is contended that Hamilton's theory was anticipated by Mr. George Bentham in his *Outline of a New System of Logic* (1827). Bentham enumerates the eight forms of Hamilton, which he reduces to six. Indicating *identity* by the sign  $=$ , *diversity* by  $\asymp$ , *universality* by the letter  $t$  (*in toto*), and *partiality* by  $p$  (*ex parte*), these forms are as follows:—

$X \text{ in toto} = Y \text{ in toto.}$	$tX = tY.$
$X \text{ in toto} = Y \text{ ex parte.}$	$tX = pY.$
$X \text{ in toto} \asymp Y \text{ in toto.}$	$tX \asymp tY.$
$X \text{ in toto} \asymp Y \text{ ex parte.}$	$tX \asymp pY.$
$X \text{ ex parte} = Y \text{ ex parte.}$	$pX = pY.$
$X \text{ ex parte} \asymp Y \text{ ex parte.}$	$pX \asymp pY.$

*Outline, &c.*, p. 133.

The reduction of propositions to these forms renders, he points out, the rules for distribution unnecessary. But see Venn's *Symbolic Logic*, Chapter I., p. 8, footnote.

expression. Every proposition in logical form should be the exact equivalent of one of these, which is not so with Hamilton's types; and these types should express the most elementary judgments, which some of them do not, being complex in character.

**Import of Propositions.** § 32.—By the *Import* of a proposition is meant the most generalised expression of the relation between the subject and the predicate. As to the import of the subject and predicate themselves, they may be looked upon in different lights, according to the views taken of Logic as a science:—

(1) As concepts, not necessarily corresponding to really existing things, or having any counterpart among really existing things.

(2) As concepts corresponding to really existing things.

(3) As standing for really existing things themselves. This latter view, taken as a basis for the treatment of Logic as a whole, is not easy to follow consistently throughout, and need not be dwelt on further.

As to the relation between the subject and the predicate, considered in its most general meaning, the principal theories held by logicians are the following:—

(1) *Hobbes* compares the subject and predicate as to denotation, and considers that the meaning of a proposition is the belief of the speaker that the predicate is a name for the same thing of which the subject is a name. This account, according to Mill, is sufficient only in the case of those propositions of which both the subject and predicate are proper names, since it overlooks the connotation of names and, in *class-names*, the real meaning is to be found in the connotation.

(2) The so-called *Class Theory* also is based upon the denotation of the subject and predicate. It consists in referring something (whether itself a class or an individual) to a class,<sup>1</sup> or in excluding it therefrom. For example, "Man

<sup>1</sup> The term *Class* has two meanings, the *class definite*, an enumerated group of individuals, each possessing some common character; and the

"is mortal" asserts that the class "man" is included in the class "mortal," or enrolled in what we may call the list or register of "mortal beings," and so on. Between this theory and that of Hobbes there is, as Mill points out, no real difference; and its incorrectness is manifest, seeing that a complete list of the sub-classes, or of the individuals forming a class, is not the first thing in the order of thought, nor can we say it is even certainly within our reach. We decide generally whether a thing belongs to a class by testing it for a given attribute, or group of attributes, not by referring to a class-list.

(3) The *Equational Theory*. This is closely related to the preceding. According to it, "A is B," means that the things denoted by A are equal to the things denoted by B. But propositions are distinguished from algebraical equations in that algebraical terms include all beneath them taken collectively, logical terms, distributively; so that while "A is B," read as equivalent to the equation, "A = B," means that "the total of things making up A is equal to the total making up B"; read as an ordinary proposition, it means that "every individual that is A is identifiable with some one or other of the individuals that are B."

(4) The *Inhesion Theory*, in which the subject is taken in denotation, the predicate in connotation. According to this theory propositions express the relation of substance and attribute; the attribute, in Aristotle's way of expressing it, *inhering* in the subject. Thus, in the proposition, "Man is an animal," "Man is animal," the predicate, "animal," may be said to inhere in the subject, "man." So, in saying, "Birds are warm-blooded," the attribute, "warm-blooded," may be said to inhere in the subject "birds." But it is objected that this theory is not sufficiently far-reaching, and

*class indefinite*, an unenumerated group of individuals or things, each distinguished by the class-mark or marks. We may regard predication of either class-name, however, as, in the analysis of its import, predication of an attribute, or group of attributes; without prejudice to any view that would look beyond such analysis to a more complete generalisation.

that propositions cannot all be regarded as expressing merely the relations of substance and attribute. They may express various other relations, such as those of time and space, of resemblance and difference, and so on, leading up to other types of reasoning besides those founded upon the relation forming the basis of this theory.

(5) The *Connotation Theory*, based upon the connotation of the subject and predicate. According to the Connotation Theory, which is that held by Mill, we arrive at the highest generalities of predication by referring the subject to one or other of the classes of nameable things. Thus, when we predicate of anything a name, whether abstract or concrete, we affirm in propositions, considered generally, not an agreement or disagreement between the meanings of two names, or a relation between two ideas, but, that the case is one of existence, co-existence, sequence, causation, or resemblance. It is not established, however, that this classification is exhaustively inclusive, a point which completeness demands. Mr. Bain modifies Mill's account of predication by laying down, that by predication attributes of the mind, or the subject, and of matter, or the object, "are declared as *agreeing* or *disagreeing in quantity*, as *co-existing*, or as *successive*.

(6) *Hamilton's View*. According to Hamilton, predication consists in pronouncing that of two notions thought of as subject and predicate, the one does or does not constitute a part of the other, (a) in the quantity of extension, or (b) in that of comprehension. Thus, if we say, "A is B," we mean that, in comprehension, B constitutes a part of A, and in extension A constitutes a part of B. Of course the part merges in the whole when A and B are but two names for the same thing.

**Verbal and Real Propositions.** § 33.—As has been explained, a *Verbal*, or, as it is also called an *Analytic*, *Explicative*, or *Essential Proposition*, is one in which the connotation of the predicate is a part, or the whole, of the conno-

tation<sup>1</sup> of the subject, and which unfolds the nature of the subject without determining anything new concerning it, as, "A triangle is a three-sided figure." It simply gives the meaning of a name, or couples a name and a thing, not two things, or two facts. A *Real, Synthetic, Ampliative, or Accidental* Proposition, on the other hand, is one which predicates of the subject some attribute not connoted by it, or couples two things or two facts not hitherto associated, thereby enlarging our knowledge; as, "The works of Hobbes contain some mathematical disquisitions"; "Heat is a mode of motion."

Mill points out that there are two different aspects in which we may regard such real propositions as are general, namely, either (1) as portions of speculative truth, or (2) as memoranda for practical use. For example, such a proposition as, All men are mortal, may be understood as meaning, (1) The attributes of man are always accompanied by the attribute, mortality; or, (2) The attributes of man are *evidence of*, or *a mark of* mortality. The latter mode of exhibiting the import of the proposition serves best, in Mill's opinion, to express the function propositions perform in reasoning, as most distinctly suggesting the manner in which they may be made available for advancing to other propositions.

The distinction between verbal and real propositions has been impugned on two grounds:—

(1) That real propositions are merely the result of a more complete examination of the general notion than such as are merely verbal; and, therefore, that whatever they assert lies

<sup>1</sup> According to the definition, propositions respecting individuals, as "Homer wrote the Iliad," cannot be regarded as verbal, since proper names have no connotation. This proposition, however, Mr. Bain classes as verbal, on the ground that we know nothing of Homer except as having the authorship of the Iliad attributed to him. The names, Essential, and Accidental, as applied to Verbal and Real propositions, respectively, are relics of the older metaphysical doctrines, according to which everything has its *Essence*, or that which makes it what it is; so that an Essential proposition, or one that unfolds the essence, was supposed to go deeper into the nature of the thing than any other. Accidental propositions predicated but properties or accidents of the thing, which were supposed to have little or nothing to do with its inmost nature.

in the notion as brought before the mind by the subject of the proposition, and are discoverable in it if it be examined with sufficient closeness. For example, it might, for this reason, be held that the difference between the verbal proposition, "A right-angled triangle is a triangle one of whose angles is right," and the real proposition, "The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides," is merely one of degree, having reference to the greater or less completeness of our notion of a right-angled triangle.

(2) That a real proposition is merely a step on the way towards the most complete analysis attainable of the subject-term, and that, when this is reached, connotation of the subject will, as a matter of ordinary knowledge or belief, come to include the predicate, and so the proposition will become analytical.

As to the first of these objections, it may be answered that when, as in most cases, the connotation of the subject-term is indefinite, each person forms his own notion of that connotation, independently of the information conveyed in a given real proposition; and, therefore, in such a proposition, the predication goes beyond the range of the subject to him. As to the second objection, that, with advances in knowledge the subject acquires a wider meaning, and a proposition once real becomes analytical, it may be urged that though the words remain the same, the essence of the proposition, namely the statement of fact conveyed by it, is so altered that the proposition becomes virtually a new one.<sup>1</sup>

<sup>1</sup> See T. H. Green's *Philosophical Works*, vol. ii., "On Verbal and Real Propositions," Monck's *Introduction to Logic*, p. 119, and Bain's *Logic*, i., 69.

Verbal and Real Propositions are also respectively called *à priori* and *à posteriori* propositions. Kant asserts that some Real (synthetical) propositions are virtually *à priori*, but yet introduce into the predicate something not to be found in the subject; as, for example, "A straight line is the shortest line between two points." "7+5=12." Some logicians maintain, however, that, while these examples may be accepted as *à priori*, they are also analytical or verbal in character. It may be pointed out that Kant's expression as rendered, "something

**Modal Propositions.** § 34.—The *Modality* of a proposition may be defined to be the degree of certainty we attach to the judgment expressed by it. A *Modal Proposition* is one which emphasizes by the introduction of a qualifying word, or otherwise, the degree of certainty with which the judgment expressed by it is made and maintained, or the mode or measure in which we hold it to be true; as, "A is B"; "A is certainly B"; "A is probably B."

In a modal proposition the assertion is called the *dictum*, and the word indicating the kind of modality, or the kind of modality itself, the *mode* or *modus*. Thus, in the proposition, "A is certainly B," "A is B," is the *dictum*, and "certainly" is the *modus*. And the given proposition may, from one point of view, be regarded as implicitly compounded of the following:—(1) "A is B"; (2) "That A is B is certain."

Some logicians exclude the subject of modals altogether from Pure Logic, as being a subject concerned with the matter of propositions and not with the form. Of those who admit it, some contend that the modality properly belongs to the copula, some, that it belongs to the predicate. Dr. Fowler considers it simpler and more scientific to attach the modality to the predicate; Mill considers it most proper, in expressing distinctions of time, as the sun *did* rise, the sun *is* rising, the sun *will* rise, to attach it to the copula, inasmuch as what we

not to be found in the subject," affords an immediate basis for an argument in favour of a thus far divergent view.

A Verbal is to be distinguished from what is called a Tautologous proposition. The former gives the *matter* of the subject in a new *form* in the predicate; the latter gives us the same matter in the same form, as "Whatever is, is." Purely tautologous propositions are valueless in thinking; but a very slight change of meaning given to the predicate, whether by emphasis or by conventional implication, may alter the character of the proposition, so as to render it fit for certain uses a convenient formula. In the light of a convention of this kind, even "A is A," or, "Whatever is, is," may cease to be tautologous.—The observation of Marcus Aurelius, that "Matter is ever in a flux," might suggest another application. But Thomson's criticism on such propositions (*Laws of Thought*, p. 143), in respect of their mere logical import is, of course, perfectly just. See Locke, *On the Human Understanding*, I, ii., 4, IV., vii., 4, IV., viii. (Trifling Propositions), and Locke's *Second Reply to the Bishop of Worcester* (Stillingfleet).

affirm in these propositions is not what the subject signifies, nor what the predicate signifies, but specifically and expressly what the predication signifies (*Logic*, vol. i., p. 89). Bain, however, would regard all such propositions as Mill has in view as compound ; asserting, first, a fact, and, secondly, the time of its happening,—a treatment which may be extended to many other kinds of modals as well.

Aristotle distinguishes three kinds of Modals, namely, the *Assertorial*, the *Apodictical* (*ἀποδεικτική*, demonstrative), and the *Problematical*.<sup>1</sup> The scholastics divide such propositions into four classes, the *Necessary*, the (*Fortuitous* or) *Contingent*, the *Possible*, and the *Impossible*, which, however, may be regarded as all in reality included in the first two classes—the *Necessary* and the *Contingent*. Having regard to the scholastic, or four-fold division, a modal proposition, when it predicates of the subject what is necessary and unchangeable, as for example, in asserting a geometrical truth, is distinguished as necessary ; when it predicates what happens to be true at a given time, but which might have been otherwise, as contingent,—as, in the case, for example, of a proposition in physical science ; when it predicates that which, though not true at a given time, may be, at some other time, it is distinguished as possible ; and when it predicates that which cannot be true at all, impossible. This classification is evidently objective in character.

Kant, after Aristotle, distinguishes modals into the *Assertorial*, the *Apodictical*, and the *Problematical*. The *Assertorial* are propositions established by experience, and true as far as experience extends, as, “A is B,” “This colour is red,” “Man is mortal” ; the *Apodictical* or *Demonstrative*, are propositions universally and necessarily true ; as “A *must be* B,” “An equilateral triangle is (by demonstration, *must be*) equiangular” ; the *Problematical* are propositions that may be true under certain circumstances, not under others. These several kinds of modality, it is observed, add nothing new to

<sup>1</sup> Πάσα πρήτασίς ἔστιν ἡ τοῦ ὑπάρχειν ἡ τοῦ ἐξ ἀνάγκης ὑπάρχειν ἡ τοῦ ἐνδέχεσθαι ὑπάρχειν.—*An. Or.* I. i., 2.

the subject or to the predicate, affecting only the copula; and the classification, as contrasted with the scholastic, is to be regarded as subjective in character.

Dr. Venn considers that Modality can be adequately treated only as a chapter in the Logic of Chance or Probability.<sup>1</sup>

<sup>1</sup> See Venn's *Logic of Chance*, 3rd edition, pp. 295-300.

## CHAPTER II

### ON THE PREDICABLES, DEFINITION, AND DIVISION

Aristotle's Classification of Predicables. § 35.—The Predicables are to be carefully distinguished from the Categories: the former, in scholastic phraseology, refer to what have been called *second intentions*, the latter to *first intentions*.

The Predicables may be regarded as a classification of predicates according to the relation in which they stand to their respective subjects in affirmative categorical propositions.

According to Aristotle, the Predicables are four in number, namely, Genus (*γένος*), Definition (*ὅρος*), Property (*ἴδιον*), and Accident or Concomitant (*συμβεβηκός*). For, predicates are either capable of becoming subjects in their respective propositions without limitation, or they are not. If they are, either they express the whole connotation of the subject, or they do not. In the former case, the predicate in relation to the subject is a Definition,<sup>1</sup> in the latter a Proprium or Property. If they are not capable of becoming the subjects without limitation, either they express part of the connotation of the subject, or they do not. In the former case, the predicate in relation to its subject is a Genus, in the latter, an Accidens or Accident. Aristotle's four classes are thus reducible to two, namely, Substitute and Attribute; for,

<sup>1</sup> Sometimes the whole proposition, and sometimes only the predicate is called in relation to the subject, a definition. It will be seen that here, by definition is to be understood, the single term, or predicate of the proposition the subject of which is the term to be defined.

Definition and Property may be classed under the head of Substitute, and Genus and Accident under that of Attribute.

Considered in connotation, *Definition* is a mark, or group of marks or attributes, unfolding the entire nature of the subject ; *Property* is a mark or attribute that belongs to the subject peculiarly, or exclusively, but which is not among those explicitly set forth in its definition ; *Genus* is a mark or attribute the subject has in common with other subjects ; *Accident* is an attribute that belongs to the subject, but not exclusively, and which does not *appear* to depend in any way on the attribute or attributes named in its definition.

We shall presently see the precise relation of this classification to the later views on the Predicables.

**Five-fold Classification.** § 36.—The Doctrine of Predicables, as it is more usually presented, owes its special features to Porphyry, a philosopher of the third century, and to the scholastic followers of Aristotle. It will be seen that, according to Aristotle's distinctions, the class of things bearing each the common mark or attribute, Genus, was called in respect of the subject, a Genus ; and that definition was to be regarded as a class co-extensive with the subject. But it came now to be maintained that nothing could be defined but a species, or sub-class, and, therefore, Definition was rejected as a predicate, and in its stead were substituted the two heads, Species and Difference. The Doctrine, besides, assumed a thorough-going Realistic<sup>1</sup> character. It was held that to every common

<sup>1</sup> We have already (in § 11 and its footnote) drawn attention to the distinguishing features of Nominalism and Realism. The scholastic controversy is said to have owed its origin to the following passage in Porphyry's Introduction to Aristotle's Categories :—"On such questions as these, whether genera and species exist in nature, or are only conceptions of the human mind ? and (on the supposition that they exist in nature) whether they are inherent in the objects of sense, or disjoined from them ? he declines to give any determination. But Realism existed long before. Plato is said to have held what is called the *Ultra-Realistic* doctrine that Universals are the only real existences ; the schools before Roscelinus, and the *Realist* schoolmen after him, that Universals exist, independently of things and of our ideas of them, in the Divine Intellect. The *Moderate Nominalists*, on the other hand (including the *Conceptionalists* as an offshoot), have held that Universals

name there corresponds what was called an *Essence*, possessed of a real existence, and that a predicate represents either the whole essence, or part of the essence, or something conjoined to the essence. Regarded thus, the Predicables, five in number, *Genus*, *Species*, *Difference*, *Property*, and *Accident*,—commonly alluded to in scholastic literature as the five Universals, have been defined as follows:—

*Genus* is a predicate which is considered as the common material part of the species of which it is affirmed.

*Species* ( $\epsilon\iota\delta\sigma$ ) is a predicate which is considered as expressing the whole essence of the individuals of which it is affirmed.

*Difference* ( $\delta\iota\alpha\phi\rho\alpha$ ) is the formal or distinguishing part of the essence of a species.

*Property* is a predicate which denotes something essentially conjoined to the essence of the species (*de inesse*, that is in our conception of it).

*Accident* is a predicate which may be present or absent, the essence of the species remaining the same.

Of these predicates, Genus and Species are, in scholastic language, said to be predicated *in quid*, that is to say, to answer to the question, *What?*—Genus, *in quid incomplete* and Species, *in quid complete* (as, for example, What is man? An animal; What is Caesar? A man); Difference *in quale quid*; Property and Accident, *in quale*—Property *in quale necessario*, as necessarily joined to the whole species, and Accident *in quale contingentur*, as belonging to *some* individuals only of the species. When we inquire what kind of animal is Man? the answer, *rational*, is the *Differentia* or *Difference*. As a *Proprium* of Man, Aldrich gives *risibility*. Of Man, to be *walking*, or a *native of London, York*,

exist as a product of the mind only, as notions constructed by the aid of language; the Ultra-Nominalists, that Universals are mere names. Of Aristotle, whether a Realist, a Conceptualist, or a Nominalist, the precise position is not very easy to determine.

Whately holds that no common terms have any real thing in nature corresponding to them; each of them being merely a sign denoting a certain inadequate notion that our minds have formed of an individual.

*Edinburgh*, or other place, are Accidents,—the former, Separable, the latter Inseparable.<sup>1</sup>

**Kinds.** § 37.—The Aristotelian logicians distinguished between Species and Genus in the following way:— When any class differs from another by *an infinite series of differences, known and unknown*, common and peculiar to all its members, they considered the distinction as one of *Kind* and called it an *Essential Difference*.<sup>2</sup> Thus, *man* is distinguished from *brute* by a multitude of differences which make up a distinction in Kind. And, considering the Individual, the proximate or lowest kind to which it is referable is its Species. Thus, we should refer the individual, John Locke, to the species *man*, excluding such sub-classes as Englishmen, metaphysician, which are marked by attributes that can be clearly enumerated. And it is to be noted that though the number of distinguishing attributes should be very large, yet if a definite number of primary differences account for all the rest, the distinction is not accounted one in Kind, and the class so distinguished is not a real Kind. Thus, we may say that, *Every class which is a real Kind*, that is, which is distinguishable from all other classes by an indeterminate number of properties not derivable from one another, *is either a Genus or a Species*. Of these, the proximate Kind, or *Infima Species*, is a kind not divisible into Kinds; and every other kind predicable of the individual, being a larger class including the proximate Kind and more, is a *Genus*. As to kinds that admit of division into real kinds, they are genera to all Kinds below them, but species to all genera in which they are themselves included; that higher class which

<sup>1</sup> See Zigliara's *Summa Philosophica*, vol. i. (Logica, p. 23), and Whately's *Logic*, p. 84.

<sup>2</sup> See Mill's *System of Logic*, vol. i., p. 139. Differences that extended only to a certain number of properties of things and terminated there, the schoolmen regarded as *differences only in the accidents*; hence difference in kind was much more radical, and in Mill's view justified them in drawing the line between the two classes. Again, while, to the *naturalist*, two things are not of different kinds, or species, if they have come from a common stock, to the *logician* they are, if the differences between them are innumerable, and not referable to a common cause.

is not considered a species of any higher genus being called the *Summum Genus*. A *Subaltern Species* or *Genus* is one which is both a species of some higher genus, and a genus in respect of the species into which it is itself divided. *Cognate* or *Co-ordinate* species are those that fall immediately under the same genus. A *Cognate Genus* is any one of the superior genera under which the species falls. A *Specific Difference* is one that distinguishes a given species from its cognate species; a *Generic Difference* is one that distinguishes it from any of its cognate genera; a *Specific Property* is one derived from the attributes of the given species; a *Generic Property* is one derived from any of its cognate genera. If, for example, we take *Plane Figure* as our *Summum Genus*, we have, as cognate species, *Curvilinear Figure*, and *Rectilinear Figure*; and of the latter, *Rectilinear* is the specific difference. Passing on downwards to the species under *Rectilinear Figure*, namely, *Triangle*, *Quadrilateral*, *Pentagon*, *Hexagon*, and so forth, and taking under *Triangle*, the species *Equilateral*, *Isosceles*, *Scalene*, we see that for *Equilateral Triangle*, *Three-sided* is a generic, and *Equilateral*, a specific difference. Again, it is a generic property of an *Equilateral Triangle* that its three angles are together equal to two right angles, and a specific property that the circular measure of any of its angles is one third of  $\pi$ . And, *Equilateral Triangle*, is the lowest general term we have in the series. *Summum Genus* and *Individual* are, then, the extremes of the classification; and the series of classes from *Summum Genus* to *Infima Species*, inclusive, form what has been called a *Predicamental Line*.

**Essence of the Species.** § 38.—Glancing once more at the list of *Predicables*, and the definitions following, we are reminded that, according to the realistic view, Difference must be of the essence of the species. This supposes one distinct and distinguishing quality, which, added to the genus, makes up the species. When we say "Man is an animal," there are various attributes by which we might distinguish the species *man* from the others that go to make

up the genus *animal*, but not every one of these would be accounted by the schoolmen the differentia. *Rationality*, however, is the usually accepted differentia of *man*; and this one, separate, and distinguishing quality the realists regard as of the essence of the species. *Proprium* and *accidens*, in the language of the schoolmen, form no part of the essence, and are predicated of the species only *accidentally*. *Proprium* is predicated *accidentally*, indeed, but *necessarily*; that is, it signifies an attribute which is not part of the essence, but which flows from or is a consequence of it, and is inseparably attached to the species. It belonged to the whole species of which it is predicated, and to that alone, and at all times. *Proprium convenit omni soli et semper. Accidens est quod abest est adest sine subjecti interitu.* Accident may be present or absent, the essence of the species remaining the same; as, for example, when we assert of a person that he is *reading*, or that he is an *American*.

Mill on the § 39.—Mill regards the Predicables as five different varieties of class-name based on the relation the predicate bears to the subject of which it happens to be predicated. The distinction between Genus and Species is founded to an extent on the nature of things; not so, that between Difference, Property, and Accident, it being founded on the connotation of names. Noting that the terms, Genus and Species, are taken in a *popular sense* to denote any two classes one of which includes the whole of the other and more, Mill adopts, as the more philosophic, the more restricted meaning with which these terms were employed by the Aristotelians. As to the essence of a class, in the scholastic sense, it is represented by the attributes of the class that are involved in the signification of the class-name; in other words, by the connotation of the name; or the real is represented by what Locke calls the nominal essence. To say that Genus and Difference are of the essence of the subject means in this view nothing more than that the properties signified by the Genus and those signified by the Difference form part of the

connotation of the name denoting the Species. Difference<sup>1</sup> or *Differentiae* is that part of the connotation of the name which distinguishes the species in question from all other species of the genus to which, on the particular occasion, we are referring it. The plural form, *Differentiae*, is employed, if the distinguishing part consists of more than one attribute. If we regard as completely determinable the attributes of the Species and those of the Genus, the *Differentiae* will consist of those attributes which with those in the connotation of the Genus make up the connotation of the Species. Property is an attribute which belongs to all the individuals included in the species, and which, though not connoted by the specific name, yet follows from some attribute that name either ordinarily or specially connotes. It may follow from that attribute in either of two ways, namely, as a conclusion follows premisses, or as an effect follows a cause<sup>2</sup>. Accident is an attribute of a thing neither involved in the signification of the name, nor having, so far as we can see, any necessary connection with the attributes so involved. Accidents are of two kinds—Separable and Inseparable. Separable Accidents are those which are sometimes found to be absent from the Species; being not only not necessary, but also not universal, as the *colour* of a European. Inseparable Accidents are properties which are universal to the Species, but not necessary to it; as, for example, the *blackness* of a crow.

<sup>1</sup> Difference, as we have seen, does not occupy a distinct position in Aristotle's list of Predicables. It is said, however, to belong naturally to Genus ( $\omega\varsigma\ o\delta\pi\alpha\ \gamma\epsilon\nu\iota\kappa\hbar\tau\iota\varsigma$ ). This observation Thomson supposes to mean, that species, instead of being regarded as Genus *plus* Difference, is more properly to be regarded as covered by the overlapping parts of two Genera of which the Difference is one.

<sup>2</sup> In either case, it follows *necessarily*; that is to say, its not following would be regarded as inconsistent with some law relating either to mind or matter. Dr. Fowler, defining the heads of predicables as so many kinds of terms, of which genus and species are common terms, and differentia, property, and accident, attributives, appears to find his nominalistic treatment a shade inadequate in defining Property: it is an attribute, and, therefore, a *mere word*, and yet it is said to follow from part of the connotation of the common term "either as an effect from a cause, or as a conclusion from premisses." (*Deductive Logic*, p. 45.)

**Definition.** § 40.—Definition is an expression that explains the meaning of a word, the nature of an object, or the way in which a thing is produced. By such expression, that which is defined (the *Definutum*) is separated as by a boundary from every other object of thought. It may also be regarded as the analysis of a whole of comprehension<sup>1</sup> or intension ; that is to say, the resolution of such a whole into its elements ; one word which connotes a set of attributes collectively being replaced by two or more which connote the same attributes singly or in smaller groups.

According to the Logic of the schools, definitions are of two sorts, Nominal and Real ; the latter being divided into Accidental and Essential, and the Essential into Metaphysical and Physical.

1. A Nominal Definition is one that explains only the meaning of a Name ; as, " Probity is honesty."

2. A Real Definition is one that explains the nature of the Thing defined, beyond what is necessarily understood by the mere Name, or its synonym.<sup>2</sup>

(1). An Accidental Definition, otherwise called a Description, is one that assigns the properties of a Species, or the accidents of an Individual. It is the only way by which we can define an Individual.

(2). An Essential Definition is one that assigns, not the properties or accidents of the Thing defined, but what are regarded as its essential parts, whether physical or logical.

(a). A Physical Definition is one that unfolds the extension

<sup>1</sup> Also called a metaphysical whole, as contrasted with a whole of extension, which is called a logical whole. Thomson enumerates six sources of definition :—(1) The analysis, and (2) the synthesis of the intension of a concept ; (3) the analysis, and (4) the synthesis of its extension ; and the accidental coincidence (5) of a symbol, and (6) of notation. The first of these gives Definition Real, the fifth and sixth, respectively, Definitions Nominal and Accidental.

<sup>2</sup> A peculiar kind of Real Definition, called *Genetic Definition*, applies only to quantities in time and space. The following is an example in mathematics :—"A circle is formed when we draw around always at the same distance from a given point a movable point which leaves its trace, until the termination of the movement coincides with the commencement.

of the concept of the Thing, or assigns the parts into which the Thing can be actually divided ; as, *Tree*, into Root, Trunk, and other parts, *Proposition*, into Subject, Predicate, and Copula.

(b). A Metaphysical or Logical Definition is the definition of a Species by the naming of its proximate genus and specific, or essential, difference. This is the regular Real Definition, *per genus et differentiam*, of the scholastics, and into it there enter the first three of the predicables. If *differentia*, however, be taken to mean, not the whole of the peculiarities in the species over and above the genus, such a definition would be *incomplete*, and the complete definition would be, *per genus et differentias*, instead of *differentiam*. Of Summum Genus, which has no class superior to itself, we cannot have a Real, though we may have a Nominal Definition.

**Mill on Definition.** § 41.—According to Mill, the simplest and most correct notion of a Definition is that of a proposition declaratory of the meaning of a word. Hence, words which have no meaning are unsusceptible of definition ; and, accordingly, proper names—regarded as not connotative—cannot be defined. The only non-connotative names (§ 21) which can be defined are the names of single attributes. In the case of connotative names, the meaning is the connotation ; and the definition of a connotative name is the proposition which declares its connotation. The only adequate definition of a name is one which declares the facts, and the whole of the facts, which the name involves in its signification. It follows that a distinction of Definition more in harmony with modern views than that into Nominal and Real is one into *Complete* and *Incomplete*.<sup>1</sup> A Complete

<sup>1</sup> A Definition may undertake to explain (a) the content of an ordinary current conception, (b) a conception not current, but which the definer undertakes to constitute, as (1) in Law, for example, manslaughter, (2) in Morals, various duties, (3) in Mathematics, as for example that of the circle, representing the mental act of constructing it, (c) a conception representing the definer's discovery of facts of nature, or his analysis of metaphysical conceptions. In the case of (a) and (c), completeness of definition is impossible.—See Mr. T. H. Green's *Philosophical Works*, vol. ii., p. 236.

Definition, *in the philosophical sense*, is, as has been just observed, a full statement of all the properties connoted by the name. Such is Mr. Bain's view as well, but with the implied qualification that we must consider the properties connoted by the name to include not all the common properties of the class named but such as are ultimate and irresolvable into one another.

Incomplete Definitions are of two kinds:—

(1) Such as define a connotative term by *a part* only of the connotation, but a part sufficient to mark out correctly the bounds of its denotation. These, *in a popular sense*, are usually regarded as perfect definitions.

(2) Such as define the name of a class by accidents—that is, by attributes not included in its connotation. These are popularly excluded from perfect definitions and named *Descriptions*.

Sometimes, however, what would otherwise be regarded as mere descriptions are raised to the rank of definitions by the object the writer has in view. Scientific definitions, for example, are generally descriptions, serving as landmarks of scientific classification: thus, Cuvier towards his classification of the animal kingdom defines *man* as, "a mammiferous animal having two hands." Moreover, since scientific classifications are being continually modified as science advances, such definitions are liable to constant variation; and this holds true, not merely with regard to terms in any given science, but to the definition of the science itself. Hence, the definition of a science must necessarily be of a provisional kind.

As to the distinction drawn by the Aristotelians between definitions of names and definitions of things, Mill holds that *all* definitions are, strictly speaking, of names only; but yet that the distinction in question has a meaning in that the so-called definition of a thing, along with the meaning of the name covertly asserts, as a postulate, or implies, a matter of fact, namely, the actual or possible existence of a thing.

possessing the combination of attributes set forth by the definition.

As to the definition *per genus et differentiam*,—the Real Definition, or Définition proper of the Aristotelians, interpreted in connotation it can no longer be regarded as the only definition; neither can it be regarded as possible to fulfil by definition one of the objects they seem to have had in view in insisting on this form, namely, to express in words the nature of a Kind, seeing that the very meaning of a Kind implies that the properties that distinguish it cannot be enumerated.

In the framing of a definition for objects which are to be denoted by a general name, we must give a distinct connotation to the name by ascertaining and specifying the common attributes; and such of these are to be chosen for the purpose as are of the greatest importance for their number, or the consequences that result from their presence in the objects. There should be chosen “such *differentiae* as lead to the greatest number of interesting *propria*”; for these, and not the more obscure qualities, determine the groups into which the objects naturally fall, and are the landmarks of definition.

**Rules for the Definition.** § 42.—The following are substantially the Rules usually laid down for definition:—

- (1) A definition must recount the essential attributes of the definitum, or thing defined (*Definitio fiat per notas rei essentiales*).
- (2) A definition must be precisely adequate to the species defined (*Definitio sit adaequata, neque latior neque angustior suo definito*).
- (3) A definition must not contain the name of the thing defined.
- (4) A definition must not be expressed in obscure, or figurative, or ambiguous language.
- (5) A definition must not be negative when it can be affirmative.

The matter of these rules is given by Whately as follows:—

A definition must be (*α*) adequate, and (*β*) plainer than the thing defined ; the latter characteristic including what is sometimes given as a third rule, namely, (*γ*) a definition should be couched in a convenient number of appropriate words. And he notes that a defect in definition arising from inadequacy cannot be remedied by making an *arbitrary exception*,<sup>1</sup> as for example, by defining "Capital" as any "property of such and such a description, *except* land."

Rule (1) points to the general process of defining as being by the genus and the differentia or differentiae. The essential attributes are the connotation of the term that stands for the thing. The rule is isolated if an attribute or attributes, not essential, or not forming part of the connotation, be stated in the definition, as "Man is a cooking animal,"—an *accidental* definition or description. Of the like character are the following :—"Man is a featherless biped," "A tree is a vegetable organism, having roots, branches, leaves, &c." The rule is, of course, violated also, if either more or less than the essential attributes be included in the definition. The term, Description, is applied to cases in which part of the connotation is combined with properties and accidents ; as, "Magnesium is a metal capable of ignition."

Rule (2) is violated, if the terms constituting the definition, taken together, connote more or less than the term defined, or if the definition—in the Aristotelian sense of the term, namely as a predicate—and the definitum are not convertible or reciprocal, that is to say, such that we may interchange them as subject and predicate at pleasure. "Man is a civilised rational animal" is at once seen to involve a violation of this rule. Again, if each of the terms constituting the definition has not a connotation distinct from, or underivable from the others, there will be a redundancy or tautology,<sup>2</sup>

<sup>1</sup> That is, one without a sufficiently clear ground.

<sup>2</sup> Tautology consists in asserting too much, not in mere words but in sense,—stating something *already implied*. The effect of such a statement, on the principle of "exceptio probat regulam" (an exception proves a rule), would be to lead to a supposition that the implicatory part might be independent of the coexistence of the implied part as

and a violation of this rule ; as if we should define a parallelogram as, "a four-sided figure whose opposite sides are parallel *and equal*."

Rule (3) is violated by such an example as, "A triangle is a figure that has any of its exterior angles equal to the interior and opposite angles" ; Rule (4) by such as, "The soul is the first *entelecheia* (*ἐντελέχεια*, actuality) of a natural body" ;<sup>1</sup> and Rule (5) by such definitions as, "Evil is that which is not good," "A point is that which has no parts and no magnitude," supposing it possible to find others of the affirmative form.

In addition to the foregoing rules which relate specially to Real Definition the scholastics give the following obvious cautions relative to Nominal Definition :—No attempt should be made to define a term which is so well understood that no more intelligible term is available to explain it ; no term in any measure obscure or equivocal should be employed without defining it ; and no terms should be used in defining except such as are perfectly understood, or have been already explained. Real Definition, like Nominal, has its limitations. Not every notion is susceptible of definition. What is perfectly clear in itself might be but obscured by definition. The simple notions, and certain very general or universal notions—called *transcendental*, by the ancients, do not admit of definition *per genus et differentiam* ; the first, such as *Existence*, *Unity*, and the like, because they do not include many attributes ; the second, because, like *Being*, *Truth*, *Beauty*, and so forth, we know no higher genera containing them. For the opposite reason, there can be no real definition of the individual ;—only an accidental definition or description. The individual is described ; the Species is defined.

explicitly stated. The force of the maxim just mentioned Whately explains to be that the mention of any circumstance, whether *exception* or other, introduced into the statement of a definition, a precept, a law, a remark, and the like, is presumed to be *necessary* to be inserted ; so that the definition, precept, law, &c., would not hold good if this circumstance were omitted.

<sup>1</sup> Aristotle's definition, quoted in Reid's *Inquiry*.

**Division.**      § 43.—The order of treatment between themselves of Definition and Division has long been a point of dispute. According to some logicians, Division should precede Definition ; and on material grounds this might be defended, the matter of a definition being sometimes gained by division. According to others, however, the reverse order is preferable, and this on formal grounds ; a formal division being possible only after definition.

Division is the distinct setting forth of the several parts that compose a whole. There may be various wholes ; but those in more marked contrast for our present purpose are, the verbal, or nominal wholes, and the physical, metaphysical, and logical, called also real wholes. Division is nominal or real according to the wholes it relates to. It is nominal, or verbal, when it distinguishes the different significations of an equivocal term, or a verbal whole. It is real, when it relates to physical, metaphysical, or logical wholes. A physical whole, which is always a concrete individual object, is composed of physically separable parts ; as, a sword into blade and hilt ; and the process of separation into such parts is called Physical Division, or Partition. A metaphysical whole consists of the aggregate of qualities of a concrete individual object, and the process of the separation and distinction of those qualities—possible only in thought—is called Metaphysical Division or Abstraction. A logical whole is a general notion, or concept ; and the process of separation into the several parts is called Logical Division. With this alone Logic is concerned.

Logical or Formal Division has reference, subjectively, to general notions or concepts, objectively to classes. It may be defined as the analysis of a whole of Extension ; or the analysis of a proximate Genus into the various co-ordinate species of which it is composed.

**Rules for Division.**      § 44.—The rules for conducting the process of Logical Division correctly are as follows :—

(1) The division must be made according to one principle or ground (— the *fundamentum divisionis*).

(2) The division must be adequate ;—that is to say, the constituent parts, or species, must be equal together to the whole, or genus divided (—the *divisum*) ; neither more nor less.

(3) The parts, dividing members (*membra dividentia*), or constituent species, must stand in the same order of generality ; that is, no one of them should be a higher species than the rest.

Rule (1) secures an end sometimes aimed at by an independent canon of division, namely, that the division must be distinct ; that is to say, that the constituent parts must be mutually exclusive. For, otherwise, the process would be chargeable with being a *cross-division*, that is, one in which the groups overlap ; such, for instance, as would be exemplified by the division of Mental Science into Ethics, Logic, Psychology, Ontology, and Metaphysics, since Metaphysics includes at least two of the other members. The fault of cross-division has to be more especially guarded against in natural classifications, which are so important in science, and in which a number of principles of division (*fundamenta divisionis*) are often simultaneously employed. The principle or ground (*fundamentum divisionis*) mentioned in this rule is some new conception for the marks of which we seek in the conception to be divided ; as, for example, if in making behaviour the principle of division, we divide man, according to behaviour, into just and unjust. If we were required to divide a polyglot collection of literary and scientific works according to the languages in which they are written, the *fundamentum divisionis*, or principle of division, being, accordingly, language, we might divide them into English, French, German, Italian, &c. ; and if the division were based upon the nature of the subject-matter, we might divide them into Logical, Mathematical, Ethical, &c. ; but a division into English, French, Mathematical, Ethical, Historical, Illustrated, and Octavos, would be logically inadmissible ; as some of the groups might cut into one another, or be overlapping, or what Leibnitz calls *communicant* species, thus producing a cross-division. That the principle of division should be essential

and important, Bain characterises as the golden rule alike of defining and of classifying.

Rule (2) secures completeness by insuring that no class shall be omitted, that is to say, that the division shall not be *too narrow*; as, for example, if we were to divide Mental Science into Ethics, Logic, and Psychology,—since this division omits much metaphysical inquiry; and, further, that the division shall not be *too wide*, that is to say, that no species shall be included not properly falling under the genus to be divided.

Rule (3) secures that, as we descend from a higher genus to a species, there shall be no sudden leap over any of the subaltern genera in the series (*divisio non facit saltum*). The species enumerated should be always those of the *proximate* or next higher genus. We violate this rule, if, for example, we divide quadrilateral figures into squares, rectangles, parallelograms, and rhomboids; since squares and rectangles are species of the genus parallelogram.

Of the different modes of dividing a given genus the most perfect from a logical point of view is that called Dichotomy (Greek, *δίκα*, in two; *τεμνω*, I cut), produced by dividing a genus, say A, into two species, say X and its complement, not-X, one a positive and the other a privative concept; and the privative, not-X, again, into two, say Y and not-Y; and so on, till our division is complete, or is brought to an end. For a particular concrete example, we may consider the genus, triangle, which might be divided, first, into equilateral and not-equilateral; the not-equilateral, next, into isosceles and not isosceles; and, again, the not-isosceles into scalene and not scalene; a point at which the process must stop, not-scalene being a non-existent class. The chief value of this process, however, consists in its securing exhaustiveness and mutual exclusiveness in the co-ordinate species, or dividing members (*membra differentia*); and, in certain branches of knowledge, unless we proceed in this way we can never be sure that our divisions are free from oversight.<sup>1</sup>

<sup>1</sup> It is also practically useful when not having a sufficient knowledge of a whole to identify a member whose class we are in search of, we

It is to be observed that in dividing by dichotomy the genus, A (the *totum divisum*), or the corresponding genus, triangle, Logic neither supplies us with the species we select to begin with, namely, X, or equilateral, nor does it enable us to say whether this class is coincident with the given genus or not, or indeed falls at all within it; and the same is true in every application of the process, and throughout the different steps. On these grounds it is held that Division by Dichotomy, though regarded by some authorities as *à priori*, is as much extra-logical as any other kind of division.

An individual, as the name implies, cannot be logically divided. An infima species is divided if we give all the individuals included under it, technically called an *enumeration*: their number, however, is rarely known to us.

Logical Division more or less fails in classifications with undefined boundaries such as those growing out of combination, growth, or development, owing in some cases to the impossibility of strict mutual exclusion, in others to the fact that the particulars cannot be enumerated.

Finally, from considering the respective modes in which definition and division are arrived at, it will be seen that, in a certain sense, in defining we divide, and in dividing we define. In defining by adding the differentia to the genus, we divide species from species. In dividing, we proceed from the genus to the species by selection of a differentia according to the object we have in view, so that each of the *membra dividentia* is thus virtually defined.

**Determinatio.** § 45.—We have now laid down the general logical principles that apply to the divisions and subdivisions of classes. Something more on classification we shall have to say in treating of Applied Logic. If we proceed directly downwards from a higher class to a lower we perform the reverse operation of the abstractive process by

proceed by the method called *abscissio infiniti* (cutting off of the infinite or negative part), that is, by repeated cutting off of that which the object is not to exhaust the whole and so finally arrive at what the object is.

which we originally arrived at the higher by resuming the marks laid aside. This reverse process is called *Determination*, and by it we descend from the *summum genus* through the *predicamental line* to the *infima species*.

The different grades of generality, from the *summum genus* to the *infima species*, in connection with a division by dichotomy are sometimes shown in a tabular arrangement called Porphyry's *Tree* or *Ladder* ( $\kappaλιμαξ$ ), and which, taking the *summum genus* as *substance*, and the *infima species* as *man*, may be exhibited as follows:—

Substance . . . . .		Summum Genus.
Corporeal	Incorporeal	
(Body)		
Animate	Inanimate	
(Living Body)		
Sensitive	In insensitive	
(Animal)		
Rational	Irrational	
(Man)		
Socrates	Plato	
		Individuals.

} Intermediate Genera or Species.

} Proximate Genus.

} Specific Difference.

} Infima Species.

### QUESTIONS.

1. Do Abstraction and Generalisation run *pari passu*; or can we perform either process independently of the other?<sup>1</sup>
2. Distinguish between a “conceptus formalis” and a “conceptus objectivus.”<sup>2</sup>

<sup>1</sup> Whately holds that Abstraction is independent of Generalisation. We can form an abstract notion of an individual, disregarding the separable accidents, such as of posture, dress, location, &c., which are regarded as *non-essential* to him. But Thomson thinks this view erroneous. How, we may ask, do we know that particular postures such as “sitting,” “standing,” &c., are not essential to the individual? How that a particular kind of dress is separable from him? By prior generalisation. Without this we should have been unable to distinguish between the essential and the accidental. We simply refer to a former general notion, and there is no abstraction without generalisation.

<sup>2</sup> See Zigliara's *Summa Philosophica*, vol. i., p. 14, and Ray's *Deductive Logic*, p. 4. Hamilton considers that the distinction taken, by the more modern schoolmen, of the *conceptus* into *formalis* and *objectivus*, is both in itself and in its nomenclature inconsistent and untenable. A *formal* concept or notion they defined as “the immediate and actual representation of the thing thought”; an *objective* concept,

3. Classify the propositional forms. Notice forms objected to, and add remarks.

4. Write each of the following propositions in strict logical form, resolving, where necessary, a compound into its equivalent simple propositions :—

- (a) It never rains but it pours.
- (b) Patience is a flower that grows not in every man's garden.
- (c) Neither riches nor power can give health either of body or mind.
- (d) Men, some to business, some to pleasure take.
- (e) The writers against religion, whilst they oppose any system, are wisely careful never to set up any of their own. C. S.

5. Designate by the technical symbols the quantity and quality of the following propositions :—

- To be weak is miserable.
- Not every expedient act is just.
- Only a brave man would have acted so.
- Art is silent poetry.
- Few, few shall part where many meet.
- Fame is no plant that grows on mortal soil.
- The good alone are great.
- Here were the end, had anything an end.
- Plain living and high thinking are no more.

6. Specify the subject and predicate in each of the following propositions :—

Necessity is the mother of invention.

$$3 + 4 = 7.$$

The sun set when the rain ceased.

Knowledge is only of phenomena.

Milton was born in London.

This is not the fault of the method itself.

Along with motion there is necessarily something moved.

Here stands the word. C. S.

or notion, as "the thing itself which is represented or thought." The second of these is either not a notion or concept at all, or it is indistinguishable from the first. Hamilton's *Reid*, p. 807. The term, *concept objectified*, is perhaps a form less objectionable than *objective concept*.

7. Discuss the import of the logical Copula.<sup>1</sup> L.
8. What is the function of the Copula? In what different manners has it been treated? Distinguish between the distributive and collective senses of *All*, and give examples in illustration.
9. Explain: "The term in the predicate acts upon the mind by its connotation or its comprehension; the term in the subject by its denotation or extension." R.
10. What is Modality? Discuss—"Modality belongs to the Copula." O.
11. What are the Quantities of logical terms with which Formal Logic is chiefly concerned? What is the Quality of a proposition? Illustrate the difficulty of expressing particular propositions in terms of Connotation.<sup>2</sup>
12. Discuss the doctrine that a proposition is always *an equation* of its subject and predicate. C. S.
13. Discuss the possibility of dealing with the logical relations among propositions by attending only to the Intension or Connotation of the terms. Can all kinds of propositions be exhibited in the intensive as well as the extensive form? Give reasons in support of your answer; and in the event of its being in the negative, draw up a list distinguishing between those kinds of propositions which can, and those which cannot be so exhibited. C. S.
14. Give the expressions in connotation of the following propositions<sup>3</sup> :—

<sup>1</sup> See Fowler's *Deductive Logic*, pp. 24-28, Mill's *Logic*, vol. i., pp. 85-89, Bain's *Logic*, vol. i., p. 44, Thomson's *Laws of Thought*, pp. 109, 132, and the authorities referred to by Dr. Fowler. Thomson, contrary to the view expressed in the text, § 27, observes that the copula *is*, in a proposition, expresses an existence modified or limited by the predicate. Keynes' *Formal Logic*, 3rd edition, pp. 147-210, contains very valuable matter on the subject.

<sup>2</sup> *All S is P*, read in denotation means, The denotation of S is included in the denotation of P; but read in connotation means, The connotation of S includes the connotation of P. Some S is P, read in denotation, means, The classes S and P partly coincide; but read in connotation, this proposition is not properly expressed as, The connotation of S and P partly coincide: we must obvert it, thus reading, The connotation of S does not include the connotation of *not-P*. See Keynes' *Formal Logic*, p. 159, keeping in mind his matter in §§ 12-20.

<sup>3</sup> Of (1) the expression in connotation is—The connotation of tulips

- (1) All tulips are beautiful flowers.
- (2) No roebuck was swifter.
- (3) Some solar photographs are wonderfully interesting to examine.
- (4) Some ancient volcanoes are no longer in action.

15. Is it true to say that a universal proposition represents nothing more than a hitherto uncontradicted experience? L.

16. Explain the meaning and practical application of the doctrine that "Logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought."

In the assertion that "All  $x$  is  $y$ ," is it implied that either  $x$  or  $y$  exists?

17. Discuss the admissibility of the Quantification of the Predicate into a system of Logic. M.

18. State the grounds of the usual doctrine that the quantity of the predicate is determined by the quality of the copula. State also and examine briefly the grounds of Sir William Hamilton's dissent. Explain generally the principle of the Quantification of the Predicate, and make any remarks that occur to you upon the forms of judgment that result from its application.

19. Bring out the exact meaning of the Hamiltonian proposition  $\omega$ : "Some X's are not some Y's." What is the contradictory of this proposition? Estimate its logical importance. C. S.

20. Discuss the questions, (1) whether all judgments are affirmative in character, (2) whether in hypothetical judgments the relation expressed is identical with that involved in categorical judgments. V.

21. What is predication? What is Hobbes' theory of predication? Does predication ever involve existence, or must existence always be specially predicated? Describe

includes the connotation of beautiful flowers; of (2),—The connotation of roebuck includes the connotation of then-not-swifter-animals-than-such-a-one; of (3),—The connotation of solar photographs does not include the connotation of objects-not-wonderfully-interesting-to-examine; and, of (4),—The connotation of ancient-volcanoes does not include the connotation of volcanoes-still-in-action.

shortly, (1) the class, and, (2) the inhesion theory of predication. Are these theories incompatible? C. S.

22. What are the possible relations between the subject and the predicate of a proposition in A, when the predicate is a common term, and the subject is the same? P.

23. Distinguish between Ampliative and Explicative propositions, and give several examples of each kind.

24. On what ground has the distinction between a Real and a Verbal proposition been impugned, and with what effect? E.

25. How far can we get in an analysis of the import of propositions without inquiring into the import of names? D.

26. Compare the Categories with the heads of Predicables, in respect of (1) their nature, and (2) their use. L.

27. Explain and illustrate what is meant by summum genus, cognate species, generic difference, and specific property. P.

28. Give instances of "Differentia," "Property," "Inseparable Accident," and examine with reference to your instances how far it is possible to distinguish them. What are the genus, species, difference, property, and accident, of "Examination"?

29. "The species and genus are like generals, the one of which has a small army but wholly unanimous, the other great but of diverse functions: for that collects more intensively, this more extensively." Comment upon this. Define Predicate, Predicament, Predicable. R.

30. Explain the terms:—Cross-Division, Genetic Definition, Species Infima, Proprium, Specific Difference. C. S.

31. The popular mode of employing the terms Genus and Species differs from that approved by the Aristotelian logicians. How does Mill explain and justify the practice of the latter?

32. Explain and illustrate:—*Genus est unum aptum inesse pluribus specie differentibus et predicare de illis in quid incomplete.*<sup>1</sup> R.

<sup>1</sup> See Zigliara's *Summa Philosophica*, vol. i., p. 23.

33. Give an abstract of the scholastic doctrines of Definition. How many kinds of definition were recognised by the scholastic logicians? Distinguish exactly between the Definition and the Description of a class. Examine the nature and importance of the distinction between Real and Nominal Definition, showing how your decision on this point is connected with the general view of Logic you adopt. Do you conceive that Definition can ever be regarded as complete and final? C. S.

34. Distinguish Complete from Incomplete Definition. L.

35. To what terms is logical Definition applicable?  
(a) Explain:—"Definitio est oratio quâ respondetur interroganti quid sit res." (b) When is a definition logically sound?  
(c) Examine:—"Sometimes things may be made darker by definition. I see a cow. I define her, Animal quadrupes ruminans cornutum. But a goat ruminates, and a cow may have no horns; cow is plainer." R.

36. What is the object of Definition? Define logically the following terms:—House, school, college, minister, governor, hypocrisy, treason, species, diet, money, gravitation, success, merit. O.

37. Define Logic, and state what you think of the following Definitions:—Logic is the art of expressing thoughts in words; Logic is a help to correct reasoning.

All Definitions are of Names, and of Names only. Examine this doctrine.<sup>1</sup> O.

38. Definitio alia nominalis, alia realis; realis iterum vel accidentalis sive descriptio vel essentialis. Explain and criticise this statement. O.

39. "Definition is either of a Name or a Thing." Explain and illustrate this.

40. State the canons for Definition. Under what circumstances is it applicable and useful? C. S.

41. State clearly the nature and the limit of Definition *per genus et differentiam*. E.

42. Examine and classify the following Definitions, or Descriptions, pointing out their faults, if any:—

<sup>1</sup> See Mill's *System of Logic*, vol. i., p. 163.

Method is a progressive transition.

Knowledge is the relation of reason to the universe.

Distance is judgment based upon sensation.

Gratitude is the memory of the heart.

Man is a being of large discourse, looking before and after.

Man is a hunter after truth.

Mind is the principle which thinks and moves.

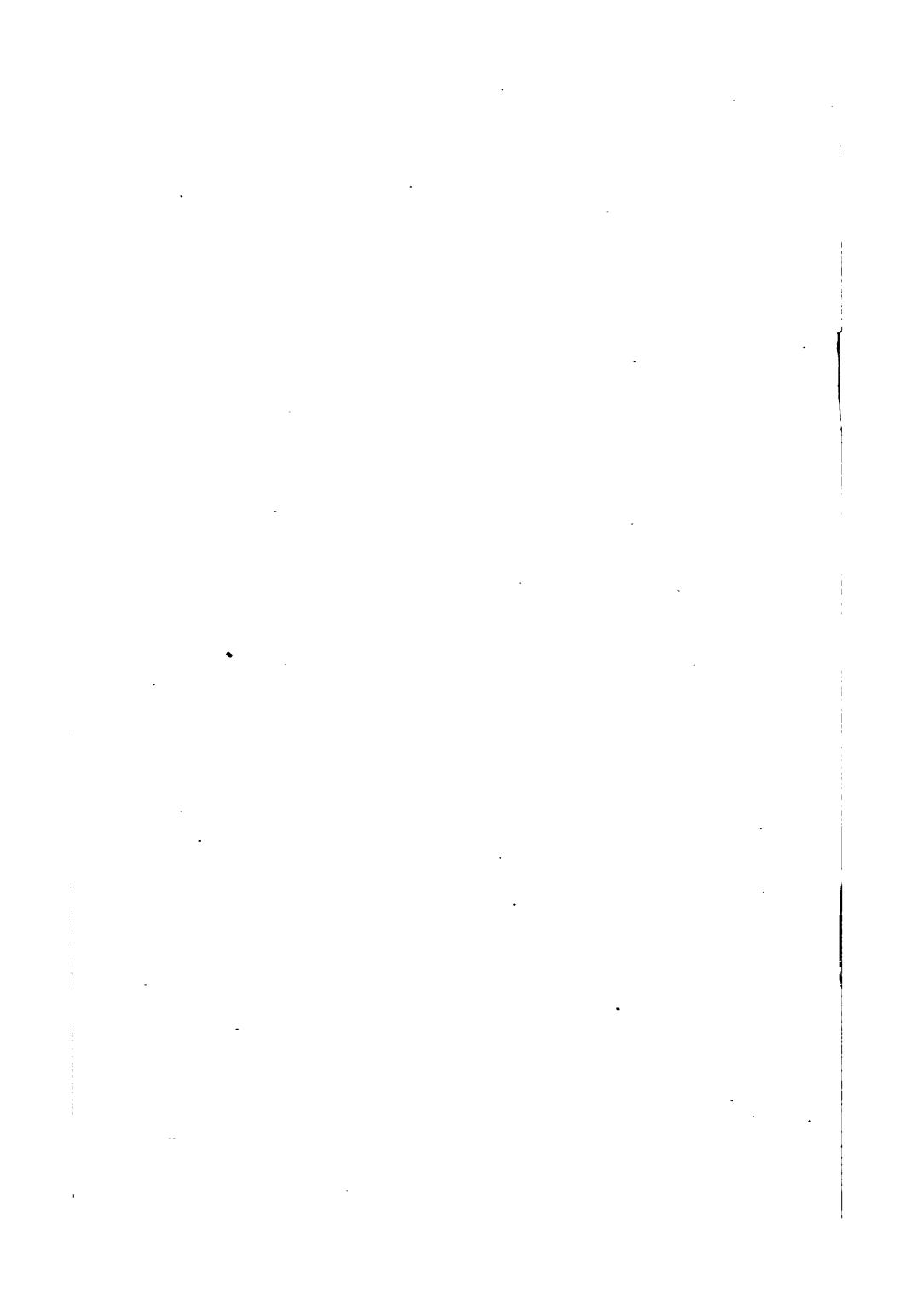
Virtue is the habit of deliberate preference consisting in a mean.

Mens est substantia cogitans.

Logic is the Art of Thinking.

Philosophy is such knowledge of effects or appearances as we acquire by true ratiocination from the knowledge we have, first, of their causes or generation, and again, of such causes, or generations, as may be from knowing, first, their effects.—(*Hobbes*.)

43. Explain the nature and object of expressing our reasoning by propositions of the form "X is a mark of Y," rather than by those of the form "A is B." L.



## BOOK III.—ON INFERENCES

### CHAPTER I

#### ANALYSIS AND CLASSIFICATION OF INFERENCES

**What Inference is ; Reasoning.** § 46.—We have, in § 14, distinguished various meanings of the term *Inference*. According to De Morgan it is “the production of one proposition as the necessary consequence of one or more other propositions.” Taken in its widest sense, Inference may be defined as the derivation of a judgment, or of a proposition expressive of a judgment, from any given elements, or as the judgment or proposition so derived.<sup>1</sup> In what sense it is to be taken in any given instance of its use will always be obvious from the proper context. When employed to denote the *process* by which from antecedent propositions or premisses we arrive at a valid conclusion, Inference may be regarded as synonymous with Reasoning ; and we may speak also of Reasonings or Inferences as the *results* of that

<sup>1</sup> Kant defines Inference as, “the derivation of one judgment from another.” When this is done without the aid of an intermediate judgment he calls the result an *inference of the understanding*, when with such aid, an *inference of the reason*. According to Hamilton, Inference is the “carrying out into the last proposition what was virtually contained in the antecedent judgments.” According to Mill, it is the “process of advancing from known truths to unknown,” to *infer* being to *reason*, in the most extensive use of the term ; there being a narrower sense, in which reasoning is equivalent to *ratiocination*, or the form of inference of which the general type is the syllogism, the sense in which we employ the term in the text of the present chapter.

process, just as we speak of Thoughts as the results of the Thought-process, or of Thinking.

**Classification of Inferences.**—In the classification of Inferences it is difficult to avoid some traces of cross-division, whatever scheme be adopted. This is due to the fact that the range and character of certain kinds of inference remain points on which discussion is not yet at rest.

One convenient way is to divide all Inferences into Inductive and Deductive, sub-dividing Deductive Inferences into Immediate and Mediate, Mediate Inferences into Syllogistic and Non-syllogistic, and Syllogistic Inferences into Demonstrative and Probable.

Inductive Inference, or Induction, is the process of reasoning from particulars to generals; or of proving general propositions from the consideration of particulars, or less general propositions, expressive of conjunctions of properties seen in individual instances. Thus, when from the *particular propositions* embodied in the statement, "The electric current passing in the copper wire A, in the silver wire B, in the platinum wire C, and so on, exerts a deflective force on a magnetic needle suspended near," we pass on to the *general proposition*, "The electric current in whatever conductor passing, exerts a deflective force on the magnetic needle," the process, including any unexpressed assumptions we feel justified in making, is called Induction, or Inductive Inference, and the conclusion<sup>1</sup> resulting therefrom an Induction or an Inductive Inference. We shall reserve the subject of Induction for treatment somewhat at length under the head of Applied Logic.

**Deductive Inference.**—Deductive Inference, or Deduction, is usually described as the process of reasoning

<sup>1</sup> When an Inference is derived from more than one proposition the result is called a *conclusion*. When the *aggregate* of the premisses (employing De Morgan's phraseology) gives the conclusion, we have Induction; when the *compound* of the premisses gives the conclusion, we have Deduction. See De Morgan's *Syllabus of a Proposed System of Logic*, p. 43.

*from generals to particulars*; or as the deducing from a general truth a particular truth therein contained. In view, however, of the classification of inferences above adopted, it may be explained to mean the inferring of a proposition from a proposition equally or more general. And the result of the process, as well, we call a Deduction, or a Deductive Inference. As an illustration, taking the general proposition, "An electric current, in whatever conductor, exerts a deflective force on the magnetic needle," in conjunction with the particular proposition, "The wire A is a conductor," we infer deductively that, "An electric current in the wire A exerts a deflective force on the magnetic needle." In this example, the conclusion is based upon the two propositions, one of which is easily seen to be attainable only by a previous induction; and the same may be observed of all similar cases. And the consideration of such examples suggests to us that in the growth of our knowledge, deductive inferences cannot be regarded as naturally prior to inductive.<sup>1</sup> Assuming, however, that such general propositions can be found as are necessary for deduction to work upon, since Deductive Inference is strictly formal, can be easily treated with a measure of distinctness and as an independent subject, and serves for the investigation of Induction, our dealing with it first in order can lead to no confusion.

Deductive Inferences may be distinguished into Immediate and Mediate. When from a given proposition we proceed to, or *draw out*, another different from it, but virtually contained in it, the process is called Immediate Inference; and when from two or more given propositions we proceed to a conclusion based thereupon, the process is called Mediate Inference.

According to Mill and Whately, the inferences we have called Immediate are apparent only, not real; what is ostensibly a new proposition being nothing more than a repetition of the assertion, or part of the assertion, contained in the

<sup>1</sup> This, however, is not a point generally conceded. See Lotze's *Logic*, p. 298.

first, and obvious to every person who apprehends that proposition. Thomson, however, taking a different view, maintains that the new proposition cannot be regarded as a bare repetition of the old ; inasmuch as there is always, either in the quality, or in the subject of thought in each, or in both together, sufficient difference to constitute them two distinct propositions.

Mediate Inferences may be distinguished into Syllogistic and Non-syllogistic. Those mediate inferences are syllogistic which can be exhibited in the strict form of syllogism. It is claimed, however, that certain other kinds of mediate inferences claim our assent no less strongly than the syllogistic. Of those, called non-syllogistic inferences, are the mathematical reasonings based upon what is known as the *argumentum a fortiori*, and other reasonings of a similar kind, of which more hereafter.

Syllogistic Inference, again, may be distinguished into Demonstrative and Probable. It is demonstrative, if both premisses are assumed to be certain ; probable, if they are, one or both, not certain, but only more or less probable. This distinction, however, may be regarded as extending beyond the field of syllogistic inference properly so-called, the principles of Induction itself resting, according to the views of Jevons and other logicians, on the theory of Probability or Probable Inference.

**Demonstra-** § 49.—To prove is to establish the truth of a **tion** ; its **various** definite conclusion. Demonstration is the proof **Kinds.** or disproof of a proposition by a chain of reasoning, or a concatenation of syllogisms, wherein all the premisses are definitions, self-evident or necessary truths, or propositions previously established.

The scholastic logicians distinguish several varieties of Demonstration. It is said to be *propter quid*, or *quia*, according as (1) it proceeds from known, primary, or indemonstrable, immediate, and adequate principles to a given conclusion ; or, (2) as the principles are remote, or the conclusion precedes them in the order of thought. Closely

related to this division is that of demonstrations into *à priori* and *à posteriori*. The first of these two kinds proceeds from cause to effect, the second from effect to cause.<sup>1</sup> Demonstration is also distinguished into *absolute* and *relative* or *ad hominem*. The former is based upon premisses that are taken as absolutely true; the latter, upon premisses the truth of which is assented to for the occasion by a disputant, whether they are actually true or not. Further, Demonstration is distinguished into *direct*, or *ostensive*, and *indirect*. By the former we proceed directly from positive principles to a conclusion; by the latter we prove a given conclusion to be true by showing that if it were false some result would necessarily follow inconsistent with admitted fundamental principles. The process of indirect demonstration is called also *reductio ad absurdum*. Finally, Demonstration is distinguished into *pure*, *empirical*, and *mixed*. In pure demonstration the premisses are *à priori* in character; in empirical demonstration the premisses are empirical, that is to say derived from experience, or *à posteriori*; and in mixed demonstration, the premisses are, some of them, *à priori*, the others, *à posteriori*.<sup>2</sup>

**Fundamentals of Demonstration.**—§ 50.—Demonstration, as observed above, rests upon premisses, all definitions, self-evident or necessary truths,<sup>3</sup> or propositions previously established. The characteristic feature claimed for it, whatever ultimate premisses it springs from, is, that *all Demonstration is absolutely certain in respect of its matter*.

<sup>1</sup> See § 5, footnote. De Morgan employs the terms *à priori* proof, and *à posteriori* proof as synonymous with deduction and induction respectively.

<sup>2</sup> See Duncan's *Elements of Logic*, pp. 256-275, and Zigliara's *Summa Philosophica*, vol. i., p. 171. Pure, Empirical, and Mixed Demonstration are more usually defined in terms indicating that the reasoning consists of no more than one step, expressed by a single syllogism; in which case, of course, the character of only two premisses has to be considered.

<sup>3</sup> That is, truths *à priori*, or such as are supposed to be independent of experience. But Mill would define *Necessary Truth* to be such as necessarily follows from assumptions, which, by the conditions of the inquiry, are not to be questioned; or, truth that may be *certainly inferred*.

Now, the proof or disproof of this general affirmative proposition, depends on the character of the premisses of the demonstration, thus overpassing the limits of Formal Logic,<sup>1</sup> and necessarily anticipates a little in relation to syllogistic inference ; but some account of the reception it has met with from philosophical writers appears essential to add for completeness' sake. If we could prove the universal affirmative proposition, that, in all cases whatever, absolute certainty attaches to the premisses of Demonstration, of course, we had established the above proposition. If, on the other hand, choosing our ground as might seem for any reason desirable, we could prove the particular negative, that, in relation to any particular class of truths, absolute certainty does not attach to the premisses, the above universal affirmative (see § 54) would be overthrown. The latter mode of proceeding is of course what suggests itself at once to the logician ; and as mathematics have become the model for the establishment of philosophical and all special truths, especially since the time of Descartes, the sciences of extension and of number have been chosen as the ground, and the discussion has been made to turn upon the fundamentals of these sciences, the conclusions of which have been accepted for ages as a typical body of demonstrated truths. The importance of the controversy, thus limited to a particular kind of subject-matter, is not difficult to appreciate. It is assumed that the mode in which the mind acquires knowledge is an index to the certainty of the knowledge acquired ; and, hence, principles, if there be such, that are independent of observation and experience so liable to create drawbacks of various kinds in respect of certainty, carry with them a convincing force of an irresistible kind, and to which *à posteriori* principles cannot pretend. It is claimed for them that they are necessary

<sup>1</sup> "Inference has nothing to do with the truth or falsehood of the antecedents, but only with the necessity of the consequence. When the inference from the antecedents is preceded by showing their truth, the whole is called *proof* or *demonstration*." De Morgan's *Syllabus*, p. 43. A more popular meaning of demonstration is any argument or reasoning regarded as proving an asserted conclusion.

truths, co-equal in certainty with the certainty of existence of the human mind itself. "To learn a proposition by experience," says Dr. Whewell, "and to see it to be necessarily true are two altogether different processes of thought."<sup>1</sup> In the former case, we learn that a proposition is true, within, however, only the limits of our experience. In the latter case, we not only learn that a proposition *is* true, but see why it *must* be true, and universally. Hence, it is argued that to establish the existence of any one body of necessary truth is to establish the existence of any one body of a higher kind of evidence than the experiential. On the other hand, to dis-establish any of the most important bodies of such principles from its place would be to make a great breach in the bulwarks of pure reason and certain belief, and to leave us the more open to the doubt and uncertainty usually attaching to the mere evidence of the senses.

Some of the best-known chapters in Mill's *System of Logic* are those that deal with demonstration and necessary truths, and aim at overthrowing the views of Dr. Whewell and other philosophers who maintain for truths of mathematics the character of necessity.

Whewell's position is that necessary truths derive their necessity from the ideas they involve which, in pure mathematics, are number and space; number being a modification of the idea of repetition which belongs to time, and neither time nor space being ideas derived from experience. The idea of space is exhibited for mathematical purposes by the definitions and the axioms<sup>2</sup> of geometry; the former necessarily referring to, and agreeing with, conceptions that can be distinctly framed in our thoughts, and the latter being truths perceived *a priori*, owing to the constitution of the mind itself, and without any necessity for verifying them by repeated trials, as in the case of truths really ascertained

<sup>1</sup> See Whewell's *History of Scientific Ideas*, vol. i., pp. 65-67, and Mill's *System of Logic*, vol. i., p. 273.

<sup>2</sup> In mathematics the word *axiom* is taken to mean a self-evident truth, which depending on no simpler proposition, is taken as the basis of reasoning.

by observation. Thus, the character of necessity attaches to the truths of mathematics, as being in their nature independent of experience.

Mill's position is that the character of necessity ascribed to mathematical truths is an illusion; inasmuch as the conclusions, in so far as they rest (1) on the *definitions*, are based, in reality, on the implied assumption that there exist real things conformable to the definitions, an assumption not strictly true; and in so far as they rest (2) on the *axioms*, depend ultimately on experience and observation. To regard, therefore, the science constructed on these elements as *a priori* he pronounces to be a psychologically incorrect doctrine, however satisfactory it might be held towards saving the credit of a supposed system of necessary truths. He, consequently, holds that the characteristic feature of Demonstrative Science, as represented by mathematics is, that it is hypothetical; a conclusion to be extended to Demonstrative Science in general, and with the resulting overthrow of the universal affirmative stated above, namely, that in respect of its matter all Demonstrative Science is absolutely certain.

Recent writers, however, continue to support the view that mathematical truths are cognisable by the mind as necessary; some writers, in the somewhat modified form that these truths are taken in by the mind in virtue of a certain peculiar and delicately graded expenditure of energy; and are, at the same time, intuitive, inasmuch as every mind has an intuition of the expenditure of its own energy,—a very interesting and suggestive new departure.<sup>1</sup>

<sup>1</sup> See the article on Axioms, by Mr. G. C. Robertson, in the ninth edition of the *Encyclopaedia Britannica*.

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## CHAPTER II

### THE NECESSARY LAWS OF THOUGHT, AND THEIR APPLICATION TO LOGICAL PROCESSES

**The Necessary Laws of Thought.** § 51.—We find on reflection that consistent thinking exhibits an accordance with certain simple, self-evident, or axiomatic principles, *à priori* in character, which are illustrated in even its least complex results, and which, by some logicians are called Principles of Inference, by others, Fundamental or Necessary Laws of Thought. Of these principles the most important are the following:—

(1) The Principle of Identity: Whatever is, is ; A is A ; or, according to Plato, the Idea is equal to itself. The first form of the principle occurs in Parmenides. To obviate the objection that, *A is A*, is a mere tautologous proposition, and, therefore, unfruitful in character, it is variously explained to mean (a) that the same term must in the same reasonings be always employed in the same sense, unless notice has been given of a change ; (b) that the same thing must always be regarded as possessing the same attributes ; (c) otherwise, that an individual group of things, distinguished by a certain mark or group of marks, is always assignable to the part of the universe of discourse allocated to such groups of things ; (d) that every object is its own nature ; and (e) that a concept, as distinguished from its objectifications, being a persistent subjective impression, is always absolutely one and the same, of whatever it may be predicated.

(2) The Principle of Contradiction. This principle is

stated in various forms, such as : Nothing can both be and not be ; The same individual cannot be both A and not-A ; The same subject cannot have two contradictory or inconsistent attributes ; Nothing can be both asserted and denied of the same thing at the same time ; and other equivalents.

The principle follows at once from the consideration that to every object of thought which is completely distinguishable from all other things there corresponds an idea which is completely distinct from all other ideas. No thing can have two distinct ideas corresponding. No one thing can be both A and not-A ; so that the principle is immediately seen to be necessary.

As applied to any given subject-matter we must be sure that we are speaking of the same thing, or part of the same thing, with reference to the inconsistent attributes. Thus, the same water may feel cold to one hand and warm to the other, if our hands had been previously immersed, the one in water at a higher temperature, the other in water at a lower ; to either hand, however, water, *at a given point of its volume*, cannot feel both cold and warm at the same time. And so of any other attributes which are inconsistent with one another. It is worth noting that both *cold* and *warm* here contend for applicability to a certain *state* of water, namely its temperature. Thus, the temperature of water at a given point may be either cold or not-cold, warm or not-warm, or generally of any definite temperature A, or not-A.

(3) The Principle of Excluded Middle. Everything must either be or not be ; A is either B or not-B. The meaning is that any given attribute either belongs to a thing or does not belong to it. A middle course, or predication, is excluded.

Thus, water, as to its state, is either solid or not-solid ; rock is either hard or not-hard ; cadmium, as to its chemical nature, is either a metal or a non-metallic substance ; and so on. Jevons remarks (*Elementary Lessons*, p. 119) that "some readers may not know what a cycloid is, or what an isochronous curve is ; but they must know that a cycloid is

either an isochronous curve or it is not an isochronous curve" ; and various other similar examples will readily suggest themselves. In the examples under consideration, however, we do not get beyond the ideation of a mere name ; and so in all like cases. It is objected by some logicians, in relation to this principle, that, if we take, say, such a subject—an immaterial subject—as *virtue*, and such an attribute as *triangular*, we are enabled to assert that "Virtue is either triangular or not-triangular," a proposition which is false and absurd. Jevons, however, defends the application of the principle even in this case, for the attribute, *not-triangular*, includes both things that have figure other than triangular, and things, among them immaterial things, that have not the properties of figure at all, and to which latter class virtue belongs.

Ueberweg (*Logic*, p. 263) observes that some objectors to the validity of the axiom consider that we may, in certain cases, admit the possibility of a mean between two contradictory terms ; as, for example, between "guilty" and "not guilty" there may be "not proven." Such an objection as this, however, arises from a misconception as to the true meaning of the principle. When we allege that A, as to his *proven* position, is "guilty" or "not-guilty," there is no middle course. Again, A, in his *non-proven* position is either "guilty" or "not-guilty" : there is no middle course. As regards our knowledge of the fact, it may be either proven or non-proven that A has done a certain act or has not done it ; but it must be the one or the other.

In what precedes, we have assumed that A, in the expression, A is either B or not-B, means an individual thing. When, however, A is taken to mean a class of things, since some members might coincide, and others not, with the attribute B, it is obvious that the principle must be inapplicable.

In addition to the three fundamental or necessary laws of thought some logicians have held that a fourth should be admitted called the Principle of Sufficient or Determinant Reason. The statement of this principle is due to Leibnitz,

who regards it as the foundation of all propositions that are not identical. The understanding, according to the philosopher, avails itself of this principle to connect the predicate of such propositions with the subject, to affirm such propositions, when this connexion can be discovered, and to deny them, when it cannot. His statement of the principle (*Théodicée*, § 44) is as follows:—"Nothing ever happens without a cause, or at least a determinant reason, that is, something which serves as a reason *a priori* why such a thing exists rather than not, and why such a thing is so, rather than otherwise."<sup>1</sup>

From these principles are educed such applications as these:—1. Granting the reason, we must grant what follows from it. On this depends syllogistic inference. 2. If we reject the consequent, we must reject the reason. If we admit the consequent, we do not of necessity admit the reason.

Mansel and Hamilton, however, consider that this principle, if it can be admitted as axiomatic, or self-evident, is extra-logical in character; and Thomson regards it as most fittingly assigned to Applied Logic.

General  
Canon  
of the  
Syllogism. § 52. The necessary laws of thought, as just explained, teach us the nature of the agreement or disagreement of the ideas or things that are brought together in a judgment, as expressed by a proposition. To proceed, however, by an intermediate idea as a step from judgment to judgment, we require, in syllogistic reasoning, what is called the Fundamental Principle, or General Canon, of the Syllogism. Of this principle, some of the statements most frequently referred to are as follows:—

Aristotle's:—The *Dictum de omni et nullo*. Whatever is predicated distributively, whether affirmatively, or negatively, of any class of things, may be predicated in like manner of anything comprehended in that class.

Leibnitz's:—*Contentum contenti est contentum continentis*; viewing the extension of the judgments. Whatever is contained in any contained thing is contained in that which contains this thing.

<sup>1</sup> See Gerhardt's *Leibnitz (Die philosophischen Schriften)*, b. vi., p. 44.

Kant's :—*Nota notae est nota rei ipsius*; viewing the intension. Whatever is a mark of any mark is a mark of that of which this last is a mark.

Hamilton's :—(1) For syllogisms in *extension*. What belongs to the genus belongs to the species and individual; what is repugnant to the genus is repugnant to the species and individuals; or, more briefly, what pertains to the higher class pertains also to the lower. (2) For syllogisms in *intension*. What belongs to the predicate belongs also to the subject; what is repugnant to the predicate is repugnant to the subject.

Mill's :—(a) Things co-existing with the same thing co-exist with one another; (b) A thing co-existing with another, with which a third thing does not co-exist, does not co-exist with that third thing. If, however, propositions are regarded chiefly as memoranda for practical guidance, Mill would translate these, for universal premisses, into the form just given after Kant.

Thomson's :—The agreement or disagreement of a subject and a predicate is ascertained by a third conception, predicate to the former and subject to the latter, inasmuch as this wholly or by the same part agrees with both, or with one only, of the conceptions to be compared.

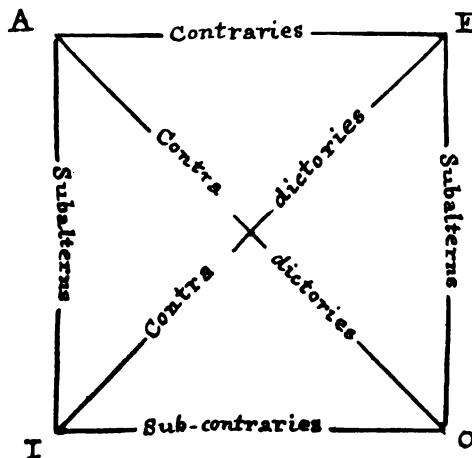
Whately's and Jevons' :—(a) Two terms agreeing with one and the same third term agree with each other; (b) Two terms, of which one agrees and the other does not agree with one and the same third term, do not agree with each other; (c) Two terms, both disagreeing with one and the same third term may or may not agree with each other.

To proceed in mathematical reasoning from judgment to judgment by an intermediate step, we make use of some one or other of the mathematical axioms; such as, (1) Things that are equal to the same thing are equal to one another, (2) Things that are the sums of equals are equal, (3) the principle known as the *Argumentum à fortiori*—A thing which is greater than a second, which second is greater than a third, is greater than the third; and others of a like character.

Some remaining peculiar forms of non-syllogistic reasoning, with the principles underlying, shall be noticed in the next chapter.

**The different kinds of Immediate Inferences.** § 53.—The principal kinds of Immediate Inferences are (1) Opposition, (2) Conversion, (3) Obversion, Aequipollence, or Permutation. They are all inferences of one proposition from one another.

§ 54.—Opposition consists in inferring from **Opposition**, the truth or falsity of a given proposition the truth or falsity of another having the same subject and predicate as the first, but differing from it in quality, or quantity, or both. The opposition between A and O, or E and I, is called Contradictory Opposition ; that between A and E, Contrary ; between I and O, Sub-contrary ; and between A and I, or E and O, Subaltern.<sup>1</sup> These relations are exhibited in the subjoined scheme :—



<sup>1</sup> Mansel regards Subalterns as improperly classed as *opposed* propositions, and Ueberweg (after Aristotle) also excludes them from such. Aldrich, however, takes the same view of opposition as that in the text, and Dr. Fowler follows in a like use of the term, though he suggests that the inference between A and I, E and O were better called Sub-alternation, or Subordination.

The Laws of Opposition are :—

1. For *Contradictries*. If any proposition be true, its contradictory is false; if false, its contradictory is true.
2. For *Contraries*. Any proposition and its contrary may be both false, but cannot be both true.
3. For *Sub-contraries*. A proposition and its sub-contrary can never be both false.
4. For *Subalterns*. (a) The truth of the particular follows from the truth of the universal; (b) the falsity of the universal does not involve the falsity of the particular; (c) the truth of the particular does not involve the truth of the universal; and (d) the falsity of the particular involves the falsity of the universal.

Thus, the inferences we can draw by Opposition are the following :—

1. If A be true, or *posited*, E is false, or *sublated*, I is true, and O is false.
2. If A be false, E is undetermined, unknown, or doubtful, I is undetermined, and O is true.
3. If E be true, A is false, I is false, and O is true.
4. If E be false, A is undetermined, I is true, and O is undetermined.
5. If I be true, A is undetermined, E is false, and O is undetermined.
6. If I be false, A is false, E is true, and O is true.
7. If O be true, A is false, E is undetermined, and I is undetermined.
8. If O be false, A is true, E is false, and I is true.

The legitimacy of these inferences may be regarded as depending exclusively on the three fundamental laws of thought, which for convenience' sake may be re-stated here in the following form :—

- (a) The Law of Identity, *A is A*.
- (b) The Law of Contradiction, *A is not not-A*.
- (c) The Law of Excluded Middle, *A is either B or not-B*.

Thus, given A as true, it follows by (a) that I is true, by (c) that E is false, and by (b) that O is false.

The inferences we can draw by Opposition are sometimes stated in the following way :—

In *necessary* matter (namely, where the subject-matter is such that A may be affirmed) A and I are true ; E and O are false.

In *contingent* matter (namely, in which universal statements cannot be made) I and O are true ; A and E are false.

In *impossible* matter (namely, in which E may be affirmed) E and O are true ; A and I are false.

Such statements, however introducing, as they do, the nature of the subject-matter, are pronounced to be extra-logical.

In subaltern opposition, we can infer from the position of the universal to the position of the particular, but not *vice versa*. In sub-contraries, we can infer from the falsity of the one to the truth of the other, but not *vice versa*. In contradictory opposition, however, we can infer from the truth of one of the two related propositions to the falsity of the other, or from the falsity of the one to the truth of the other. Inasmuch, therefore, as there is, thus, inference both ways, contradictory opposition is said to be the most perfect.

The contradictory as compared with the contrary has a value towards rebutting a universal in disputation which is sufficiently obvious. It can be more easily proved than the full contrary ; and it is equally effective for disproof of the proposition advanced.

**§ 55.—Conversion**, as a kind of immediate **Conversion**, inference, is the process of deriving one proposition from another, such that the subject and predicate of the one shall be, respectively, the predicate and subject of the other.<sup>1</sup> The original proposition is called the *Exposita*,

<sup>1</sup> To conversion, and to all other forms of immediate inference in which the new proposition differs from the old in subject, or predicate, or both, the term *Education* is applied in some recent treatises on Logic.

or *Convertend*, and the derived proposition, the *Converse*. That the latter may be a legitimate inference from the former, the following rules must be observed.

1. The quality of the converse must be the same as that of the convertend.
2. No term must be distributed in the converse that was undistributed in the convertend.

If we take, for example, such a proposition as, All B is C, the subject and predicate of the converse must be C and B, respectively. C, however, must not be distributed in the new proposition, since being the predicate of an affirmative it was undistributed in the old. The converse must therefore be, Some C is B, a proposition of the same quality as the convertend, with the terms transposed ; the caution against undue distribution of terms being observed. Conversion thus, of a universal affirmative, is called *Conversion by Limitation*, or *Per Accidens* ; the latter name being given to it, according to Mansel, because it is not a conversion of the universal *per se*, but by reason of its including the particular.

That the proposition, Some C is B, is immediately inferable from All B is C, will be obvious from the following considerations :—To say that All B is C, may be taken as equivalent to saying that B, viewed with reference to a certain attribute or group of attributes, lies wholly within C, the universe of the attribute or group of attributes. Hence, Some C is B.

From the proposition, Some B is C, we, on similar grounds, immediately infer, Some C is B ; and, looking to exclusion, from No B is C, we infer, No C is B. Conversion thus, of a particular affirmative, or a universal negative, the quantity being unchanged, is called *Simple Conversion*.

According to Whately and many of the older logicians, a particular negative, such as, Some B is not C, may be converted simply by attaching the negative particle to the

It is rather vague, and does not indicate the *immediate* character of such inferences.

predicate, so that we have virtually to do with a particular affirmative. We thus get, Some B is not-C, and for the converse, Some not-C is B. This process has been called *Conversion by Negation* or *Contra-position*. As it cannot, however, be regarded, properly speaking, as a single inference, and as the methods of the preceding examples are inapplicable, particular negatives are declared by most recent writers to be inconvertible; having regard to the definition above given of conversion.

In brief, universal affirmative propositions are converted *per accidens*, the quality being preserved, but the quantity diminished; universal negatives and particular affirmatives are converted simply, the quality and quantity being preserved. A is converted into I, E into E, I into I, and O is not convertible at all in the ordinary sense.<sup>1</sup>

As illustrative examples we may consider the propositions, All metals are elements (A), John struck the ball (A), No knowledge is useless (E), Some Africans are albinos (I). Of these, the converses are, respectively:—Some elements are metals (I), Some person that struck the ball is John (I), No useless matter is knowledge (E), Some albinos are Africans (I).

§ 56.—*Obversion*.—Obversion is the process of inferring from a given proposition, called the *obvertend*, another proposition, called the *obverse*, having the same subject as the original proposition but differing from it in quality, the new predicate being the contradictory of the old.

Obversion is also called Infinitation (—from the *nomen infinitum*, or *indefinitum*, namely, the contradictory, used as the new predicate), Permutation (Karslake), Immediate Inference by Privative Conceptions or Concepts (Thomson), and Aequipollence (Ueberweg). According to Bain, however, the

<sup>1</sup> The schoolmen embodied the rules for conversion in the following lines, the vowels of the nominative words designating A, E, I, and O:—

Fexi simpliciter convertiter; eva per accid.;  
Faxo (or asto) per contra; sic fit conversio tota.

new proposition, or obverse, expresses simply the same fact as the old, seen from its *other side*, and is no inference properly so called, but only the same fact in a different wording.

The obverses of the four propositional forms, A, E, I, and O, may be exhibited as follows :—

<i>Original Proposition.</i>	<i>Obverse.</i>
A. <i>All S is P.</i>	E. <i>No S is not-P.</i>
E. <i>No S is P.</i>	A. <i>All S is not-P.</i>
I. <i>Some S is P.</i>	O. <i>Some S is not not-P.</i>
O. <i>Some S is not P.</i>	I. <i>Some S is not-P.</i>

As to the original proposition A, *P* and *not-P* make up a universe, and are wholly exclusive. Therefore if *All-S* be included in *P*, it lies without *not-P*; that is to say, *No S is not-P*. As to E, since *No S is P*, *All-S* lies in the other part of the universe, namely, in *not-P*; that is, *All S is not-P*. As to I, since *Some S is P*, this same *Some-S* lies wholly without the portion of the universe *not-P*. That is, *Some S is not not-P*. Lastly, as to O, since *Some S is not P*, it lies wholly without *P*; that is to say, in the part of the universe *not-P*; so that this same *Some-S* is *not-P*, or, *Some S is not-P*. These all follow, by the law of excluded middle.

**Contra-  
position.** § 57.—**Contraposition** is the process of inference from a given proposition another called the *contrapositive*, differing from the original proposition in having for its subject the contradictory of the original predicate. The contrapositive is, thus, the converse of the obverse of the original proposition. The process has been alluded to under the head of *Conversion* in relation to O propositions, a class declared not to be convertible at all, taking the term *conversion* as employed in the ordinary sense. If we compare the contrapositive or contrapositive converse of O with the converses of A, E, or I, we see that the new subject and predicate are, *not-P* and *S*, instead of *P* and *S* respectively. Apart, therefore, from the duplex character of the process employed, the result does not satisfy the definition of ordinary conver-

sion. Contraposition, however, stands on its own basis as a legitimate procedure in immediate inference.

As a general illustration of obversion and contraposition, taking for our fundamental proposition, *All S is P*, we have, by obversion, *No S is not-P*; and hence, by conversion, *No not-P is S* (contrapositive), whence, by obversion, *All not-P is not-S*; and, therefore, by conversion, *Some not-S is not-P*, whence, by obversion, *Some not-S is not P*, with a difference in quality: and so on.

If, employing the notation suggested in § 13, we denote the A proposition by *SaP*, the E by *SeP*, the I by *SiP*, and the O by *SoP*, also *not-S* by *S'*, and *not-P* by *P'*, we may exhibit compactly the obverses and contrapositives of A, E, I, and O, in the following tabular form:—

<i>Original Proposition.</i>	<i>Obverse.</i>	<i>Contrapositive.</i>
A. <i>SaP</i> .	E. <i>SeP'</i> .	E. <i>P'eS</i> .
E. <i>SeP</i> .	A. <i>SaP'</i> .	I. <i>P'iS</i> .
I. <i>SiP</i> .	O. <i>SoP'</i> .	None.
O. <i>SoP</i> .	I. <i>SiP'</i> .	I. <i>P'iS</i> .

§ 58.—Very closely associated with the fore-  
Inversion. going is the process of immediate inference called Inversion (see Keynes' *Formal Logic*, § 69), which may be defined, the inferring from a given proposition of another having for its subject the contradictory of the original subject. Of the two propositions so related, the original proposition may be called the *invertend*, and the inferred proposition, the *inverse*. Thus, the inverse of A, or *SaP*, is *S'oP*, of E, or *SeP*, is *S'iP*, of I, or *SiP*, is wanting, and of O, or *SoP*, is also wanting.

Hence, with *S* and *P* as the subject and predicate, respectively, of the original proposition, the *predicate* of the obverse is *not-P*, or *P'*, the *subject* of the contrapositive is *not-P*, or *P'*, and the *subject* of the inverse, *not-S*, or *S'*.

If we apply the processes of immediate inference thus far discussed to the development of the meanings of the A, E, I, and O propositions, we find, employing abridged notation, that from the A proposition, or *SaP*, are derivable the forms, *SeP'*,

*PiS, PoS', PeS, P'aS', S'oP, and S'iP'*, and from the *E* proposition, or *SeP*, the forms, *SaP', PeS, PaS', PiS, PoS', SiP, and S'oP'*, called, respectively, the obverse, the converse, the obverted converse, the contrapositive, the obverted contrapositive, the inverse, and the obverted inverse<sup>1</sup> of the original proposition ; from the *I* proposition, or *SiP*, the forms, *SoP', PiS, and PoS'*, or its obverse, converse, and obverted converse ; and from the *O* proposition, or *SoP*, the forms, *SiP', PiS, and P'oS'*, or its obverse, contrapositive, and obverted contrapositive ; the *A* and the *E* thus each giving seven, and the *I* and the *O*, each, three, inferable forms from the original proposition.

**Mutual Relations of certain Propositions.** § 59.—If we consider two propositions into which only the same terms, say *S* and *P*, or their contradictories, *not-S*, and *not-P*, enter, such propositions may have one of six different kinds of relation one to the other.<sup>2</sup>

1. They may be mutually inferable  
one from the other ; as, for example, *All S is P*, and *No S is not-P*, or again, *All S is P'*, and *No S is P*. In such a case the relation is expressed by saying that the propositions are equivalent.

2. They may hold the relations of the *A* and *I*, or the *E* and *O* of the square of opposition, either (a) explicitly, that is to say, as they stand, or (b) implicitly, in being reducible to equivalents holding these relations. In these cases, it will be possible formally to infer the second from the first, but not *vice versa* ; and the propositions are said to be (a) *explicitly*, or (b) *implicitly subaltern*.

3. They may hold the relations, either explicitly or implicitly, of the *I* and *O* propositions of the square of opposi-

<sup>1</sup> It will be seen that the inverse is obtained only when the given original proposition is universal. It is proper to observe that the validity of *S'oP*, the inverse of *SaP*, or *A*, has been impugned by some logicians, on the ground that *P*, undistributed in *SaP*, is distributed in *S'oP*. See Ray's *Deductive Logic*, p. 313, and Keynes' *Formal Logic*, § 70.

<sup>2</sup> See Keynes' *Formal Logic*, § 57.

tion; in which cases they are said to be (a) *explicitly*, or (b) *implicitly subcontrary*.

4. They may be entirely *independent* one of the other; the truth or the falsity of either not affecting the truth or the falsity of the other.

5. They may (a) hold the relations of the A and E of the square of opposition, or (b) be reducible to propositions holding these relations, in which cases they are said to be, (a) *explicitly*, or (b) *implicitly contrary*.

6. They may (a) hold the relations of the A and O, or the I and E, or be reducible to equivalent propositions holding these relations, in which cases they are said to be (a) *explicitly* or (b) *implicitly contradictory*.

As illustrations, suppose the question were (1) to state the logical relation of *All X is Y* to *No not-X is not-Y*, since these propositions have different subjects and predicates our first business is to reduce them to *equivalents*, if possible, having the same subject and predicate; the mark of equivalency being mutual inferability. The second of the given propositions becomes by simple conversion, *No not-Y is not-X*, which, a glance at Fig. 1 shows, is equivalent to, *All not-Y is X*. Again, the first of the given propositions is seen from Fig. 2 to be equivalent to, *No X is not-Y*, that is to *No not-Y is X*. Consequently the two given propositions hold to each other the relation of the E and A of the square of opposition, and are *implicit contraries*. Again, suppose the question were (2) to state the logical relation of *All X is Y*, and *Some not-Y is X*, we have for an equivalent of the first of these, *No X is not-Y*, and for an equivalent of the second, *Some X is not-Y*; and consequently, the two given propositions hold to each other the relations of the E and the I of the square, and are *implicit contradictories*.

Not-X

X

FIG. 1.

Not-Y

Y

FIG. 2.

Not-Y

Y

FIG. 3.

**Other modes of Immediate Inference.** § 60.—Besides the foregoing, there are other kinds of immediate inference that must be briefly noticed. Of these, immediate inference by (1) *Added Determinants* consists in joining an attribute to the subject and predicate of a proposition, so that the new proposition thence resulting shall follow from the old. Thus, from “A negro is a fellow-creature,” it follows, that “A negro in suffering is a fellow-creature in suffering.” In some cases, however, the addition of a determining word, may not have the same effect upon both subject and predicate; and, as a result, the new proposition is not to be taken as following from the old. For example, it does not follow that, because “A cottage is a building,” “A large cottage is a large building,” since what may lie close to the greater extreme of the class *cottages* may lie very remote from the greater extreme of the class *buildings*. Immediate inference by (2) *Complex Conceptions* consists in employing the subject and predicate of a proposition as parts, or marks, of a new and more complex conception. For example, “Oxygen is a gas; therefore, the solidification of oxygen is the solidification of a gas.” Obviously, this kind of inference is very closely related to the preceding. It is not validly drawn when the expression of the new complex conception has a different meaning in both subject and predicate. Thus, it does not follow that, because, senators are voters, a majority of senators is a majority of voters.

Boole and De Morgan consider that inference in Hypotheticals is not different from immediate inference; and Mr. Bain discusses it under the head of Equivalent Propositional Forms. We postpone the treatment of it, however, to a later chapter.

## CHAPTER .III

### ON SYLLOGISM

Reasoning : § 61.—Reasoning may be defined as the Argument; process,<sup>1</sup> or the expression of the process, of Syllogism. deriving one judgment from another. The expression of a reasoning is also called an *Argument*. A reasoning or argument may be of various degrees of length, and composed of one, or of several steps. Each step of a reasoning must be, if completely set forth, what we are going to define as a syllogism. A syllogism is a kind of mediate inference or reasoning, composed, when fully expressed, of three propositions of such quality, and quantity, and united in such a way, as that the third, which is called the conclusion, follows of necessity, from the two first, called the premisses (*prae missae*). The term is derived from the Greek *συλλογισμός*, computation, and appears to have acquired the meaning it bears in Logic from the resemblance between computation, understood in the mathematical sense of gathering the result of a reckoning, and that gathering the result of other judgments which is called reasoning. A syllogism may be defined otherwise, after Aristotle, as a sentence or thought, in which from something laid down, and admitted, something distinct from what we have laid down follows of necessity; or, it may be defined as, the explicit setting forth of the legitimate procedure in reasoning from one proposition

<sup>1</sup> “The term, *Discourse*, is often used for the reasoning process, strictly considered; and *discursive*, for mediate, in relation to inference.”

to another by means of an intermediate proposition distinct from either. The grounds of the legitimate procedure in reasoning from one proposition to another by means of an intermediate proposition will appear as we go on; so that we shall be able to distinguish syllogisms from groups of propositions that might without sufficient examination be mistaken for such. The following are examples of syllogisms:—

1. Every man is mortal ;  
Socrates is a man ;  
∴ Socrates is mortal.
2. Magnesium is a metal ;  
All metals combine with oxygen ;  
∴ Magnesium combines with oxygen.
3. The boundaries of solids have no thickness ;  
Surfaces are the boundaries of solids ;  
∴ Surfaces have no thickness.

Syllogism and Mediate Inference are often taken as synonymous. There are valid mediate inferences, however, which do not appear to be easily thrown into the long-accepted form of syllogism properly so-called. But syllogism, as illustrated by the examples just given, may be regarded as the final form in which the ultimate steps of most mediate reasoning can be exhibited; hence the importance of noting its distinguishing features and essential conditions.

§ 62.—The elements of a syllogism, called by <sup>Elements</sup> and Form <sup>the scholastics,</sup> *materia syllogismi*, are of two of Syllogism. kinds, propositions and terms.

In a syllogism, fully written out, there are, as will be understood from the definition, two propositions called the Premisses, from which can be inferred a third proposition, called the Conclusion. These three propositions form what is called the *proximate matter* of the syllogism. In the first of our illustrative examples, the premisses are: All men are mortal; Socrates is a man; and the conclusion is, Socrates

is mortal. The other elements of a syllogism are the terms of the propositions composing it, which constitute the material elements of these propositions, considered in themselves, and are called the *remote matter* of the syllogism. It will be seen that, in the syllogism under consideration, there are in the given premisses three, and only three, objects of thought before us, as expressed by the three terms, man, mortal (being), Socrates ; and of these, the term, man, occurs twice. Thus, one of the premisses enables us to predicate mortal (being) of every man, and, therefore, of any man whatever ; and, since, by the other premiss, Socrates is a man, we may therefore, in accordance with the *Dictum de omni et nullo* predicate it of Socrates ; whence the conclusion.

Of the three terms, two, known as the *Extremes*, appear in the conclusion, one as the subject of it, and the other as the predicate. That term which is the subject of the conclusion is called the *minor term*, and that which is predicate, the *major term* ; because in affirmative syllogisms of the First Figure (§ 66) if the case be not one of mere co-extensiveness, these terms are, respectively, of *less* and of *greater* extent than that term which is repeated in the premisses, and hence called the *middle term*.

Of the two premisses, that which contains the major term is called the *major premiss* ; and when the parts of the syllogism are written in order, by convention, stands first. The premiss containing the minor term is called the *minor premiss*, and in every such case stands second ; the conclusion being written last in order.

The *form* of a syllogism, and all that is actually affirmed in it, is the necessary consequence of the conclusion from the premisses. It is the logical connexion between the conclusion and the premisses, or the logical connexion and concatenation of the three propositions *inter se*. The form or essence of a syllogism consists, not in the truth of the propositions laid down, or of that which is arrived at, but in the necessary consequence of the latter from the former ; implying that a new and distinct proposition has been produced, not a mere

repetition of the antecedents, the truth of which cannot be denied without impugning the admitted truth of such antecedents. The conclusion itself is one of the material elements of the syllogisms ; the necessary consequence of the conclusion from the premisses is the form.

Sometimes, a syllogism is so written that the conclusion is stated first, as for example, All S is P, because, All S is M, and, All M is P. Thus stated, the conclusion was by the older logicians called the *Quaestio* or *Problema* ; and the middle term, however the syllogism was written, and not, as now, the whole process, was called the *Argument*.

Sometimes one of the premisses, as being easily supplied, is left unexpressed, in which case the syllogism is called by Aldrich an *Enthymeme* ; as if we should say, Every man is mortal, therefore Socrates is mortal. A discussion of the different orders of enthymemes will be found towards the end of the present chapter.

In the first of the three examples of syllogisms given above, the terms are viewed in *Extension* ; the class 'mortal' contains under it the class 'man,' and the class 'man' contains in it the individual, Socrates. When the terms are viewed in *Intension* or *Comprehension*, the reasoning stands thus : Socrates is a man, Every man is mortal, ∴ Socrates is mortal. In this form, the syllogism may be read :—The attributes of Socrates contain the attributes of a man ; The attributes of a man contain the attribute, mortal ; Therefore, the attributes of Socrates contain the attribute, mortal.

**Different Kinds of Syllogisms.** § 63.—Syllogisms, according to the character of their premisses, may be classified in several ways.<sup>1</sup> The most suitable classification for our

<sup>1</sup> Kant's Categories serve as bases for a variety of classifications. According to Kant, Judgments or Propositions differ in four respects, namely, in respect of Quantity, Quality, Relation, and Modality. In respect of Quantity, propositions are divisible into Universal, Particular, and Singular (in Logic, virtually Universal) ; in respect of Quality, into Affirmative, Negative, and Infinite (Affirmative in form, but Negative in meaning) ; in respect of Relation into Categorical (the relation subsisting between the predicate and the subject being unconditioned affirmation, or negation), Hypothetical (the relation being cause and

purpose here, however, as enabling us to group them in such a way as to conveniently bring into prominence the greatest number of their most important peculiarities, is a classification according to *relation*; that is to say, one based on the classification of their premisses according to relation, in other words, according as they are categorical, hypothetical, or disjunctive propositions,—a distinction based, in the ultimate, on the relation of the predicate to the subject in these different kinds of propositions. The first broad classification of syllogisms, according to relation, is into *Pure* and *Mixed*. Syllogisms are said to be *Pure*, when the premisses are (1) both categorical, or (2) both hypothetical, or (3) both disjunctive, giving *Pure Categorical*, or *Pure Hypothetical* or *Pure Disjunctive Syllogisms*. Syllogisms are said to be *Mixed*, (1) when the major premiss is hypothetical, the minor, categorical, giving *Mixed Hypothetical Syllogisms*, or (2) when the major premiss is disjunctive, the minor, categorical, giving *Mixed Disjunctive Syllogisms*, or, (3) when the major is hypothetical, the minor, disjunctive, giving *Dilemmas*.

**General Rules of Syllogism.**—We have already given various ex-

pressions based on the necessary laws of thought for the fundamental canon of syllogism. Of these, Aristotle's, or the *Dictum de omni et nullo*, is:—

*Quicquid de omni valet, valet etiam de quibusdam et de singulis; quicquid de nullo valet, nec de quibusdam valet, nec de singulis;* Whatever is predicated distributively, whether affirmatively or negatively, of any class of things, may be predicated in like manner of anything comprehended in that class. The *Dictum de omni et nullo*, however, applies directly only to syllogisms of the following form:—

effect, or condition and conditioned), and *Disjunctive* (the relation of disjunctive predication); and in respect of Modality, into *Problematical*, *Assertorial*, and *Apodictical*. Relation in propositions is understood in another sense, namely, as having to do with the *coincidence*, or *non-coincidence*, of the predicate and the subject, and leading to a division of affirmative propositions into *attributive*, or *de inesse*, and *substitutive*; but with relation in this sense we are not concerned here.

$$\begin{aligned} M &\dots P; \\ S &\dots M; \\ \therefore S &\dots P; \end{aligned}$$

that is to say, in which the middle term is subject of the major premiss, and predicate of the minor ; but it applies indirectly to all other syllogisms, since they are reducible to this form ; and from it are easily derived the general rules of syllogism, which may be stated as follows :—

(1) In the premisses, the middle term must be distributed once at least.

If we have given to us, for example, that *Some M is P*, *All S is M*, there can be no inference as between *S* and *P*. For, *All S is M*, means *All S is some M* ; but as we are not entitled to assume that this particular *some M* is the *some M* of which *P* is predicated, *S* and *P* are mutually unpredicable.

The value of the two propositions such as those before us as a basis for inference is to be estimated according to the least favourable interpretation of *M* in each case (Law of Parsimony<sup>1</sup>). This is equivalent to saying, that *M* is, in these propositions, to be considered as taken in two different parts of its extent ; so that we have no medium of comparison, and no real middle term. In fact, the propositions, in reality, contain *four terms*,<sup>2</sup> instead of three. A conclusion purporting to follow from two premisses of this kind involves a fallacy of undistributed middle.

(2) No term is to be distributed in the conclusion which was not distributed in one of the premisses.

This is obvious : for, otherwise, we should be arguing from a part to the whole. Violations of this rule, as for example,

<sup>1</sup> The law or principle of Parsimony forbids assuming more than is necessary ; and in the case of a choice being offered to us, in a given case, between a more and a less favourable interpretation, leads us to choose the latter as the safer for purposes of inference.

<sup>2</sup> Whately's first and second general rules are :—(a) Every syllogism has three and only three terms ; and (b) Every syllogism has three and only three propositions. These, however, are structural in character, and, assuming the structure of the syllogism as determined once for all by the definition, can hardly be regarded as appropriately included among the general rules.

in the quasi-syllogisms, *Some M is P*; *No S is M*;  $\therefore$  *No S is P*; and, *All M is P*; *Some S is M*;  $\therefore$  *All S is P*, involve, if the subject is unwarrantably distributed in the conclusion, *Illicit process of the Minor Term*, if the predicate is unwarrantably distributed, *Illicit process of the Major Term*.

(3) *If both premisses be negative no conclusion can be drawn.*

A pair of negative premisses assert that there is no agreement between the middle term and either of the others. Agreement can in no way be affirmed from perfect disagreement. We have, therefore, no basis for an assertion of relation between the major and minor terms.

(4) *If one of the premisses be negative the conclusion must be negative.*

For example, taking for premisses, *No P is M*; *All M is S*, the universe of *P* and the universe of *M* lie wholly apart; and this is a basis but for the predication of pure disagreement. This disagreement extends to whatever either coincides with, or lies within, either universe. Hence, as *Some S is M*, we may assert that *Some S* is wholly excluded from *P*, that is, *Some S is not P*. In general terms, since one premiss is assumed to be negative, the other must be affirmative; and, if in the one premiss we affirm agreement between the middle term and one of the extremes, and in the other deny any agreement between the middle term and the other extreme, there can be no agreement between the two extremes, that is to say, the subject and the predicate of the conclusion. Hence, if one of the premisses be negative, the conclusion must be negative.

(5) *If both premisses be particular no conclusion can be drawn.*

By the third rule, from two particular negatives nothing follows. From two particular affirmatives, in like manner, there can be no conclusion, owing to an undistributed middle. And, in the remaining case, namely, when we are given a particular negative and a particular affirmative, the negative

distributing the middle term as its predicate, there can be no conclusion; for any quasi-conclusion should be negative, and this would involve illicit process of the major term.

(6) *If one of the premisses be particular the conclusion must be particular.*

The premisses must be either both affirmative, and therefore, A and I; or, one affirmative and the other negative, that is to say, O and A, or I and E. With the affirmative combination, if we allot the one place of distribution to the middle term, there can be no distribution of the extremes; hence the conclusion must be I. With the negative combinations, there must be, as well as a distributed middle, a distributed major term; and, as there are but two distributions possible in the premisses, there must be an undistributed minor term, and hence the conclusion must be I.

**Some obvious Deductions.** § 65.—From the foregoing general rules the following deductions are easily drawn:—

1. The minor premiss must be affirmative, if in the major premiss the major is predicate.
2. The conclusion cannot be a universal affirmative, if the minor term is predicate of the minor premiss.
3. If the conclusion be universal, the middle term can be but once universal in the premisses.
4. If the middle term be twice universal, the conclusion must be particular.
5. If either premiss be particular, the middle term is but once distributed.
6. When the minor term is predicate of the minor premiss, or the major term is subject of the major, the conclusion cannot be A.
7. When the minor term is universal in the minor premiss, both premisses must be universal.
8. If the minor premiss be negative, the major must be universal.
9. If the conclusion be negative, one of the premisses must be negative.

10. If the conclusion be negative, the major premiss cannot be a particular affirmative.

11. If one premiss be O, the conclusion must be O, and the other premiss, A.

12. If the major premiss of a valid syllogism be affirmative, and the major term distributed both in premisses and conclusion, while the minor term is undistributed in both, the major premiss must be A, the minor, O, and the conclusion, O.

**Of the Four Figures.** § 66.—The *Figure* of a syllogism is the arrangement or disposition it exhibits of the middle term and the extremes.

As we have already shown, there are four possible arrangements of the major, middle, and minor terms in a syllogism ; and the four figures thus arising may be defined by specifying the positions occupied in each case by the middle term.

In the First Figure, the middle term is the subject of the major premiss and the predicate of the minor ; in the Second Figure, it is the predicate of both ; in the Third Figure it is the subject of both ; and in the Fourth Figure it is the predicate of the major premiss and the subject of the minor.

The following are illustrations of the figures, taking S, M, P, as the minor, middle, and major terms, respectively :—

	First Figure.	Second Figure.
Major Premiss,	Every M is P ;	Every P is M ;
Minor Premiss,	Every S is M ;	No S is M ;
Conclusion,	Every S is P ;	No S is P.

	Third Figure.	Fourth Figure.
Major Premiss,	Every M is P ;	Every P is M ;
Minor Premiss,	Every M is S ;	Every M is S ;
Conclusion,	Some S is P ;	Some S is P.

The respective dispositions of the terms will, perhaps, be

the more easily remembered, if we omit all in these syllogisms, but what is distinctive of figure, thus :—

First Figure. Second Figure. Third Figure. Fourth Figure.

M . . . P	P . . . M	M . . . P	P . . . M.
S . . . M	S . . . M	M . . . S	M . . . S.
S . . . P	S . . . P	S . . . P	S . . . P.

**Dicta of the Figures.** § 67.—The first figure was called by Aristotle the *perfect figure*, owing to the peculiar clearness of the reasoning in it, this figure being, as already observed, the only one to which the *Dictum de omni et nullo* is directly applicable. The second and third were called by Aristotle the *imperfect figures*. The fourth was called the *Galenian*, after Galen, who is thought to have been the first to formulate it. Aristotle did not recognise this figure; but in the sense explained it may be classed with the second and third as imperfect as well.<sup>1</sup> Logicians have given for the imperfect figures special dicta corresponding to the *Dictum de omni et nullo* for the first figure. Of these, the dictum of the second figure, called the *Dictum de diverso*, as enunciated by Mansel, is: “If a certain attribute can be predicated affirmatively or negatively of every member of a class, any subject of which it cannot be predicated does not belong to that class.” For the third figure there are: (a) the *Dictum de exemplo*: “If a certain attribute can be affirmed of any portion of the members of a class, it is not incompatible with the distinctive attributes of that class,” and (b) the *Dictum de excepto*: “If a certain attribute can be denied of any portion of the members of a class, it is not inseparable from the distinctive attributes of that class.” For the fourth figure there is the *Dictum de reciproco*, in its two parts—(a) “If no M is B, no B is this or that M”; and (b) “If C is or is not that B, there are B's which are or are not C.”

<sup>1</sup> The fourth figure has been objected to, on the grounds (1) that it is but an inverted and clumsy way of stating what would much more naturally fall into the first figure, and (2) that it is inferior to the first figure, since we can prove an I only from premisses which, by transpositions, would in the first figure give a conclusion in A. But see Keynes' *Formal Logic*, § 209.

Hamilton gives the following dicta or canons for the first, second, and third figures:—

*Fig. 1.* In as far as two notions are related, either both positively, or one positively and the other negatively, to a third notion to which the one is subject and the other predicate, they are related positively or negatively to each other as subject and predicate.

*Fig. 2.* In as far as two notions, both subjects, are, either each positively, or the one positively and the other negatively related to a common predicate notion, in so far are those notions positively or negatively subject and predicate of each other.

*Fig. 3.* In as far as two notions, both predicates, are each positively, or the one positively and the other negatively, related to a common subject notion, in so far, are those notions positively or negatively subject and predicate of each other.

**§. 68.**—From the general rules of syllogisms established in § 64, certain special rules for each figure are easily deduced, when we have regard to the positions of the terms in the respective premisses. Such special rules, however, are to be taken in conjunction with, not by way of substitution for, the general rules already laid down.

I. The special rules of the First Figure are:—

(1) *The minor premiss must be affirmative.*

For, if in the first figure, we suppose the minor premiss negative, we easily see that, with a necessarily affirmative other premiss, we should have an illicit process of the major.

(2) *The major premiss must be universal.<sup>1</sup>*

For, if not, let it be particular. Then, its subject,—that is to say, the middle term, being undistributed in the major premiss,

<sup>1</sup> Regarding mediate inference as proceeding from the subsumption of a particular case under a general rule, or sumption, Hamilton, and some logicians after him, call the ground proposition, or major premiss, the *sumption*, the applying proposition, or minor premiss, the *subsumption*, and enunciate the special rules accordingly.

would have to be distributed in the minor premiss, and as the predicate of that premiss. But this is impossible, since the minor premiss is affirmative. Hence, the major premiss cannot be particular.

II. The special rules of the Second Figure are :—

(1) *One of the premisses must be negative.*

Else, there would be an undistributed middle.

(2) *The conclusion must be negative.*

This is obvious, since one of the premisses is negative. Some logicians, therefore, dispense with this special rule, as superfluous.

(3) *The major premiss must be universal.*

Else, there would arise an illicit process of the major.

III. The special rules of the Third Figure are :—

(1) *The minor premiss must be affirmative.*

Else, there would be an illicit process of the major.

(2) *The conclusion must be particular.*

Else, there would be an illicit process of the minor.

IV. The special rules of the Fourth Figure are :—

(1) *If the major premiss is affirmative, the minor premiss must be universal.*

This, to avoid an undistributed middle.

(2) *If one premiss is negative, the major premiss must be universal.*

This, to avoid an illicit process of the major term.

(3) *If the minor premiss is affirmative, the conclusion must be particular.*

This, to avoid an illicit process of the minor term.

The special rules of this figure are hypothetical, because we cannot, as in the other figures, absolutely determine the *quality* of either premiss ; as they may be both affirmative, or either negative.

**Character-  
istics and  
Special Uses  
of the  
Figures.**

§ 69.—The first figure is the only one in which conclusions of all the forms, A, E, I, O, can be proved ; no other giving an A conclusion. It is necessarily, therefore, of the greatest importance in deductive science, the object of which is to draw conclusions in A. In the second figure, only negatives can be proved. It has been called the *exclusive*, or *exclusory* figure, for it is the figure suitable to be employed in the process called *abscissio infiniti*, by which we go on excluding one by one, certain suppositions, or certain classes of things, from that the real nature of which we seek to ascertain. In the third figure, only particulars can be proved. This figure best answers our purpose if we seek to establish *exceptions* to what an opponent maintains to be a law or rule, or seek to disprove the asserted universality of a proposition. It suggests itself also as the suitable figure to employ when the middle is a singular term. Accordingly, Lambert observes that the first figure is suited to the discovery or proof of the properties of a thing ; the second, to the discovery or proof of the distinctions between things ; and the third, to the discovery or proof of instances or exceptions. The fourth figure, which, by the majority of logicians is not recognised at all, Lambert considered to be suited to the discovery or exclusion of the different species of a genus.

**Some Exer-  
cises on the  
General and  
the Special  
Rules.**

§ 70.—The following will be found useful exercises on the general rules of syllogism, and the special rules of the figures :—

1. What kind of proposition does not occur as one of the premisses of the first figure ?
2. What peculiarity distinguishes the conclusions of the third figure ?
3. What kind of proposition cannot be proved by the fourth figure ?
4. What figure gives but negative conclusions, and why ?
5. Show that there can never be more than two universal terms in the premisses more than in the conclusion.

6. Show that there can never be more than one particular term more in the premisses than in the conclusion.

7. Show that if there be one universal term more in the premisses than in the conclusion, either but one of the premisses, or the conclusion, shall be universal.

8. Prove that a particular conclusion, without a particular premiss, points to the assumption in the premisses of something superfluous.

9. When the conclusion of a syllogism in the third figure is substituted for the major premiss, what figure results?

10. Prove that O cannot be a premiss in the first or fourth figure, and that it cannot be the major in the second, or the minor in the third.

**§ 71.**—The *Mood* of a syllogism is the group of symbols that represents in the conventional order the premisses and conclusion of that syllogism respectively, as to quantity and quality. According to the conventional order, the letter representing the major premiss stands first in the group, that representing the minor premiss, second, and that representing the conclusion, last.

The number of conceivable moods is the number of permutations of four things three at a time ; the repetition of any of the things up to three times being admissible. To determine this we first find the number of permutations of four things two at a time, repetitions being allowed ; we thus get  $4^2 = 16$  ; and combining with each of these the four things in turn, we get  $4^3 = 64$ , the total number. There are thus sixty-four conceivable moods ; but, of these, all except eleven must be rejected, irrespective of considerations of figure, as violating the general rules of syllogism. The eleven remaining moods which have to be still further examined are :—AAA, AAI, AEE, AEO, AII, AOO, EAE, EAO, EIO, IAI, OAO.

Testing these remaining moods by the special rules of the figures, we find in harmony with these rules, in the first figure, the moods, AAA, AAI, EAE, AII, EAO, EIO ; in the second, the moods, EAE, EAO, AEE, AEO, EIO, AOO ;

in the third, the moods, AAI, EAO, IAI, OAO, AII, EIO ; and, in the fourth, AAI, AEE, IAI, EAO, EIO, AEO.

But though the special rules have done their work, a still further reduction is possible, seeing that, the moods AAI, EAO, in the first figure, EAO, AEO, in the second, and the mood AEO in the fourth, being included respectively in AAA, EAE ; in EAE, AEE ; and, lastly, in AEE, in these respective figures, though their conclusions are valid, are superfluous, since for the particular conclusions in these moods, the premisses justify universals.

Removing, therefore, these superfluous moods, there remain the following nineteen valid moods, four in the first figure, four in the second, six in the third, and five in the fourth ; namely, in the first figure, the moods, AAA, EAE, AII, EIO ; in the second figure, the moods, EAE, AEE, EIO, AOO ; in the third figure, the moods AAI, IAI, AII, EAO, OAO, EIO ; and in the fourth figure, AAI, AEE, IAI, EAO, and EIO.

**Subaltern Moods and Weakened Conclusions.** § 72.—We have seen that when, by the assistance of the general and the special rules, the conceivable moods of syllogism are diminished by those that must be regarded as invalid, there remain 24 perfectly legitimate moods, six in each figure. Five of these legitimate moods, however, we have omitted as superfluous, since for the particular conclusions in them the premisses justify universals ; so that the number of moods of practical importance is reduced to 19. The moods thus omitted, namely, AAI, EAO, in the first figure ; EAO, AEO, in the second ; and AEO, in the fourth, are called *subaltern moods*, their conclusions being the subalterns of the corresponding universals. They are also called *weakened syllogisms*, and their conclusions, *weakened conclusions*. It is obvious that in the third figure in which no universal conclusion can be inferred, there can be no such syllogisms or conclusions.

**Fundamental and Strengthened Syllogisms.** § 73.—De Morgan calls syllogisms *fundamental* and *strengthened*, of which neither of the premisses is stronger than is necessary to produce the con-

clusion. Of the 19 valid moods of the final enumeration in § 71, but 15 can be regarded as fundamental in the sense thus explained, since in four of them, AAI, and EAO, in the third figure and also in the fourth, the same conclusion can still be obtained, if for one of the premisses we substitute its subaltern. Syllogisms such that one of the premisses is unnecessarily strong for the conclusion are called *strengthened syllogisms*. It is obvious that the conclusions in such syllogisms must be no other than I and O; and that since these are not necessitated by the rule that, the conclusion follows the weaker part, both premisses in such syllogisms must be universals. Our search for strengthened syllogisms must be made, therefore, in the list of legitimate moods with two universal premisses and a particular conclusion. In fact every such mood is a strengthened syllogism, with the exception of AEO in the fourth figure.

**Examples on § 74.**—The following are examples for the Figures and Moods. exercise :—

1. Why, by the general rules of syllogism, is IEO to be rejected?
2. What general rules are violated by AIA, EIE, IOO, IIA, and OIO?
3. Why is IEO to be rejected in the third figure, and why AII in the fourth?
4. Why are EEE, EAA, AAE, III, IAA, to be rejected in all the figures?
5. Show that, if by *mood* we mean barely the particular group of symbols, without reference to its recurrence in the respective figures, the 19 moods of the final enumeration in § 71 are reducible to ten; and that A may be said, on this understanding, to be proved in one mood, E, in two moods I, in three, and O, in four.
6. Arrange in correct logical order the following syllogisms and exhibit the moods to which they belong:—  
(1) Some B's are A's; No C's are B's; ∴ Some A's are not C's. (2) All A's are B's; No B's are C's; ∴ No A's are C's.

7. In what figures are the following to be regarded as satisfactory premisses :—AA, EA, OA, AI?

8. Under what circumstances, and in what relations, may the same syllogism be regarded as at once a strengthened and a weakened syllogism?

9. Prove that a universal conclusion of either quality can follow only from universal premisses in which the middle term is differently quantified in the premisses.

10. Show that moods in which a double distribution of the middle term occurs have premisses needlessly wide for the conclusion.

**Reduction.** § 75.—Although distinct canons have been given for the figures, and each has by some logicians been regarded as an independent illustration of the fundamental laws of thought, those who, following Aristotle, regard the first as the only perfect figure, and its dictum the clearest, have felt it desirable to establish the validity of the moods of the other figures by suitably transforming reasoning in them to the first figure. This process is called *Reduction*, and is of two kinds. When the moods of the second, third or fourth figure, are reduced to the first figure, by conversion, obversion, and the transposition of premisses, we employ what is called *Ostensive Reduction*. When, instead, we show by means of the first figure and the laws of opposition that the contradictory of the conclusion is false, and, therefore, that the conclusion itself is true, the process is called *Indirect Reduction, Reductio ad impossibile* (or *ad absurdum*). Either of these methods may be employed, as seems most suitable, for bringing reasoning in the second, third, and fourth figures under the first, and so basing it immediately on the *Dictum de omni et nullo*. But reduction, in a wider sense, may be to any required figure.

**The Mnemonic Verses.** § 76.—Omitting the subaltern moods, there remain, as we have shown, 19 valid moods, four in the first figure, four in the second, six in the third, and five in the fourth. These are indicated for each figure by the vowels in the words in italics in the following mnemonic verses :—

*Barbārā, Celārent, Dārīi, Ferīōque prioris ;  
 Cēdārē, Cāmēstres, Festinō, Bārōcō, secundae ;  
 Tertia, Dārapti, Dīsāmīs, Dātīsī, Fēlapton,  
 Bōcārdō, Fēriōn, habet : Quarta insuper addit  
 Brāmanīp, Cāmēnes, Dīmāris, Fēsāpō, Frēsīson.*<sup>1</sup>

Thus, the moods in the first figure, AAA, EAE, AII, EIO, are called respectively the artificial names, Barbara, Celarent, Darii, Ferio ; and so of the rest.

On considering the valid moods in each figure, as thus displayed, the following points which had been before us in § 69 will be readily noted :—

(a) In the first figure we have the four conclusions, A, E, I, O.

(b) In the second figure we have but the negative conclusions, E and O.

(c) In the third figure we have but the particular conclusions, I and O.

(d) In the fourth figure, of the four conclusions, A, E, I, O, there is wanting but the universal affirmative.

Other peculiarities also will be readily seen ; and these mnemonic lines, besides their other uses, afford convenient materials for the construction of exercises involving the determination of mood and figure by the general and the special rules, or the limitation of certain moods to determinable figures, and the like, all presenting themselves on mere inspection.

The main function of these mnemonics, however, is to show how moods of the second, third, and fourth figures can be *reduced* to corresponding moods of the first figure. To this end they are adapted by ingenious contrivances. The moods of the first figure are distinguished by the initial letters, B, C,

<sup>1</sup> It will be interesting to compare with these the following, being the earlier form of the mnemonic verses. The moods of the fourth figure, with, however, the premisses transposed, are written after those of the first as indirect moods of that figure :—

Barbara, Celarent, Darii, Ferio, Baralipton,  
 Celantes, Dabitis, Fapesmo, Friesomorum,  
 Cesare, Camestres, Festino, Baroco, Darapti,  
 Felapton, Datisi, Disamis, Bocardo, Ferison.

D, F ; and the occurrence of one of these as the initial letter of any of the moods of the other figures indicates that such mood, or *reducend*, has for *reduct* the mood of the first figure correspondingly indexed. The other consonants that occur in the moods of the first figure are non-significant.

The letters, *m*, *s*, *p*, and *k*, that occur in the moods of the second, third, and fourth figures, are, however, directive, or operative in character, and show how the reduction to the first figure in each case is to be performed.

Of these, *m* indicates that the premisses between which it stands, whether affected by any of the remaining symbols or not, are to be interchanged to form the reduct in the first figure.

The letter *s*, which it will be observed, stands in the moods as above enumerated always after an E, or an I, indicates that the proposition denoted by the preceding vowel is to be converted *simply*.

The letter *p*, after a premiss,—and it follows of premisses but A, indicates that such premiss is to be converted *per accidens*. When it follows the vowel of the conclusion, as it does in one case, namely, in Bramantip, it indicates the process by which from the new conclusion obtained in the first figure, the original conclusion, which is that sought to be established, is to be derived. The letter *p*, in Bramantip, indicates that the A conclusion obtained in the first figure is to be converted *per accidens*, to arrive at the I conclusion of the given syllogism in the fourth figure. The letter *s*, in a similar situation, as in Camestres, Disamis, Camenes, and Dimaris, has a similar meaning. With *s*, however, the quantity of the new conclusion remains unaltered in changing to the old ; with *p*, it is weakened. It will be seen that in the premiss-syllables *p* occurs in the minor only of the names of the moods. Its effect in this position is to preclude *metathesis*, by weakening the minor, so that we may affirm it will hold its place as a particular affirmative minor premiss in the reduct.

The letter *k*, after a premiss, shows that the mood containing it was by the older logicians reduced by the

indirect method ; the first step being to omit the premiss preceding it, and to join the other to the contradictory of the conclusion. This letter only occurs in two moods, Baroko and Bokardo ; but it is equally applicable to the other moods of the imperfect figures. If, however, we should regard *k* as a symbol denoting obversion, so that *ks* should be taken to denote contraposition, by altering the names of these moods to Faksoko, and Doksmavsk, respectively, a direct mode of reducing them would be completely indicated.

Of the other letters, *b*, except as initial, *d*, except as initial, *l*, *n*, *r*, and *t*, are non-significant.

**Examples of** **§ 77.**—The following are examples of direct. **Reduction.** reduction :—

1. Bramantip to Barbara.

All P is M ; } become { All M is S ;  
All M is S ; } { All P is M ;  
∴ All P is S.

Whence, Some S is P.

2. Camenes to Celarent.

All P is M ; } become { No M is S ;  
No M is S ; } { All P is M ;  
∴ No P is S.

Whence, No S is P.

3. Darapti to Darii.

All M is P ; } become { All M is P ;  
All M is S ; } { Some S is M ;  
∴ Some S is P.

∴ Some S is P.

4. Baroko, as Faksoko, to Ferio.

All P is M ; } become { No not-M is P ;  
Some S is not M ; } { Some S is not-M ;  
∴ Some S is not P.

∴ Some S is not P.

5. Bokardo, as Doksmavosk, to Darii.

Some M is not P ; } become { All M is S ;  
 All M is S ; } { Some not-P is M ;  
 . . . . . ∴ Some not-P is S.

Whence, Some S is not-P.

. . . . . ∴ Some S is not P.

**Reductio ad impossibile.** § 78.—To exemplify the indirect method of reduction, let us take Baroko :

All P is M ; (A)  
 Some S is not M ; (O)  
 . . . . . ∴ Some S is not P. (O)

This conclusion must be true. For, if not, suppose it to be false. Then, its contradictory must be true.

. . . . . ∴ All S is P.  
 But all P is M ;  
 . . . . . ∴ All S is M.

This syllogism, transposing the premisses, is Barbara ; and granting the premisses to be true, the conclusion, also, must be true. But if the conclusion be false, one of the premisses must be false. And the conclusion, All S is M, is false since it is the contradictory of one of the original premisses, namely,

Some S is not M.

. . . . . ∴ One of the premisses must be false ; not, however, All P is M,—this being one of the original premisses, but the other.

. . . . . ∴ All S is P, is false.

. . . . . ∴ Its contradictory, that is to say, Some S is not P, is true.

And it can be easily shown that the other moods of the imperfect figures can be established by means of reasoning in

the first figure, and the principles of opposition, in a similar way.<sup>1</sup>

In the general application of the indirect method to the imperfect moods, it is of practical use to be able to say at once which premiss should be suppressed in order that the retained premiss, with the contradictory of the conclusion, may form premisses in one of the moods of the first figure. Since the order of the terms in the conclusion is the same as in its contradictory, it is obvious that in the second and third figures the premiss to be suppressed is that in which the major or the minor term occupies the same position that it does in the conclusion. Hence, in the second figure, the minor premiss must be suppressed, and the major retained ; while, in the third figure, the major premiss must be suppressed, and the minor retained. As to the fourth figure, whichever premiss we suppress, the retained premiss, with the contradictory of the conclusion, will form premisses in the first figure.

If, further, we enquire when will the reduct syllogism in the first figure be *valid*, and when will its conclusion be *inconsistent* with the truth of the suppressed premiss, we shall find that the necessary and sufficient conditions are those expressed in the special rules of the second, third, and fourth figures, respectively.

As to the question, which term of the reducend will form

<sup>1</sup> Spalding notes two points of difference between the logical uses of indirect demonstration on the one hand, and the scientific and popular uses of it on the other :—

(1) The impossibility of the falsehood of a theorem in any of the exact sciences is demonstrable through the inconsistency of the falsehood with *any* of the truths which, either as axioms or as proved propositions, have previously received a place in the system ; but, in the indirect reduction of a syllogism the logician can assume nothing except his two premisses.

(2) In mathematics always, in other kinds of matter not seldom, the inconsistency dwelt on lies between propositions logically describable not as contradictions but as contraries—that is to say between A and E. This, because, in the exact sciences, all propositions are either universal or singular ; and, it is sufficient ; because contraries cannot both be true : if we can hold one to be true, we may infer the falsity of the other.

the middle term of the reduct, it is clear that in the second figure it will be the major term, in the third, the minor, while in the fourth it may be either the one or the other.<sup>1</sup>

**Some additional Examples in Reduction** will serve still further to familiarise its **Reduction.** cesses :—

1. Construct Disamis, Camestres, and Fesapo, and reduce them *ostensively*, and *per impossibile*.

Disamis, Fig. 3.      Camestres, Fig. 2.      Fesapo, Fig. 4.

Some M is P ; (I)	All P is M ; (A)	No P is M ; (E)
All M is S ; (A)	No S is M ; (E)	All M is S ; (A)
. . . Some S is P. (I)	. . . No S is P. (E)	. . . Some S is not P. (O)

These moods are reduced *ostensively* to the first figure, that is to say, by conversion, obversion, and the transposition of premisses, as follows :—

Disamis to Darii.

Some M is P ;	become	All M is S ;
All M is S ;		(Some P is M ;
. . . Some S is P.	. . . Some P is S.	

Camestres to Celarent.

All P is M ;	become	No M is S ;
No S is M ;		(All P is M ;
. . . No S is P.	. . . No P is S.	

Fesapo to Ferio.

No P is M ;	become	No M is P ;
All M is S ;		(Some S is M ;
. . . Some S is not P.	. . . Some S is not P.	

They are reduced *per impossibile*, in showing, by means of the first figure, and the laws of opposition, that the con-

<sup>1</sup> See Monck's *Introduction to Logic*, pp. 178-198.

tradictory of the conclusion of each is false, and therefore, that the conclusion itself is true, as follows:—

Disamis, reduced *per impossibile*.

Some M is P;

All M is S;

∴ Some S is P.

For, if this conclusion be false, then,

No S is P.<sup>1</sup>

But, All M is S;

∴ No M is P.

But, this is false, since, Some M is P, is true.

∴ No S is P, is false.

And, ∴ Some S is P, is true.

Camestres reduced *per impossibile*.

All P is M;

No S is M;

∴ No S is P.

For, if this conclusion be false, then,

Some S is P.<sup>1</sup>

But, All P is M;

∴ Some S is M.

But, this is false, since, No S is M, is true.

∴ Some S is P, is false.

And, ∴ No S is P, is true.

Fesapo, reduced *per impossibile*.

No P is M;

All M is S;

∴ Some S is not P.

For, if this conclusion be false, then,

All S is P.<sup>1</sup>

<sup>1</sup> On arriving at this proposition, that one of the original premisses is to be taken in conjunction with it which is suitable to form with it, either as major or minor premiss, a syllogism in the first figure. In the case of Fesapo, as is true generally of the fourth figure, whichever premiss we suppress, the proposition before us and the retained premiss will form premisses in the first figure.

But, No P is M ;

∴ No S is M ;

And, ∴ No M is S.

But, this is false, since, All M is S, is true.

∴ All S is P, is false.

And, ∴ Some S is not P, is true.

Or, we might have proceeded thus :—

All S is P ;

But, All M is S ;

∴ All M is P.

∴ Some P is M.

But, this is false, since, No P is M, is true.

∴ All S is P, is false.

And, ∴ Some S is not P, is true.

2. Show that the moods of the first figure to which those of the second are reducible by reduction *ad impossibile*, are respectively : Cesare to Ferio, Camestres to Darii, Festino to Celarent, and Baroko to Barbara.

3. Show that the modes of the first figure to which those of the third are reducible by reduction *ad impossibile*, are respectively : Felapton and Bokardo to Barbara, Darapti and Disamis to Celarent, Ferison to Darii, and Datisi to Ferio.

4. Show that the modes of the first figure to which those of the fourth are reducible by reduction *ad impossibile*, are respectively : Fesapo to Barbara or Celarent, Fresison to Darii or Celarent, Bramantip and Dimaris to Celarent, and Camenes to Darii.

5. Explain the following peculiarities of the named moods :

(1) The letter *p* never occurs in the first syllable of any of them.

(2) The letters *m* and *p* never occur simultaneously in the first and second syllables of any of them.

(3) The letter immediately preceding *m* in the first or second syllable of any of them is always an *A*, or an *I*.

**Reductio ad impossible, and the principle of the Syllogism.** § 80.—In a syllogism, the truth of the conclusion necessarily follows from the truth of both premisses; and consequently, if the conclusion be false, one, or other, or both, of the premisses must be false, for, otherwise, the conclusion would have been true. This principle lies at the foundation of *reductio ad impossibile*, and has been assumed throughout in such of the preceding worked examples as are intended to illustrate the process. It follows, of course, that, if the conclusion be false and *one* of the two premisses be true, we are warranted in limiting the false, or the source of vitiation of the syllogism, to the remaining premiss. In explicitly stating the principle here, it may be pointed out, at the same time, that the falsity of the premisses does not prove the falsity of the conclusion, a not unnecessary reminder relative to the syllogism. Every case of the truth of the premisses is a case of the truth of the conclusion. But to argue that, because a given case is *not* a case of the truth of the premisses, it must necessarily *not* be a case of the truth of the conclusion would be to involve oneself in an illicit process of the major term. So, in like manner, to argue, on similar grounds, that the truth of the conclusion proves the truth of the premisses would be to involve oneself in the fallacy of undistributed middle.

**The Unfigured Syllogism.** § 81.—In what precedes we have treated syllogism as of mode and of figure. The distinction between the *figured* and the *unfigured* syllogism is due to Sir William Hamilton. The unfigured syllogism arises from such a rigid quantification of the predicate as enables us to interchange subjects and predicates at pleasure, so that the syllogism takes no determinate figure. Garden describes it as produced by two substitutive propositions of which, as contrasted with the figured, the terms stand in no relation to each other of breadth and depth, of containing and contained, of major and minor, but merely that of equivalency. Its canon has been given by Hamilton as follows:—

“In as far as two notions either both agree, or one agree-

ing, the other does not, with a common third notion, in so far these notions do or do not agree with each other.”<sup>1</sup>

The following are examples:—The preacher yesterday was Dr. A. ; This gentleman is yesterday’s preacher ; ∴ This is Dr. A. All whales and some mammals are equal ; All whales and some water-animals are equal ; Some mammals and some water-animals are equal. A and B are equal ; B and C are equal ; ∴ A and C are equal.

**§ 82.**—Sometimes a syllogism is incompletely stated, as when we say, “ Personal deformity is an affliction of nature; therefore, personal deformity is not a disgrace.” Here, we are supposed to tacitly supply the major premiss, namely, “ Disgrace is not an affliction of nature,” or its converse. A syllogism thus incompletely stated is usually called an Enthymeme. This is not to be confounded, however, with the Rhetorical Enthymeme described by Aristotle. The latter is defined as *συλλογισμὸς ἐξ εἰκότων ἢ σημειων*, “ a syllogism from probable propositions or from signs”; the “sign” being, “a fact which is known to be an indication more or less certain, of the truth of some further statement, whether of a single fact, or of a general belief.” Thus, the rhetorical enthymeme is, generally speaking, but a suggestive or *persuasive* argument, as distinguished from the demonstrations of science. The logical enthymeme, however, aims at scientific proof.

When the major premiss of the syllogism is suppressed, the enthymeme is said to be of the First Order; when the minor premiss is suppressed, of the Second Order; and when both premisses are given, the conclusion being left to be supplied, of the Third Order. As an example of this latter kind of enthymeme, Sir William Hamilton gives the well-

<sup>1</sup> For the figured syllogism, or that in which the terms are related as subject and predicate of propositions in a given order, Hamilton gives the following form which is equivalent to the canon for syllogisms in extension:—“ What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related, that relation subsists between the terms themselves.” This is equivalent to saying that the conclusion must follow the weaker part. See Bowen’s *Logic*, p. 224.

known epigram on a contemporary German scholar by the celebrated Porson :

“ The Germans in Greek  
Are sadly to seek ;  
Not five in five score,  
But ninety-five more ;  
All, save only Hermann,  
And Hermann's a German.”

The inference, here left unstated, is, of course, that even Hermann, being a German, is, as to Greek scholarship, no better than he should be.<sup>1</sup>

§ 83.—When syllogisms are so combined, that <sup>Pro-</sup>  
syllogism, <sup>Epi-</sup> one of them furnishes a reason for one of the  
syllogism ; <sup>Epi-</sup> premisses of the other, the former is called a *Pro-*  
*syllogism* ; <sup>Epi-</sup> and the latter, an *Episyllogism*. For  
*cheirema* ; <sup>Sorites.</sup> *syllogism*, and the latter, an *Episyllogism*. For  
example, in the following,—

All C's are D's ;	(1)
All B's are C's ;	(2)
All B's are D's ;	(3)
But, All A's are B's ;	(4)
∴ All A's are D's ;	(5)

we have two syllogisms in Barbara, of which (3), the conclusion of the first, or the prosyllogism, composed of the propositions, (1), (2) and (3), forms the major premiss of the episyllogism, composed of (3), (4), and (5).

When, in a syllogism, either premiss is based upon a reason implying the existence of an enthymematic prosyllogism, such syllogism is called an *Epicheirema*. Such for example, is the following—

<sup>1</sup> See Mansel's *Aldrich (Artis Logicae Rudimenta)*, Appendix, Note F, and Hamilton's *Lectures on Logic*, Lecture XX. With regard to the derivation of the term, *Enthymeme*, as usually given, from the Greek, *ἐν*, in, and *θυμός*, the mind, Mansel notes that, contrary to the explanations of some writers, it does not specially refer to a premiss in the mind, both because *θυμός*, in Aristotelian phraseology, is not “the mind,” and has nothing to do with expression or suppression of premisses, and because the word, *ἐνθύμημα*, occurs in writers earlier than Aristotle, and before it could have assumed its technical meaning.

All A's are B's, for they are X's ;  
 And all B's are C's ;  
 ∴ All A's are C's.

When an argument is resolvable, by expressing fully the steps implied, into a series of simple syllogisms, such that the conclusion of each but the last becomes a premiss in the following, such a train of reasoning is called a Sorites (Greek, *σωρός*, a heap), or accumulating argument. Consider, for example, the argument—

All A is B ;  
 All B is C ;  
 All C is D ;  
 All D is E ;  
 ∴ All A is E.

This is easily seen to be resolvable into three syllogisms, thus—

All A is B ;	All A is C ;	All A is D ;
All B is C ;	All C is D ;	All D is E ;
∴ All A is C.	∴ All A is D.	∴ All A is E.

There is another form, called the Regressive, or (from its inventor, Goclenius) Goclenian Sorites, which differs, generally, from the foregoing, called the Aristotelian, or Progressive form, in having its premisses in the reverse order. It runs thus—

All D is E ;  
 All C is D ;  
 All B is C ;  
 All A is B ;  
 ∴ All A is E.

In more particular terms, the Aristotelian differs from the Goclenian Sorites, as to (a) definition, (b) expressed premisses, (c) mode of resolution into syllogisms, (d) special rules, and (e) relation of extremes of conclusion to those of premisses.

(a) The Aristotelian Sorites is a series of propositions in which the predicate of each preceding becomes the subject of each succeeding. The Goclenian is a series of propositions in which the subject of each preceding becomes predicate of each succeeding.

(b) In the Aristotelian Sorites, all the expressed premisses are majors except the first. In the Goclenian Sorites, all the expressed premisses are minors except the first.

(c) As to the mode of resolving them into syllogisms, in Goclenian, the premisses are left as they stand; in the Aristotelian, the two first are transposed.

(d) As to Special Rules.—In the Aristotelian Sorites, no premiss can be particular except the first; none can be negative except the last. In the Goclenian, that which may be particular is the last expressed premiss; that which may be negative is the first.

(e) The conclusion of the Aristotelian consists of the first subject and the last predicate; the conclusion of the Goclenian consists of the last subject and the first predicate.<sup>1</sup>

*Certain kinds of Non-syllogistic Reasoning.* § 84.—Some logicians consider that the syllogism cannot be regarded as the sole ultimate type of all formal inference, and hence, there have been introduced into treatises on Logic schemes for the construction of what is called a *Logic of Relatives*, the function of which is to take account of relation generally, instead of those merely which are indicated by the ordinary syllogistic copula, *is*. Relation, considered generally, Dr. Venn (*Symbolic Logic*, p. 400) defines as, “a mode of thinking of two objects together; a connection, or want of connection.”

Copular relations have been divided into four classes:—  
(1) Transitive and invertible; (2) transitive but uninvertible; (3) intransitive but invertible; and, (4) intransitive but uninvertible. To the first belong equality, brotherhood, and

<sup>1</sup> From the sorites A is B; B is C; C is D; D is E, we may conclude, A is C; A is E; also, B is D; B is E; and C is E. And, generally, in any sorites, if the number of premisses =  $n$ , the number of terms =  $n + 1$ , and the number of conclusions =  $\frac{n(n-1)}{1 \cdot 2}$ .

various other relations of a similar copular nature existing between the two names both ways, and equally affecting a third similarly joined to either; thus, A equals B; A is the brother of B; A is fastened to B; A is a contemporary of B. To the second, belong inclusion, causation, sequence, greater magnitude, &c.; to the third, exclusion, difference, &c.; to the fourth, the great majority of relations not falling under the preceding heads.

The *argumentum à fortiori* is based upon copular relation of the second class: A is greater than B, B is greater than C, ∴ A is greater than C. Mansel treats this argument as a *material consequence*, meaning thereby, "one in which the conclusion follows from the premisses solely by the force of the terms," some proposition being understood by help of which the mind is able to reduce the whole to the syllogistic form. Thus, B is greater than C; *A, greater than B, is greater than C*; but A is greater than B, ∴ A is greater than C. Other logicians, however, regard this mode of treatment as a mere evasion, or, what shall be explained as a *petitio principii*, and hold that reasoning, based on the copular relation underlying this argument, may be placed side by side with the syllogism as a distinct logical form. A similar explanation they deem sufficient for such reasoning as A equals B, B equals C, ∴ A equals C; A is the brother of B, B is the brother of C, ∴ A is the brother of C; and so forth. There has not, up to the present, however, been any approach made towards a complete and comprehensive treatment of reasoning of a non-syllogistic character under the head of a Logic of Relatives; and the extreme vagueness and generality of the conception of a Relation makes here very much against progress.

**Ultra-total Quantification of the Middle Term.** § 85.—In ordinary syllogistic reasoning the recognised distributive words are *All*, in the sense of *Every*, and *Some*; and we have seen that from a pair of premisses quantified by *Some* no conclusion can be drawn. If, however, the premisses are quantified each by *Most*, a conclusion follows. Thus, from the premisses—

Most men have coats,  
Most men have waistcoats,

we necessarily infer that, Some men have both coats and waistcoats.

Again, a conclusion can be drawn in what De Morgan calls, "the numerically definite syllogism," that is to say, a quasi-syllogism in which exact numbers are given in each premiss bearing a suitable relation to the whole of a given group. For example, if 70 instances of anything out of a hundred are X's, and 50, Y's, then, at least, 20 X's must be Y's.

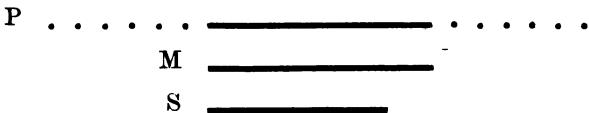
When we employ *Most*, or numerical quantifications equivalent thereto, we see that, for valid inference in such cases, "The quantifications of the middle term, whether as subject or predicate, taken together, must exceed the quantity of that term taken in its whole extent"; in other words, we require what Hamilton calls, the *ultra-total* quantification of the middle term between the two premisses.

**Systems of Diagrammatic Notation.** § 86.—The object of a system of diagrammatic notation is to represent to the eye, by figures, relations subsisting in thought between conceptions. Thus, the relation of terms in two given propositions may be represented diagrammatically, so as to enable us to see at a glance whether a conclusion can be drawn from them or not, and if it can, to show what it is. It is true, that Mansel and other logicians of conceptualistic views disapprove of systems of notation of this kind, regarding Logic as exclusively concerned with thought, and thought with concepts, the latter being from their very nature unrepresentable to the sense. In answer to this objection, it is urged, however, that the ordinary diagrammatic notation of lines and circles, and the like, is not primarily intended to represent concepts themselves but only their extension, and that, so employed, it greatly facilitates logical analysis.

**Lambert's Notation.** § 87.—Lambert, in his *Neues Organon*, makes use of the following scheme:—

A distributed term, taken by itself, is denoted by a

horizontal dark, and an undistributed term by a horizontal dotted line. When two terms are joined in a proposition, such proposition is denoted by two horizontal lines, dark or dotted, as may be necessary, written, in the case of an affirmative proposition, one just under the other ; and, in the case of a negative proposition, one a little lower than the other, and apart from it on one side, in such a position that no part of the one shall stand vertically over any part of the other. Of these lines, the lower always represents the subject, the upper the predicate. A syllogism is represented by three lines written two and two in accordance with the convention just laid down. The lowest, usually marked S, denotes the subject of the conclusion, or the minor term of the syllogism ; the middle, marked M, denotes the middle term ; and the uppermost, marked P, the predicate of the conclusion or the major term. The relations of the three terms for affirmative premisses may be thus exhibited :—



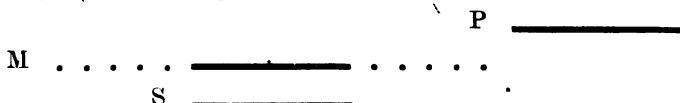
Here, the copulative connections between S and M, and between M and P, as well as their relations in extension, are sufficiently indicated by the *relative lengths* and *character* of the lines, and their *positions*. Reading from the diagram, we have, at sight, the following inferences :—

1. All M is P.
2. All S is M.
3. Some P is M.
4. Some M is S.
5. All S is P.
6. Some P is S.

It will be at once seen, however, that there are certain crudities in the scheme that at once assert themselves when, instead of reading into words a diagram such as that just

given, we translate into diagrammatic symbols a given syllogism.

Subject to like drawbacks, the following diagram represents a syllogism with a premiss negative :—



Here, we may read :—

No M is P ;

All S is M ;

∴ No S is P.

A substitutive proposition may, subject to the same convention, be represented by two equal parallel lines ; and a disjunctive proposition, by two equal lines, that representing the predicate being broken up into a suitable number of distinct parts.

This mode of notation, however, is not now very much in use for purposes of logical illustration.

**Euler's Notation.** § 88.—In this scheme described by Euler, in his *Lettres à une Princesse d'Allemagne*, the extension of a term is denoted by a circle, an affirmative

Fig. 1.

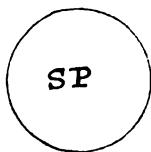


Fig. 2.

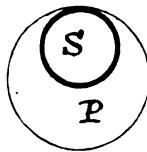


Fig. 3.

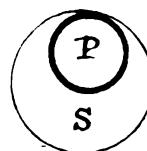


Fig. 4.

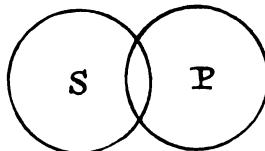
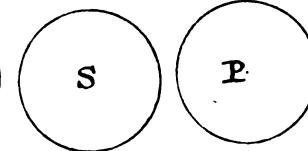


Fig. 5



*universal/* judgment by one circle wholly, or partly, contained in another, and a negative by two circles each wholly outside the other. On this understanding, the figures or diagrams before us express, respectively, the following propositions :—

- Fig. 1. *All S is P*, and, *All P is S*.
- Fig. 2. *All S is P*, and, *Some P is not S*.
- Fig. 3. *All P is S*, and, *Some S is not P*.
- Fig. 4. *Some S is P*, *Some P is S*, *Some S is not P*, and, *Some P is not S*.
- Fig. 5. *No S is P*, and *No P is S*.

Any one diagram may be constructed so as to represent but a single proposition, by a suitable convention with regard to dotted circumferences, shading, and other simple contrivances that at once suggest themselves ; and the mode of representing any given syllogism in this notation will be found so obvious as not to necessitate here any more lengthened treatment of details. (For some interesting recent developments see Dr. Venn's *Symbolic Logic*, pp. 421-438.)

§ 89.—Sir William Hamilton devised an ingenious scheme of notation (see Thomson's *Laws of Thought*, pp. 186-188) which enables us to read each syllogism with equal facility according to extension and intension. He employs the letter M to denote the middle term of the syllogism, and C and  $\Gamma$  (the Greek capital form of the letter Gamma) to denote the two extremes. The positive copula he indicates by a wedge-like line increasing in thickness from the predicate to the subject in extension ; the negative, by a similar line, crossed by a vertical stroke. A colon is placed between the term and the copula, when such term is distributed ; a comma, when it is not. A syllogism is represented by placing M, the middle term, in the centre, connecting it at each side with the other terms by the means indicated, and placing beneath the whole the copula representing the conclusion. And so, the notation is complete. Instead of the copulative symbols employed by the inventor of the scheme, however, we might with advantage employ, for

the positive copula,  $\beta\alpha$ , or  $\alpha\beta$ , according as we read from the subject in extension to the predicate, or from the predicate to the subject ; the corresponding forms of the negative copula being  $\beta n\alpha$ , or  $an\beta$ , and the conclusion being divided from all but the extremes, with their quantification marks attached, by a horizontal line. With this slight modification introduced, the following expressions illustrate the use of Hamilton's notation :—

$C : \beta\alpha, M ; C, \beta\alpha, M ; C : \beta n\alpha : M ; C, \beta n\alpha : M$  ;—to be read, respectively, All C's are some M's (A) ; Some C's are some M's (I) ; No C is any M (E) ; Some C is not any M (O).

The syllogisms, Barbara, Celarent, Darii, Ferio, are expressed, respectively, as follows :—

$$C, \frac{\alpha\beta : M, \alpha\beta : \Gamma}{\alpha\beta} (AAA) ; \quad C : \frac{an\beta : M, \alpha\beta : \Gamma}{an\beta} (EAE) ;$$

$$C, \frac{\alpha\beta : M, \alpha\beta, \Gamma}{\alpha\beta} (AII) ; \quad C : \frac{an\beta : M, \alpha\beta, \Gamma}{an\beta} (EIO).$$

In the Unfigured Syllogism we may substitute for  $\alpha\beta$ ,  $\alpha\alpha$ , or  $\beta\beta$ .

The proposition,  $C\beta\alpha\Gamma$ , read extensively, or in extension, means that C is included under  $\Gamma$  ; read intensively, or in intension, that  $\Gamma$  is included in C.

## CHAPTER IV

### ON COMPLEX SYLLOGISMS

**Complex Syllogisms.** § 90.—A Complex, or Non-Categorical Syllogism, is one that has for its premisses one or more complex or compound propositions.<sup>1</sup> Complex Syllogisms are Pure, or Mixed. They are pure, if both premisses are hypothetical, or both disjunctive propositions. They are mixed, if one premiss be hypothetical and the other categorical, or one disjunctive and the other categorical, or one a hypothetical and the other a disjunctive proposition. Or, adopting a slightly different mode of approach, such syllogisms are fundamentally hypothetico-disjunctive syllogisms or dilemmas ; the first and distinguishable into hypothetical and disjunctive syllogisms, and second classes being pure or mixed, according to the character of the major premiss in each ; the third class being always mixed, and admitting a categorical or a disjunctive conclusion. Either way, however, we encounter some slight confusion of terminology. For, such syllogisms are also sometimes called conditional syllogisms, and the major premiss in each a con-

<sup>1</sup> In a complex proposition, a complement accompanies the subject, or the predicate, or both ; in other words, such a proposition has for subject, or predicate, or both, a complex term, that is to say, a term which is a combination of simple terms. In a compound proposition, the subject, or the predicate, or both, are multiplex ; that is to say, consist of copulative, or alternative combinations of other propositions. Such a proposition, for example, as, "If metals are heated, they expand," being plainly equivalent to, "Metals under heat expand," belongs to the class of complex propositions. Such a proposition as, "If ignorance is bliss, 'tis folly to be wise," implying that the truth of "'Tis folly to be wise," depends, assertorially, or inferentially, on the truth of "Ignorance is bliss," belongs to the class of compound propositions. As applied, however, to the syllogism, in the sense of the present chapter, the term *complex* is not in competition with *compound* : the former term is convenient and unambiguous.

ditional proposition ; by the latter being understood, two or more categoricals united by a conjunction. The conditional proposition, in this purely formal sense, includes both the hypothetical and the disjunctive proposition. Again, the term, conditional, is sometimes applied to a proposition not to be distinguished in the abstract from the hypothetical, the distinction being one of import. A conditional proposition, in this latter sense, is regarded as expressing a relation between two phenomena, being the result of a generic judgment based on intension or content of attributes ; a hypothetical, a relation between two propositions of independent meaning ; the former, also, being expressible in the concrete by a complex, the latter being expressible by a compound proposition. (See Keynes' *Formal Logic*, § 127.)

**Hypothe-** § 91.—Both the hypothetical and the disjunc-  
**ticals.** tive propositions make their assertions under a condition, but their assertions are made in different ways. In the hypothetical (called by Dr. Fowler, conjunctive) proposition, the condition is introduced by the conjunction, *If* ; in the disjunctive, by *Either*. In the hypothetical, two propositions are declared to be so related that the truth of one follows from the truth of the other. In the disjunctive, two propositions are declared to be so related as to be alternatively predicate. A hypothetical syllogism is a syllogism in which the premisses are, one, or both, hypothetical propositions. In the pure hypothetical syllogism, both are hypothetical propositions ; in the mixed, one is hypothetical, the other, categorical. Considering, first, the mixed form of hypothetical syllogism, according as the categorical premiss in this form is affirmative, or negative, the argument is said to be a *Constructive*, or a *Destructive*, hypothetical syllogism—varieties illustrated, respectively, by the following examples :—

Constructive.	Destructive.
If A is B, C is D ;	If A is B, C is D ;
But A is B ;	But C is not D ;
Therefore, C is D.	Therefore, A is not B.

These examples are types of the most common and familiar kinds of hypothetical or conditional argument. The major premiss is, in each case, "If A is B, C is D." Of the two parts of which this is composed, "A is B," is called the *Antecedent, Reason, or Protasis*, and, "C is D," the *Consequent, or Apodosis*. The Canons of this kind of argument, called the Laws of Reason and Consequent, are :—1. Granting the reason, we must grant what follows from it. 2. If we reject the consequent we must reject the reason. 3. If we admit the consequent we do not of necessity admit the reason. Accordingly, the valid modes of reasoning in hypothetical syllogisms are : in Constructives, to *affirm the antecedent*, whence the argument is said to be the *modus ponens* or the mood that posits or affirms ; and, again, in Destructives, to *deny the consequent*, whence the argument is called, the *modus tollens*, or the mood that removes the consequent. This will be easily seen, by stating the major premiss in the form, "The case of A's being B is the case of C's being D." Then, in Constructives, we proceed to say that, "This is the case of A's being B," whence there follows the conclusion that, "This is the case of C's being D." In Destructives, on the other hand, the minor premiss above stated is equivalent to, "This is not the case of C's being D," whence there follows, as a conclusion, that, "This is not the case of A's being B." For such syllogisms, therefore, we have the following rule : *Either we must affirm the antecedent, or we must deny the consequent*, that the argument may be conclusive. If we deny the antecedent, or affirm the consequent, no conclusion follows : *sublato antecedenti nil, positio consequenti nil*. The former proceeding is equivalent to illicit process of the major in categoricals ; the latter, to an undistributed middle term.<sup>1</sup>

<sup>1</sup> The element of modality present in a hypothetical proposition, and relating to the concrete existence of its elements, precludes us from satisfactorily reducing such a proposition to the categorical form ; but such an element disappears from the hypothetical syllogism, which, therefore, can be reduced to the categorical form without injury to its contents.

In the pure hypothetical syllogism, since both premisses are hypotheticals, and, as such, equivalent to universal affirmatives, the syllogism, to be valid, must have its equivalent in Barbara. Thus, the syllogism,

If A is B, C is D ;  
 If C is D, E is F ;  
 ∴ If A is B, E is F,

is valid ; not so, the following,

If A is B, C is D ;  
 If A is B, E is F ;  
 ∴ If C is D, E is F.

**Disjunctives.** § 92.—A disjunctive syllogism is a syllogism in which the premisses are, one or both, disjunctive propositions (but, compare with Dr. Fowler's definition, *Deductive Logic*, p. 117). Pure disjunctive syllogisms, that is to say, such as have two disjunctive premisses, and of which, S is either P or Q, S is either not-P or R, ∴ S is either Q or R, is an example, are so little in use that it is sufficient to point out that they are not a non-existent class. As for syllogisms with such premisses as, Either A is B or C is D, Either A is B or E is F, they are in reality, hypotheticals. The mixed disjunctive syllogism,—the disjunctive syllogism in more common use, is a syllogism with a disjunctive major premiss, and a simple, or categorical minor. Since the latter may either affirm or deny one of the alternatives stated in the major, we have, thus, two valid moods in mixed disjunctives, the *modus ponendo tollens* (the mood which, by affirming, denies), of which the following is an example :

A is either B or C ;  
 But A is B ;  
 Therefore, A is not C ;

and the *modus tollendo ponens* (the mood which, by denying, affirms), illustrated by the following :

A is either B or C ;  
 But A is not B ;  
 Therefore, A is C.

The *modus ponendo tollens* proceeds on the assumption that, if one alternative of a disjunctive proposition be admitted, the other cannot. Whately, Mill, and some other logicians, however, object that such an assumption is not always legitimate. For, they say, if we take such an example as, "A good book is valued for the usefulness of its contents, or the excellence of its style," it does not follow that, because the contents of the book are useful, its style may not be excellent. This objection does not apply to the *modus tollendo ponens*, a mode which is always cogent. If a book be valued but for the usefulness of its contents, or the excellence of its style, and the book, A, which is valued, be not valued for the latter reason, it must be for the former ; and *vice versa*.

**Character of the Inference in Hypotheticals.**—So much, as to the more usual treatment of Complex Hypothetical and Disjunctive Syllogisms. Boole, and De Morgan, however,

are of opinion that the inference in hypotheticals is not different from immediate inference. The obvious differences between the hypothetical syllogism and the categorical are (1) the absence of a middle term,—the so-called major of the hypothetical contains all the terms; (2) the interchangeability of the minor premiss, and the conclusion; (3) the fact that the so-called major of the hypothetical consists of two propositions, that of the categorical of two terms. Boole observes that, inasmuch as the hypothetical syllogism need contain no more than two terms, it is no syllogism at all. De Morgan says that "the law of thought connecting hypothesis with necessary consequence is of a character which may lay claim to stand before syllogism, and to be employed in it rather than the converse. Sir William Hamilton, also, in his more recent writings, prefers to treat hypothetical reasoning as a case of immediate inference. And, Mr. Bain considers that, so-called conditional

inference comes most appropriately under the head of equivalent propositional forms. In such an example as, If A is B, C is D, the equivalent is, A being assumed to be B, it follows that C is D. Here, he holds, there is no inference. Accepting, A is B, we accept, C is D ; that is to say, another expression for the same fact. Again, in disjunctives, considering the example, A is either B or C, we have for equivalent forms, not being B, A is C, and, not being C, A is B. Here the supposed reasoning consists, merely, in electing an alternative according to the facts of the case. Mr. Minto, however, on the other side, contends that we cannot argue that the conclusion is directly educible from either the major, or the minor premiss alone. For, considering such a major premiss as, If A is B, C is D, it merely affirms the relation of reason and consequent between the two propositions ; but it does not *concede* the minor premiss, and without the concession of the minor premiss we cannot proceed to assert the conclusion. And, considering such a minor premiss as A is B, we cannot proceed from this proposition to the proposition, C is D, unless it is *conceded* that the relation of reason and consequent holds between them, that is, unless the major premiss, also, is conceded. And Dr. Fowler (*Deductive Logic*, p. 125) is of like opinion, holding that hypothetical syllogisms are syllogisms rightly so-called, and forms of mediate inference.

§ 94.—A Dilemma is defined by Mansel as, **The Dilemma**, “a syllogism having a conditional major premiss with more than one antecedent and a disjunctive minor” ; by Whately, as, “a conditional syllogism with several antecedent in the major and a disjunctive minor” ; and, by Thomson, as, “a syllogism with a conditional premiss in which either the antecedent or the consequent is disjunctive. According to Hamilton, “If the sumption (major premiss) of a syllogism be at once hypothetical and disjunctive, and if, in the subsumption (minor premiss) the whole disjunction, as a consequent, be sublated (denied) in order to sublate the antecedent in the conclusion, such a reasoning is

called a hypothetico-disjunctive syllogism, or dilemma." Dr. Fowler defines the dilemma as, "a complex syllogism, of which one premiss is a conjunctive (hypothetical), and the other, a disjunctive proposition." He differs from Mansel and Whately, principally on one point, namely, in admitting as a dilemma, a syllogism with but a single antecedent in the major premiss. He is of opinion, also (*Deductive Logic*, p. 123), that, as having a disjunctive minor, a dilemma cannot properly be regarded as a conditional (conjunctive) syllogism. The term, Dilemma (Greek  $\delta\iota$ , two, and  $\lambda\bar{\eta}\mu\mu\alpha$ , an assumption), strictly implies, as Whately explains, but *two* antecedents (hence, it is common to speak of "the horns of a dilemma"); but it is evident that there may be either two or more. Following the analogy of the term, dilemma, an argument of the kind with a single antecedent, might not inappropriately be called a monolemma, one with three antecedents, a trilemma, with four, a tetralemma, and so on.

The following are the three principal forms of the Dilemma :—

1. The *Simple Constructive Dilemma*. In this form, the major premiss contains several antecedents with one common consequent; the minor grants these antecedents disjunctively; and the conclusion expresses the inference of the common consequent. Thus :—

If A is B, C is D; and, if E is F, C is D;  
 But, either A is B, or E is F;  
 ∴ C is D.

2. The *Complex Constructive Dilemma*. This form has a major premiss containing several antecedents, each with a different consequent; the minor grants the antecedents disjunctively; and the conclusion expresses the inferences of the consequents disjunctively. Thus :—

If A is B, C is D; and, if E is F, G is H;  
 But, either A is B, or E is F;  
 ∴ Either C is D, or G is H.

In the preceding dilemma, the conclusion is a simple proposition, in this, a complex ; whence the respective names of these two forms.

3. The *Destructive Dilemma*. When the major premiss contains several antecedents, each with a different consequent, and the minor denies these consequents disjunctively, we may then, in the conclusion, express, disjunctively, the denial of the antecedents. Thus :—

If A is B, C is D ; and, if E is F, G is H ;  
 But, either C is not D, or G is not H ;  
 ∴ Either A is not B, or E is not F.

This form is always *Complex*. It is easy to show that there cannot be a *Simple Destructive Dilemma*. Consider, for example, the following :—

If A is B, either C is D, or E is F ;  
 But, neither C is D, nor E is F ;  
 ∴ A is not B.

We see at once that this is not properly a dilemma,—in the sense of Mansel's definition : it has not *two* or more antecedents in the major premiss, and the minor is not disjunctive.

“Dilemmatic arguments,” Jevons observes, “are more often fallacious than not ; because it is seldom possible to find instances of two alternatives exhausting all possible cases, unless, indeed, one of them be the simple negative of the other, in accordance with the law of excluded middle.” Consider, for example, the following : “For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary ; and, for the idle, and such as are indifferent to mental improvement it is ineffectual ; therefore, the incitement of academical honours is either unnecessary or ineffectual.” Here, there is understood the disjunctive minor premiss : “But those who might aspire to academical honours are either the diligent or the idle” ; and the two alternatives, evidently, do not exhaust all the possible cases.

A dilemma, it is pointed out, can often be retorted, or rebutted, by producing as cogent a dilemma to the contrary effect. Take, for example, the address of the Athenian mother to her son : “ Do not enter into public business ; for, if you say what is just, men will hate you, and if you say what is unjust, the gods will hate you ” ;—to which Aristotle suggests the following retort : “ I ought to enter into public affairs ; for, if I say what is just, the gods will love me ; and, if I say what is unjust, men will love me.” Now, if we regard the *just* and the *unjust* as an exhaustive classification of human acts, and the moods of the gods and of men as necessarily opposed in their contemplation of such acts, and *love* and *hatred*, as an exhaustive enumeration of moods, the effect of the retort is clearly the same as that of the address, that is to say, “ If you enter into public business you cannot command universal love.” And, if all other motives for entering public business were absent, the determination would depend on this, of love or hatred. But all other motives are left out of sight in the maternal counsel given above.

Another well-known example is that of Protagoras, the Sophist, and his pupil Euathlus. Protagoras engaged to teach Euathlus dialectics, on condition that half the fee should be paid at once, and the other half when Euathlus should have won his first cause. But as Euathlus neither appeared as an advocate, nor paid his master the out-standing half-fee, Protagoras sued him for the balance. In the court, Protagoras addressed Euathlus as follows : “ Most foolish young man, however the judges may decide, you must pay me the out-standing half-fee ; for, if they decide in my favour, you must pay me by the decision of the court, and, if against me, by our bargain ” : to which Euathlus replied by the rebutting dilemma, “ Most sapient master, however the judges may decide, you must lose the outstanding half-fee ; for if they decide in my favour, you shall lose it by the decision of the court, and if against me, by our bargain, for I shall not have won my cause.”

It was open to the judges to adopt any one of *three* courses :

to decide in favour of Protagoras ; to decide against him ; or not to decide at all. Either of these dilemmas takes account but of two of the three possible courses.

To insure the validity of a dilemma as a form of argument, there have been laid down the following canons, called the three rules of the dilemma :—

1. The alternatives in the disjunctive premiss must exhaust the possible cases.

2. The consequence declared to follow from the admission of one or other of the alternatives must be indisputable.

3. It must not be possible to retort the dilemma under consideration by as cogent a dilemma to the contrary effect.

To the same purpose, the conditions that must be satisfied by a valid dilemma have been laid down by Hamilton as follows :—

1. A veritable consequence must subsist between the antecedent and the consequent of the sumption.

2. The opposition in the consequent must be thorough-going and valid.

3. The disjunctive members in the sumption must be legitimately sublated.

§ 95.—The following examples and exercises *Studies in Hypothetical* will be found useful in the further study of *cals.* conditional and hypothetical propositions, and of complex syllogisms :—

1. Is the reasoning in hypothetical (conjunctive) inference immediate or mediate ? Aristotle calls the *ἀπαγωγὴ εἰς τὸ ἀδύνατον* a hypothetical syllogism ?<sup>1</sup> R.

<sup>1</sup> See Trendelenberg's *Elementa Logices Aristoteleæ*, for explanations of Greek logical terms generally. The *ἀπαγωγὴ εἰς τὸ ἀδύνατον* is *reductio per impossible*. In the application of this process to Baroko and Bokardo, we employ two oppositions, (1) the falsehood of the conclusion of the reduct is inferred from the truth of the suppressed premiss ; (2) the truth of the reducend conclusion is inferred from the falsehood of the substituted premiss. We employ also, the conditional argument that, in the first figure, if the conclusion be false, the premisses are not true ; and, to prove the falsity of the substituted premiss, the disjunctive syllogism : In the reduct, either the substituted premiss or the retained premiss is false ; But the retained premiss is not false ; ∴ The substituted premiss is.

2. Distinguish the treatment, as bases of immediate inference, of (a) If A is true, then B is true, or, A then B,— being given no more than that A and B represent two *propositions*; and (b), If A is B, C is D, or, If S is M then it is P, in which the symbols represent *terms*.

The first of these forms (a), in which A and B stand for two propositions, may be called the pure hypothetical proposition. In this form we are not permitted to analyse or compare the contents of A and B respectively, or to proceed from the proximate to the remoter form. A is, however, the antecedent, in the usual sense, and B is the consequent. In applying opposition to this proposition, such considerations must be excluded as are not offered to us by the very general form in which the proposition is expressed. And, as no circumstance associated with A, the antecedent in this proposition, can be regarded as rendering the distinction between *All*, and *Some*, *All instances*, and *Some instances* in keeping, A being true, or not true, in an absolute sense, the distinction between universal and particular, or distinctions of quantity, may be left out of consideration as inapplicable to propositions of this kind when subject to the restriction laid down. Every proposition, however, has a contradictory; and there remains to determine the contradictory of A. And, in respect of this, the question suggests itself from the mere form of the proposition. Do we imply that the truth of B logically depends on the truth of A, or do we merely assert the bare dependency of the one upon the other? If we adopt the former, or modal interpretation, then the contradictory will be, If A is true, B may (possibly) be untrue; since the proposition may be read as a modal in the sense that, If A is true, then it is impossible (logically inconsistent) that B is untrue. If the latter, or assertorial, the contradictory is, A is true, but C is untrue; the given proposition being regarded as a simple denial of this assertion.

With regard to (b), the second of these forms, the symbols in the two expressions given above for it stand for *terms*.

This form, in which we permit ourselves to proceed to a more remote analysis, or a step further in the direction of the concrete, with all that it implies, is the hypothetical form more commonly so called, and which may for distinctness' sake be termed the conditional form. It is held by some logicians that, whenever the proposition, If *A* is *B*, *C* is *D*, is of any real value in predication, we can always show, if we look into the mode in which such a proposition takes account of things and attributes, that it is expressible in the simpler form, If (any) *S* is *M*, then it is also *P*; that, further, this form contains, if we examine into it, a reason why *S* is also *P*; and, that such a reason is either ostensively or illatively contained in the proposition, *S* is *M*. In considering how far hypothetical or conditional propositions may, on this latter understanding, be made the bases of immediate inference, it will be readily seen that such a proposition as, If any *S* is *M* it is *P*, admits of *All* and *Some* before the antecedent, in such shapes as, *Always*, *Sometimes*, *Whenever*, and the like; and again, that there may be a corresponding negative form; so that propositions of this class taken with the latitude explained, appear to admit distinctions of quantity and quality. It will be found, however, that though particular affirmatives and particular negatives are thus obtainable, they are modal in character, our treatment yielding as true hypotheticals only the two universal forms. But, if with these universals, we admit the two modals, we have a complete set of forms corresponding to the categoricals of the square of opposition, namely:—

- A. Always if any *S* is *M* that *S* is also *P*.
- E. Never if any *S* is *M* that *S* also *P*.
- I. Sometimes if an *S* is *M* that *S* is also *P*.
- O. Sometimes if an *S* is *M* that *S* is not also *P*.

And, if we make use of abridged notation, taking for our fundamental proposition, *A*, or, *SMaP*, and employing the symbol *M'* (or, if we prefer it,  $\bar{M}$ ) for not-*M*, and *P'* for

not-P,—a convenient way of denoting negative terms, the obverse, converse, obverted converse, contrapositive, obverted contrapositive, inverse, and obverted inverse, of such A, or  $SMaP$ , may be written respectively as follows:— $SMeP'$ ,  $SPiM$ ,  $SPoM'$ ,  $SP'eM$ ,  $SP'aM'$ ,  $SM'oP$ ,  $SM'iP'$ . And the remaining propositions of the square, E, I, O, may be treated in a similar manner; the inferences in each case corresponding to those obtained in a former section, when the original propositions worked from were categoricals.

3. Show that from the two propositions, Either A is B or C is D, and, Either A is B or E is F, we may draw four conjunctive (hypothetical) conclusions.

4. Show that the reasoning involved in the dilemma:—If A is B, C is D; and, if A is not B, E is F;  $\therefore$  Either C is D, or E is F, may be exhibited in a single categorical syllogism.

5. To what hypothetical proposition is the disjunctive, Either A is B, or C is D, equivalent?

Show that the reasoning involved in the complex dilemma, If A is B, C is D; and, if E is F, G is H; But, either A is B, or E is F;  $\therefore$  Either C is D, or G is H, may be expressed as a hypothetical sorites.

6. If A is not B, it is C; but it is not C; what is the conclusion?

Exhibit the reasoning in a categorical form.

7. Explain, and examine:—"The *modus ponendo tollens* is valid when the alternatives are exclusive, invalid, when they are not."

8. Is a disjunctive syllogism rightly defined as, "an argument in which there is a disjunctive judgment"? Explain fully.

9. How may we contradict a conjunctive (hypothetical) proposition? "The contrary of the hypothetical judgment is, as usual, a judgment of the same type"?

10. Show that the proposition, "If  $z$  is not  $y$  it is  $x$ ," is the converse by contraposition of the proposition, "If  $z$  is not  $x$  it is  $y$ ."

11. How far are distinctions of quantity and quality applicable to disjunctive propositions?

Distinctions of quantity may be regarded as applicable, in certain cases; not, distinctions of quality. But see Keynes' *Formal Logic*, § 142.

12. What are the four possible varieties of the most common form of conjunctive syllogism? How many of them are valid, and why?

13. Construct a valid conjunctive inference with an E conclusion, a valid sorites with an O conclusion, and a valid dilemma with an A conclusion.

14. Construct hypothetical (conjunctive and disjunctive) inferences with an O premiss.

15. State accurately the difference between a disjunctive syllogism and a dilemma. Give examples to illustrate your answer.

16. Give examples of the simple constructive, of the complex constructive, and of the destructive dilemma.

17. Explain the terms, *antecedent*, *consequent*, *modus ponens*, and *modus tollens*, as used in connection with conditional syllogisms.

18. It has been contended that a *modus ponendo tollens* is not admissible in disjunctive syllogisms. Examine this view.

19. What are the principal immediate inferences from the universal disjunctive proposition, Every S is either P or Q, or, in abridged notation,  $\overline{S}aPQ$ ?

Denoting not-P, by  $P'$ , not-Q, by  $Q'$  and not-S by  $S'$ , the obverse, converse, obverted converse, contrapositive, obverted contrapositive, inverse, and obverted inverse, of the given proposition may be written as follows:— $\overline{S}e\overline{P}'\overline{Q}'$ ,  $\overline{P}\overline{Q}iS$ ,  $\overline{P}\overline{Q}oS'$ ,  $\overline{P}'\overline{Q}'eS$ ,  $\overline{P}'\overline{Q}'aS'$ ,  $S'o\overline{P}\overline{Q}$ ,  $S'o\overline{P}'\overline{Q}'$ .

20. Exemplify the different ways of dealing with a dilemma.

21. Give, and illustrate, various definitions of the dilemma.

22. Examine the following :—

If virtue were a habit worth attaining, it must insure either power, or wealth, or honour, or pleasure ;

But virtue insures none of these ;

Therefore, virtue is not a habit worth attaining.

(Hamilton's *Lectures*, iii. p. 352.)

## CHAPTER V

### FUNCTIONS AND LOGICAL VALUE OF THE SYLLOGISM

§ 96.—Mill's view of inference, and of the functions of the syllogism, is as follows:—"All inference is from particulars to particulars: general propositions are merely registers of such inferences already made, and short formulae for making more. The major premiss of a syllogism, consequently, is a formula of this description: and the conclusion is not an inference *from* the formula, but an inference drawn *according to* the formula; the real logical antecedent, or premiss, being the particular facts from which the general proposition was collected by induction. Those facts, and the individual instances which supplied them, may have been forgotten; but a record remains, not indeed descriptive of the facts themselves, but showing how those cases may be distinguished respecting which the facts, when known, were considered to warrant a given inference. According to the indications of this record we draw our conclusion, which is, to all intents and purposes, a conclusion from forgotten facts. For this, it is essential that we should read the record correctly; and the rules of the syllogism are a set of precautions to insure our doing so." (*Logic*, i. p. 221.)

If we conclude from the death of the individuals, A, B, C, and every other person we ever heard of in whose case the experiment had been fairly tried, that X is mortal like the rest, we may indeed pass through the generalization, All men are mortal, as an intermediate stage: but it is not in the

latter half of the process—the descent from *All men* to *X*—that the inference resides. The general proposition is at once an embodiment of our notes of a number of particular facts, all of which have been observed, and of the inference we feel warranted in drawing with regard to all cases similar to those we have been considering, past, present, and future, however numerous they may be. The inference is finished when we have asserted that, All men are mortal. What remains to be performed afterwards is merely the deciphering of our own notes.

Thus, in the ordinary course of our reasoning, syllogism is only the latter half of the process of travelling from premisses to a conclusion. There are some particular cases, however, in which it is the whole process; as, for example, if our knowledge comes from testimony, or, if the generalization be not an assertion at all, but a command,—a law, in the moral and political sense of the term. In both cases the generalities are the original data, and the point to be determined is whether the authority which declared the general proposition intended to include a given case in it, and whether the legislator intended his command to apply to a given case or not. The operation is here, according to Mill, not in reality, a process of inference, but a process of interpretation. And this last phrase appears to him to characterize, more aptly than any other, the function of the syllogism in all cases. When the premisses are given by *authority*, the function of reasoning is to ascertain the testimony of a witness, or the will of a legislator, by interpreting the signs in which the one has intimated his assertion, and the other his command. When the premisses are derived from *observation*, the function of reasoning is to ascertain what we (or our predecessors) formerly thought might be inferred from the observed facts, and to do this, by interpreting a memorandum of ours (or of theirs). When we draw a conclusion, however, with regard to a specific case, we do not infer this from the memorandum but from the former experience. All that we infer from the memorandum is our own previous belief (or that of those who

transmitted to us the proposition), concerning the inferences which that former experience warrants.

The *rules of syllogism* are, thus, rules for the interpretation of the memorandum or other formula, so as to prevent the drawing of any inferences not conformable to such memorandum or formula; and the *sole purpose* of syllogism is to maintain consistency between the conclusions we draw in every particular case, and the previous general directions for drawing them; whether these directions were framed by ourselves as the result of induction, or were received by us from an authority competent to give them. Though there is, then, always, a process of reasoning or inference where a syllogism is used, the syllogism is not to be taken as, simply, a correct analysis of that process. The inference is, when not merely from testimony, an inference from particulars to particulars, authorized by a previous inference from particulars to generals, and substantially the same with it; one, of the nature, consequently, of induction.

"The value, therefore," Mill concludes, "of the syllogistic form, and of the rules for using it correctly, does not consist in their being the form and the rules according to which our reasonings are necessarily, or even usually, made; but in their furnishing us with a mode in which these reasonings may always be represented, and which is admirably calculated, if they are inconclusive, to bring their inconclusiveness to light." (*Logic*, i. p. 228.)

The throwing of the whole body of possible inferences from a given set of particulars, into one general expression, operates as a security for their being just inferences in more ways than one. It checks rash inferences by making us sensible of the extent of the consequences involved; and, again, if the premisses are insufficient, and the general inference therefore, groundless, contradictory instances will readily suggest themselves to us. An induction from particulars to generals, followed by a syllogistic process from those generals to other particulars, is a form in which we may always state our reasonings if we please. It is not a form in which we

*must* reason, but it is a form in which we *may* reason, and into which it is indispensable to throw our reasoning when there is any doubt of its validity; though when the case is familiar, and little complicated, and there is no suspicion of error, we may, and do, reason at once from the known particular cases to unknown ones.

<sup>Views of</sup> <sup>other</sup> <sup>§ 97.</sup>—If the major premiss be considered to be, itself, the evidence by which the conclusion is proved, instead of being an assertion of the existence of evidence sufficient to prove any conclusion of a given description, Mill holds that in every syllogism there is a *petitio principii*,<sup>1</sup> since from a general principle we cannot infer any particulars but those which the principle itself assumes as known.

Dr. Brown's theory of ratiocination is peculiar. Regarding the major premiss in the light thus explained, he not only failed to see the advantage in point of security for correctness, which, according to Mill, is gained, by interposing this step between the real evidence and the conclusion, but thought it incumbent on him to strike out the major altogether from the reasoning process without substituting anything else, and maintained that our reasonings consist only of the minor premiss and the conclusion. Thus, from Socrates is a man, he proceeds to, Socrates is mortal, suppressing, as an unnecessary step in the argument, the appeal to former experience.

Dr. Brown's mode of procedure was naturally consequent on the opinion he adopted that reasoning is merely analysis of our own general notions, or abstract ideas; and that the proposition, for example, Socrates is mortal, is educed from the proposition, Socrates is a man, simply by recognizing the notion of mortality as already contained in the notion we form of a man. But, according to Mill, the word, *man*, does not connote *immortality*; and how, then, does it appear that in the mind of every person who admits Socrates to be a man,

<sup>1</sup> Assuming as certain, and established, what is precisely the principle to be proved; or, employing a proposition to prove that on which it is itself dependent for proof.

the idea of *man* must include the idea of *mortality*? To avoid this difficulty, Dr. Brown was led to re-establish under another name that step in the argument corresponding to the major, by affirming the necessity of previously perceiving the relation between the idea of *man*, and the idea of *mortal*. This, according to Mill, amounts to a surrender of the doctrine that an argument consists of the minor and the conclusion only. And, on the point of previously perceiving the relation between the two ideas in question, Mill adds that the idea of *man*, as an universal idea,—the common property of all rational creatures, cannot involve anything but what is strictly implied in the name. If anyone includes in his own private idea of *man* some other attributes, such for instance as *mortality*, he does so only as the consequence of experience, after having satisfied himself that all men possess that attribute. Thus, whatever the idea contains in any person's mind, beyond what is included in the conventional signification of the word, has been added to it as the result of assent to a proposition. If we admit this position, Dr. Brown's view which requires us to suppose, on the contrary, that assent to the proposition is produced by evolving, through an analytic process, this very element out of the idea may be regarded as sufficiently refuted, and the minor premiss must be admitted to be totally insufficient to prove the conclusion, except with the assistance of the major, or of that which the major represents, namely, the various singular propositions expressive of the series of observations of which the generalization called the major premiss is the result.

In view of the charge of *petitio principii* brought by Mill against the syllogism, Whately's admission is sometimes referred to, namely, that when the two premisses are *combined*, they do *jointly* imply and virtually assert the conclusion. A test, however, of the sufficiency of this explanation will be furnished by our answer to the question, whether, assuming it to be impossible that any person had hitherto ever been able to satisfy himself that the conclusion in a given instance, had the matter occurred to him, was true,

could the premisses employed by such or such a reasoner have been laid down.

According to Dr. Martineau, a *petitio principii* is entirely relative to the state and range of the individual understanding, and cannot be established as a fault against an argument by merely showing that the inference *might* be thought already in the assumption, but only by showing that it *must* be.

Professor De Morgan urges that Mill's view tacitly assumes the superfluity of the minor; that is to say, taking, for example, the syllogism, All men are mortal, Socrates is a man, ∴ Socrates is mortal, it tacitly assumes that we know Socrates to be a man, as soon as we know him to be Socrates. Mill rejoins, that the objection would be well-grounded, if the assertion that the major premiss includes the conclusion meant that it individually specifies all that it includes. As, however, the only indication it gives is a description by marks, we have still to compare any new individual with the marks; and to show that this comparison has been made is the office of the minor.

Professor Bowen, in relation to the alleged *petitio principii*, observes:—"Strictly speaking, all valid reasoning proceeds *ex concessis*. Two premisses must be *assumed*, or taken for granted; and these two, taken in conjunction, necessarily involve the conclusion. Thus much must be conceded to those who claim that every syllogism presupposes the truth of what it is brought forward to establish. But then it is presumed that there is no undue assumption; that the two premisses which we now posit either have been already proved, or that they are universally admitted truths, or that they have just been conceded, *pro hac vice*, by the opponent." (*Logic*, p. 295.)

§ 98.—The following questions afford useful   
**Exercises and Studies.** matter for consideration at the present stage:—

1. It has been said that the process of thought which the syllogism seeks to describe is not that by which the inference is reached, but that by which it is justified; and that, in its totality, it is not gone through at all, unless the need for justification is suggested. Explain this statement, and give your reasons for accepting or rejecting it. R.

2. Is it true that the major premiss can be established only by induction ; and that this supposes the examination and testing of every individual instance ; and, hence, that we fall into a *petitio principii* in syllogistic deduction ? R.

3. "To explain a fact you must exhibit it as the instance of a general principle" ?

4. "The 'analytic order of the syllogism thoroughly disposes of the common but superficial objection that the syllogism is a *petitio principii* ' ?

5. "Almost everyone knows Lord Mansfield's advice to a man of practical good sense, who being appointed governor of a colony had to preside in its court of justice, without previous judicial practice or legal education. The advice was to give his decision boldly, for it would probably be right ; but never to venture on assigning reasons, for they would almost infallibly be wrong." (Mill's *Logic*, i. p. 217.) Why ?

6. Discuss :—"The sole ground which a syllogism affords for assenting to the conclusion is that the supposition of its being false, combined with the supposition that the premisses are true, would lead to a contradiction in terms."

7. What, according to Mill, is the relation between Formal Logic, and Logic, in the widest sense ?<sup>1</sup>

8. Sketch Dr. Thomas Brown's theory of syllogism and Mill's criticisms thereon. D.

9. What does Mill say of Whately's assertion that the syllogism is the philosophical analysis of *the* mode in which all men reason ?<sup>2</sup>

<sup>1</sup> Logic, in the widest sense, is the entire theory of reasoned or inferred truth. Of this, Formal Logic is a very subordinate part, not being directly concerned with the process of reasoning or inference, in the sense in which that process is a part of the investigation of truth. The name seems to be properly applied to all that portion of doctrine which relates to the equivalence of different modes of expression ; the rules for determining when assertions in a given form imply, or suppose, the truth or falsity of other assertions. The end aimed at by Formal Logic, and attained by the observation of its precepts is not truth, but consistency.—Mill's *Logic*, i. pp. 238-240.

<sup>2</sup> That if from individual cases we feel ourselves justified in proceeding to a general proposition, there appears to be no good reason

10. How does Whately answer the objections, (a) that syllogism is limited to the art of reasoning in words, (b) that men may reason from a single premiss, (c) that men may reason from one individual case to another without any universal premiss, (d) that the conclusion is not inferred from the universal premiss, and (e) that the inference from induction is not syllogistic?<sup>1</sup>

11. "Syllogism always means a movement of thought that uses what is given for the purpose of advancing beyond it."

why we may not, if the end aimed at is to infer with regard to another individual case, proceed to that at once, without interpolating a general proposition which adds not one iota to the proof. He expresses himself as unable to see, since the individual cases are all the evidence we can possess, why we should be forbidden to take the shortest cut from these sufficient premisses to the conclusion, and constrained to travel the "high priori road" of the general proposition. "I cannot perceive," he says, "why it should be impossible to journey from one place to another unless we 'march up a hill, and then march down again.' It may be the safest road, and there may be a resting-place at the top of the hill, affording a commanding view of the surrounding country; but for the mere purpose of arriving at our journey's end, our taking that road is perfectly optional; it is a question of time, trouble, and danger."—*Logic*, i. p. 214.

<sup>1</sup> (a) This view, Whately observes, arises in great measure from man's not perceiving that language of some kind or other is an indispensable instrument of all reasoning that properly deserves the name. (b) Take, as an instance, "Socrates is a man; therefore, he is a living creature." Here the conclusion had been already stated in the premiss to any one who did but understand the meaning of the words: "living creature" being a part of what is signified by man. But, take the instance, "He has swallowed a cup of laurel-water; therefore, he has taken poison"; the inference is one which no one could draw who should be ignorant that laurel-water is poisonous. (c) Here, the meaning sometimes is, that no such universal premiss is expressed. For, it continually happens, that even long trains of reasoning will flash through the mind with such rapidity that the process is performed unconsciously; so that a conclusion may be supposed to be seized by intuition which, in reality, is the result of rapid inference. (d) Those who raise this objection acknowledge that the truth of the universal premiss is an indispensable condition of such inference,—an admission, Whately adds, which would satisfy most logicians. For, if any one should choose to maintain that the conclusion is drawn from the one premiss, *by* or *through* the other premiss, this would be accounted merely a needless and unimportant innovation in phraseology. (e) But, it is admitted, that we have to decide, in each case of induction, the question, whether the instances adduced be sufficient to warrant the inference; and the decision of this question in the affirmative is, if expressed in words, the very premiss necessary to complete the syllogism.—Whately's *Logic*, Introduction, § 4.

Criticise this observation, and give a statement of the relation which you think obtains between the premisses and the conclusion of a syllogism. C.

12. No M is A ;

No M is B ;

∴ Some not-A is not-B.

Does this contradict the rule that from two negative premisses no conclusion can be drawn? C. S.

13. What is the formula given by Mill as the universal type of the reasoning process?<sup>1</sup>

14. Why has immediate inference been called *apparent*? Do the reasons urged in support of this view appear to you in any measure to apply to mediate inference as well?

15. Examine these assertions :—

“In the very statement of the major proposition, the truth of the conclusion is presupposed.” (D. Stewart.)

“The insufficiency of the syllogistic system of Aristotle and his successors is as evident as its utility is real.” (J. S. Mill.) P.

<sup>1</sup> Certain individuals have a given attribute; an individual, or individuals, resemble the former in certain other attributes; therefore, they resemble them also in the given attribute.—Mill's *Logic*, i. p. 232.

## CHAPTER VI

### ON PROBABLE REASONING

**Probability.** § 99.—Allusion was made in § 48 to the theory of probability or probable inference. The term *probability* relates subjectively to a state of mind or belief, and objectively to a state or situation of things external to us. It is assumed that belief may be of various degrees of intensity, and the term *probability* is, in a technical sense, applied to any degree of belief less than the very highest. The highest degree of belief is called *certainty*, or, by writers who prefer to reserve the term *certainty* for the objective fact, and to employ a distinct term to express the subjective altitude, (absolute, as distinguished from moral) *certitude*. The view of probability just explained, contemplates, therefore, a scale of degrees of intensity of belief, and the possibility of a numerical or quantitative expression of the probability in any given instance. If we represent full belief on such a scale by 1, the various probabilities expressible by probable propositions may be represented on it by corresponding numerical fractions ranging between the limits 0 and 1, to be determined from accessible data in a way to be shown immediately. The term *probability* is, in ordinary phraseology, sometimes employed in a more restricted sense, that is to say, as meaning, “more likely than not.” Its numerical equivalent, in any given instance, as thus understood, would lie somewhere between  $\frac{1}{2}$  and 1. The term, as has been observed above, relates, subjectively, to a state of mind or belief, and objec-

tively, to a state or situation of things external to us. "The probability of an event," says Mill, "is not a quality of the event itself, but a mere name for the degree of ground which we have or somebody else has, for expecting it." The probability of an event may, therefore, subjectively regarded, be different to different persons, or to the same person after he has acquired additional evidence. But, in itself, every event is certain, not merely probable: and, if we knew the whole of the circumstances affecting it, in variety, and order, and interaction, the situation would be completely and precisely, and certainly ascertainable. Inasmuch, however, as these circumstances are usually not known with regard to certain classes of events, we reason in probabilities without reference to them, as if such events were independent of law or causation; and, hence, the term *Chance*<sup>1</sup> is usually taken as synonymous with probability; chance being usually spoken of as in direct antithesis to law; whatever, it is supposed, cannot be ascribed to any law being attributed to chance (Mill's *Logic*, ii. p. 51).

In accordance with the quantitative view above illustrated, Mill defines probability from the subjective stand-point as, "the degree of ground there is for expecting an event," or "the degree of its occurrence which we are warranted in entertaining by our present evidence," and De Morgan, as "the numerical measure of an expectation that an event will happen." Objections, however, have been raised to the position that a certain definite and measurable amount of belief is realizable with regard to any given contingent event, or that there can exist a numerical condition of mind corresponding to such event, or that the probability of a single contingent event may be regarded as partial or fractional belief; partial or fractional belief being a conception difficult to realize, and not to be justified in relation to the event, since such event must either happen or not happen, and natural belief,—supposing degree of belief capable of increase and

<sup>1</sup> "Chance," says Jevons, "is merely an expression of our ignorance of the causes in action, and of our consequent inability to predict the result or bring it about infallibly."—*Principles of Science*, ii. p. 225.

diminution, being difficult to adjust to a correctly interpretative numerical scale ; belief being, besides, so frequently mixed up with other subjective conditions that such a scale could take no account of. Dr. Venn, accordingly, concludes that to attempt to found the science of probability on a subjective basis could lead to no satisfactory result, and that it is primarily concerned with the statistical frequency of events. He considers that the term, the probability of an event, or the chance of its happening, "presupposes a series ; within the indefinitely numerous class which compose a series a smaller class is distinguished by the presence or absence of some attribute or attributes. . . . These larger and smaller classes respectively are commonly spoken of as instances of the 'event' and of 'its happening in a particular way.' . . . The probability or chance of the events happening in that particular way may be defined as the numerical fraction which represents the ratio between the two different classes in the long run, or the *limit* to which the ratio between these classes tends as we take more and more of them, proceeding on to an indefinite extent" ; the interesting fact to note being that when the series are interminable, and of a fixed type, such a limit will not fail to appear. (See Venn's *Logic of Chance*, 3rd ed., pp. 163-164.) As an illustration, suppose the statistics of births showed, taking the series of births at a given date, and assuming it to be of indefinite extent, and the series of deaths at 80 years of age of those born at the given date, that the former were to the latter as 9 to 1, then the chance or probability that any given infant now born will live to 80 is the fraction  $\frac{1}{10}$  ; the *time* when the evidence by which the question is settled might be regarded as complete being immaterial.

**Probable Reasoning.** expressive of certainties is, as has been already said, called demonstrative. Reasoning based upon one or more propositions expressive of probability is called probable reasoning, sometimes syllogism of chance.

From the union of a probable proposition, and of one expres-

sive of certainty, as premisses, we can never arrive at other than a probable conclusion. From the union of two probable propositions as premisses we arrive at a conclusion weaker than either, which in the qualitative phraseology of some writers is expressive not of a probability but of a *possibility*; but which, if we adopt a quantitative treatment, may be regarded as expressive of a probability, though one of a lower degree than that of either of the premisses. Thus, from "A is probably B," "B is probably C," we arrive at a conclusion sometimes expressed as, "A is possibly C" (Fowler's *Deductive Logic*, p. 128); but, suppose "A is probably B" to be expressed in the quantitative proportional form, "5 A's in 8 are B," and, "B is probably C," in the form, "3 B's in 5 are C," the conclusion is, "3 A's in 8 are C," and which may be read, "A is possibly C," or "A is probably C," according to our interpretation of the *modus*, the probability being  $\frac{3}{8}$ . And so in all similar cases.

**§ 101.**—The following are the most important <sup>Rules of In-</sup> fundamental rules of inference in probabilities <sup>Probabilities.</sup> treated quantitatively :<sup>1</sup>

(1) If an event may happen in a certain number of ways, and fail in a certain other number of ways, and all these ways are equally likely to occur, the probability of its happening is the ratio of the number of favourable cases to all the possible cases. On the other hand, the probability of its failing is the ratio of the number of unfavourable cases to all the possible cases. The sum of both probabilities is equal to unity.

The ratio of the number of favourable cases to the number of unfavourable cases is called the *odds for* the event, and the reciprocal of this ratio, the *odds against* the event.

In this and the following rules the term *probability* is taken as synonymous with *chance* or *expectation*.

<sup>1</sup> See Todhunter's *Algebra*, Mill's *Logic*, ii. pp. 62-78, La Place's *Théorie Analytique des Probabilités*, and Professor Crofton's article on *Probability* in the nineteenth volume of the ninth edition of the *Encyclopædia Britannica*. Compare, also, Thomson's *Laws of Thought*, § 124.

(2) If an event may happen in different independent ways, these being exclusive, or incompatible, the probability of its happening is the sum of the probabilities of its happening in the different independent ways.

(3) If there be independent events of which the respective probabilities are known, then the probability that they will *all* happen is the product of their respective probabilities of happening.

(4) When two events are dependent one on the other, the probability of their concurrence is the product of the probability of the first into the probability that when the first has happened the other will follow.

(5) The probability that out of a number of independent events one or more will happen is found by multiplying the separate probabilities against them together, and deducting the product from unity.

(6) The probability of the recurrence of an event already observed is found by dividing the number of times the event has been observed, increased by one, by the same number increased by two.

(7) The probability that an event already observed will be repeated any given number of times is found by dividing the number of times the event has been observed, increased by one, by the same number increased by one and the number of times the event is to recur.

**Examples.**      § 102.—The following examples and exercises will serve to illustrate the foregoing rules, while at the same time accentuating certain essential points of theory :—

1. There are placed before a blind man a gold, a silver, and a leaden urn, the first containing five white and two black balls, the second and third six white balls each ; what is the chance of his drawing a black ball ?

His drawing from the golden urn is  $\frac{1}{3}$  of the possible cases ;

His drawing a black ball is  $\frac{2}{7}$  of the possible drawings from that urn ;

Therefore, his drawing a black ball is  $\frac{2}{21}$  of the possible cases.

In other words, there are 19 chances to 2 against his drawing a black ball.

It is assumed that in the "long run" of an indefinitely great number of trials, the blind man would find the golden, the silver, and the leaden urn, each in its turn, an equal number of times, and, consequently, the golden urn a number of times equal on the average to  $\frac{1}{3}$  of the whole number of trials. And, again, in the long run of an indefinitely great number of drawings from the golden urn it is assumed that the white balls would be drawn with a frequency proportioned to their number, five, and the black with a frequency proportioned to their number, two. Since the white have an advantage, the drawing of a white ball might be expected to recur with a frequency proportioned to the strength of that advantage.

2. Supposing the blind man mentioned in the preceding example were to observe that his belief was  $\frac{1}{3}$  of a certainty that he would in any one trial find the golden urn to draw from, what reasonable explanation could be given of the terms of such observation?

That, on the face of it, he was prepared to adjust his conduct to  $\frac{1}{3}$  of what it would have been in the case of certainty. His belief, in the strict sense, might be regarded as objectively justified by what would take place in the long run, or in a very great number of trials. He might, for example, be said to be *certain* that in 600 trials he would, a number of times closely approximating to 200, have laid his hand on the golden urn, and his conduct might be taken as an index of such certainty. A numerical association arising from the group attaches itself to the individual event, carrying with it an adjustment or apportionment of conduct consistent with that unitary amount the entire group calls forth; and this, translated back into belief in connection with the individual event, gives rise to the notion of fractional or partial belief with regard to any one contingent event. The

blind man's observation that his belief in the given instance was  $\frac{1}{2}$  of a certainty he would, accordingly, if pressed, have been prepared to justify by his conduct; relating his conduct, in its apportionment to his conduct under full belief as associated with what would happen in the long run. At the moment and in relation to the single event, the blind man might be considered to have full belief in the disjunctive proposition that on trial he either would or would not succeed in laying his hand on the golden urn. (See Venn's *Logic of Chance*, 3rd ed., pp. 145, 151, 165.)

3. There are two urns, one of which contains three white and four black balls, the other four white and five black balls; what is the probability of obtaining a white ball by a single drawing from one of the urns taken at random?

The probability of drawing from *the first urn* is  $\frac{1}{2}$ ;

The probability of drawing *a white ball* from that urn is  $\frac{3}{7}$ ;

Therefore, the probability of drawing a white ball from the first urn is  $\frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$ .

So, the probability of drawing a white ball from the second urn is  $\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$ .

Hence, the probability of drawing a white ball at one trial is  $\frac{3}{14} + \frac{2}{9} = \frac{55}{126}$ . (See Todhunter's *Algebra*, § 724.)

4. The probability that a March morning will be a breezy morning is  $\frac{1}{2}$ ; the probability that a breezy morning is rainy is  $\frac{1}{3}$ ; hence, the probability that a March morning will be both breezy and rainy is  $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ .

5. If the probability of my being able to obtain the second volume of a certain work be represented by  $\frac{1}{2}$ , and that of obtaining the third be  $\frac{1}{3}$ , then the probability of my being able to obtain both will be represented by  $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ . And so in all like cases.

6. Of two couriers, the chance that one will reach a given destination in time is  $\frac{1}{2}$ , and that the other will reach it is  $\frac{2}{3}$ ; what is the chance that one or either of them will reach it in time if they both try?

$1 - \frac{1}{2} = \frac{1}{2}$  is the chance that the first will fail to reach the given destination in time;

$1 - \frac{2}{3} = \frac{1}{3}$  is the chance that the second will fail;

$\therefore \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$  is the chance that both will fail, and hence,

$1 - \frac{1}{6} = \frac{5}{6}$  is the chance that one or other, if not both, will succeed.

7. Why does Dr. Venn affirm (*Logic of Chance*, 3rd ed., p. 138) that "the conception of the science of Probability as a science of the laws of belief seems to break down at every point"? R.

8. "A man is to receive £1 if a die gives six, and to pay 1s. if it gives any other number. It will generally be admitted that he ought to give 2s. 6d. for the chance, and that if he does so he will be paying a fair sum" (*Venn's Logic of Chance*, 3rd ed., p. 142). What is meant by calling the sum a fair one? R.

That, in the long run, each party's gains will have counterbalanced his losses.

9. "In applying the doctrine of chances to that subject in connection with which it was invented—Games of chance—the principles of what has been happily termed 'moral arithmetic' must not be forgotten."—*Laws of Thought*, p. 264. Give the substance of Dr. Venn's comments (*Logic of Chance*, p. 153) on this passage.

10. Examine the following passages:—

"Probability as such is not true of the fact, but it always has reference to fact."

"The logic of probability is essentially concerned with statistical frequency, and not simply with the quantity of belief that we happen to possess."

"A great improbability before the event may become a very small improbability after the event" (*Logic of Chance*, chap. XII.).

"Is probability simply the quantity of belief we happen to possess? No. The amount of our belief is psychological: the probability of a fact is always logical."

"Any theory which calls the Doctrine of Chances merely objective, or merely subjective, is certainly false." R.

11. What is exactly meant when it is said that certain events are equally likely in the long run? R.

12. Given the chance of a thing's being  $a$  and  $b$ , respectively, find the chances of its being (1) both  $a$  and  $b$ , (2)  $a$  and not  $b$ , (3)  $b$  and not  $a$ , and (4) not  $a$  and not  $b$ .

13. "De Morgan has described life insurance as a bet"? (*Logic of Chance*, p. 379.) R.

14. Comment on the following passages:—

(1) "An *à priori* necessity has been assumed for this proportionate recurrence of events."

(2) "We soon come to know better; and refraining from *à priori* suppositions we trust solely to induction from a sufficiently prolonged basis of actual observation."

(Bain's *Logic*, ii. p. 92.)

Suppose we know of several events that some one will certainly happen, and that nothing in the constitution of things determines one rather than another; in such case each will recur, in the long run with a frequency in the proportion of one to the whole. Thus, in tossing a penny there is supposed to be nothing in the form of the coin or in the impulse given to it to determine one side rather than the other. In this case every second throw will in the long run be heads. The assumption of an *à priori* necessity appears to be justified in this case. We seem to be in a state of equipoise between the two possibilities of head and tail, and feel that any inequality in the result would be without reason or cause. Hence, we are apt to assume, that in virtue of an *à priori* necessity, the turning up of head and tail should be equally balanced at the end of a long trial. The fact is, however, that in cases such as the tossing up of a penny, we are exceptionally circumstanced in point of knowledge. We know what are the causes at work, and that (neglecting the differences in weight of the figures in relief on both sides) there is nothing to give a bias in the long run to either side of the penny. In the more complicated cases, however, as human life, shipwrecks, fires, etc., we should not be disposed to predict anything beforehand from such

considerations as might dispose us to forecast in such cases as tossing a penny or throwing dice. We should not consider all years, from one to ninety, as equally likely for men to die in, or that the year of age is quite indifferent. We refrain in such more complicated cases from *a priori* suppositions, and trust solely to induction based on actual observation.

**Combination of Probable Arguments.**—It often happens that several probable arguments are adduced in support of one and the same conclusion. Such arguments, when independent, instead of weakening, mutually strengthen each other; and if any one of them have the force of certainty, the conclusion may be regarded as true, or certainly established. To find the probability of a conclusion supported by several such probable arguments, we must first calculate the probability of failure as derived from each separate argument, and multiplying the probabilities of failure together the result will be the probability that the arguments taken together will fail to establish the conclusion: subtracting this fraction from unity we get the probability in favour of the conclusion from the several arguments taken jointly. Thus, suppose the probabilities in favour of a certain conclusion as derived from three independent syllogisms or arguments to be represented, respectively, by  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{3}{4}$ , the probabilities against it thence derived are  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{4}$ ; and, therefore, the probability against it based on all three is  $\frac{1}{24}$ . Hence, the probability in favour of it is  $\frac{23}{24}$ ; in other words, the odds in favour of it are 23 to 1.

**Evidence in General.**—Evidence may be defined to be that fact or association of facts that enables the mind to see truth. It is explained by Jevons to mean "any facts apprehended by the mind, and made the grounds of knowledge and belief."

Direct Evidence is that which is obtained from personal knowledge of the facts in issue.

Of Indirect Evidence, much, including what is known as circumstantial evidence, may be regarded as resting upon probabilities.

Circumstantial Evidence is "that which is obtained from circumstances that necessarily, or usually, attend facts of a particular nature, from which arises presumption." It is so called from the fact that it adduces several circumstances as all pointing to, or supporting, the same conclusion.

In estimating the value of circumstantial evidence we must weigh carefully all the circumstances brought before us bearing upon the given case. We must sift these circumstances, separating those that appear to lead to one and the same conclusion from such as seem to make against it. This being done, we shall be in a position to say how far the net result of the mass of evidence of this kind before us is in favour of any particular conclusion.

A chain, or coil, of circumstantial evidence may be presented in a series of syllogisms either in the first figure,—each having one probable and one certain premiss, or, in the second figure—involving an undistributed middle, and leading, in both forms, to but a probable conclusion. Consider, in illustration, the following set of circumstances. A man has been found lying dead on a roadside at a particular place; his death being apparently due to the effects of a peculiar wound on the head. At a certain hour on the same day as the dead body was discovered, he was seen, if not in company with, at least very closely followed along this road by a person armed with a heavy stick. The dead man was known to have had in his possession on that day a considerable sum of money in gold not since accounted for. The person who appeared to be following if not accompanying him on the road at the hour indicated of the said day was a person of very doubtful character, and known at the time to be in monetary difficulties. The wound on the head was such as could have been inflicted with such a weapon as the heavy stick seen with this person. Stick and owner had since the event disappeared; though the person in question was known to have been previously very seldom absent from the neighbourhood. Here we have a great variety of circumstances from which we can construct syllogisms

to illustrate the formal method of presenting a chain of circumstantial evidence.

We may argue, for example, that :—

A man found dead with a wound of a certain kind on the head is probably a murdered man ;

This man was so found ;

Therefore, this man is probably a murdered man.

Or, we might proceed thus :—

A murdered man would be found with a wound not self-inflicted ;

This man was found dead with a wound not self-inflicted.

Therefore, the man is probably a murdered man.—The syllogism being in the second figure, involving an undistributed middle but justifying a probable conclusion.<sup>1</sup>

And so on.

Yet notwithstanding the force with which the circumstances just enumerated seem to lead to the conclusion that a certain individual distinctly pointed at has been guilty of murder, there remains a sufficient opening for the suspected person, if he himself also had not been made away with, to completely clear himself of the charge. Though the presumption thus arising from the consideration of evidence of this kind in favour of a definite conclusion may therefore, as in this case, be very great, especially when we have a number of arguments all pointing to that conclusion, still, uncertainty is an element that always attaches to the final result ; and the cumulation of such arguments leads, after all, in the issue, to the establishment of a probability only, not to demonstration.

Of evidence in general, the two most important applications are those in legal affairs, and in matters of history. The leading rules of evidence bearing on legal affairs will be found laid down by writers on civil and criminal law. The occurrences of former times, and even those of our own time

<sup>1</sup> Commenting on this form, Dr. Fowler says, "It seems, however, extremely awkward to represent such reasonings as fallacies, and then, by way of compensation, to regard them as probable arguments."—*Deductive Logic*, p. 140.

that took place, or have taken place, under peculiar combinations of circumstances such as may never be repeated must be received on the testimony of others. And the leading rule of historical evidence is (Bain's *Logic*, ii. p. 423) that "all testimony must be *contemporary*, or received directly, or through trustworthy tradition from contemporaries."

As to evidence arising from individual testimony, we should, in estimating its value, have regard to (1) the reasonableness of the matter or thing related, and (2) the character of the witnesses by whom it is related in respect of veracity and competency. As to the witnesses, the leading points to guide us should be the following: namely, "whether the persons relating the facts or circumstances to be believed are possessed of powers of mind capable of perceiving the truth; whether they were eye- or ear-witnesses of what they relate; whether the narrators are honest, faithful, and unprejudiced persons; whether their testimony is corroborated by other persons worthy of credit; whether the facts or circumstances lay exposed to open investigation at the time they took place; whether the witnesses betray an indiscreet zeal for the establishment of their story, or bring forth as much evidence as will exactly suit the matters affirmed to be true; whether there be an outward visible appearance of honesty and simplicity of mind about them; whether the narrators belong to different nations, sects and situations in life; and lastly, whether other little incidental circumstances commonly connected with the relation of the main facts be of such a character as not to throw any doubt over the testimony of the witnesses" (Blakey's *System of Logic*, p. 112).

The channels for the transmission of testimony respecting the occurrences of former times are tradition and ancient writings. Tradition can be received only with very great caution. As to ancient writings as purporting to be the recorded testimony of certain individuals of a former generation, we have first to inquire whether they are actually the handwriting, or the composition as taken down by dictation,

or a faithful report, made at the time, of the substance of the evidence of the individuals whose names the writings bear, or to whom they are attributed. The establishment of either of these three points is the proof of what is called the *genuineness* of the writing. But a second question must be answered before we can accept the evidence furnished by an ancient document. We must be satisfied not only that the testimony is genuine,—that it was, actually given by those from whom it purports to come, but also that it is authentic, in other words, that this testimony is a true and faithful narrative of what actually happened. Proofs of the genuineness of the writing amount, at most, only to bringing the witnesses into court and establishing their identity; proofs of the *authenticity* must be found by sifting their evidence, and applying it all to the tests and means of verification we possess, in order to ascertain whether they are telling the truth. If not genuine, the document is said to be spurious; if not authentic, to be false. The tests and proofs of the genuineness and the authenticity of a writing or document may be generally divided into two classes, called respectively, the external and the internal evidence respecting the point to be proved. The external evidences of genuineness are to be found either in other admitted writings of the supposed author, or in the works of writers who were his contemporaries, or nearly so; the evidence being direct, if the disputed writing is therein explicitly attributed to him, or indirect, if these works quote as his productions passages found in the writings or documents under scrutiny. The external evidences of the authenticity of a given writing considered as a narrative of facts are too numerous to mention. They are found in allusions to the same facts, or to incidents obviously connected with them, by contemporary authors; in customs, traditions, and institutions which have come down to later times, and the origin of which cannot be accounted for, except on the supposition that the reported events actually took place; in coins, medals, and inscriptions, belonging to the same age, or to one immediately subsequent and connected

by equally close relations with the alleged facts ; in the notoriety which such incidents must have obtained, the interest which must have been felt in them, and the consequent improbability that falsifications and forgeries respecting them would ever have been attempted, or, if attempted, would have escaped immediate detection. As to internal evidence, it may be regarded as comparatively weak to establish either the genuineness or the authenticity of a writing or document, but as powerful to disprove them. To various features of the writings of an ancient author imitations and forgeries may make so close an approach as to render it exceedingly hazardous to conclude that a writing is genuine or authentic because it has such and such characteristics. On the other hand, a single undoubted anachronism in respect of events, institutions, customs, or even language, may be regarded as fatal to the claims of a writing or document to either genuineness or authenticity. (See Bowen's *Logic*, p. 436 ; and, on the whole question of Historical Evidence, Bain's *Logic*, ii. pp. 423-431, and Sir G. C. Lewis's *Methods of Politics*, vol. i., pp. 181-317.)

**Additional Examples and Studies.** § 105.—The following examples and studies for the most part relate to the matter of the preceding section :—

1. Define circumstantial evidence, and exemplify it in syllogistic form. What cautions should be observed in estimating its value ? Give instances where it is ( $\alpha$ ) conclusive ; ( $\beta$ ) doubtful ; ( $\gamma$ ) worthless. C. S.

2. Discuss the following :—

“Circumstantial evidence is better than direct, for men often lie, but facts never do.” C. S.

3. “The very concurrence and coincidence of so many evidences that contribute to the proof carries great weight.”

—Sir Matthew Hale. To what kinds, or cases, of evidence or proof, does the above assertion apply ? Does it apply, for instance, to the proof of the identity of a writer by means of his handwriting ? C. S.

4. “In what relates to human action certainty is un-

attainable." How does this statement bear upon the popular distinction between direct and circumstantial evidence?

5. Discriminate the kinds of evidence required to establish the following propositions:—

- (a) Light moves with a velocity of nearly 186,000 miles per second.
- (b) Napoleon the Third was an usurper.
- (c) The Battle of Agincourt was a victory for the English.
- (d) The Iliad is an epic poem.
- (e) Three times six are eighteen.
- (f) The injury was due to malice. C. S.

6. Discuss the applicability of the theory of Probability to the credibility of witnesses. (See Venn's *Logic of Chance*, chap. XVI.)

7. Explain and illustrate the following statements:—

"Circumstantial evidence falls naturally into a series of enthymemes of the second figure."—Thomson's *Laws of Thought*, p. 248.

"This mode of treatment, no doubt, originated in the desire to conform the argument in circumstantial evidence to those enthymematic syllogisms of Aristotle<sup>1</sup> which employ

<sup>1</sup> The Enthymeme of Aristotle (see § 82) is described as *συλλογισμὸς ἐξ ἐκδτων ἡ σημεῖον*, a syllogism from probable propositions, or from signs. The sign (*σημεῖον*) is a proposition in which some one fact, mark, or sign, that accompanies, precedes, or follows, another fact or conception is adduced as a necessary or probable indication that the other is present. As an illustration of the use of the *σημεῖον* in the second figure, consider the following quasi-syllogism: The author of "Junius" wrote a particular hand; Sir Philip Francis wrote the same kind of hand; Therefore, Sir Philip Francis is the author of "Junius." Here, the *sign* (referring to the single term merely) is the middle term. Taking other signs, such as certain mistakes in correcting proof-sheets, a particular style, an anomalous use of certain words, the employment of certain images, and the having ceased to write at a particular time—the middle terms in the string of enthymemes in Mr. Taylor's investigation of the authorship of the "Letters of Junius"—we have a series of syllogisms each logically invalid, but yet, taken all together, making a very strong case. The "probable proposition" (*εἰκός*) of the Enthymeme is merely a general statement which does not, however, amount to a universal; e.g., "Most men who envy hate." Most men who envy hate; This man envies; ... This man (probably) hates. Other examples of *εἰκότα* are: "Ignorant men will seek revenge," "Men are

the *σημεῖον* in the second figure.”—Fowler’s *Deductive Logic*, 10th ed., p. 140.

8. What is the logical defect of circumstantial evidence?
9. Discuss the general conditions on which the value of testimony depends. Examine the following dictum:—“The trial of the evidence commences when the jury retires: the evidence of eleven other men to the character of the evidence is itself part of the evidence.” C. S.
10. The authorship of a book is attributed to A. B. He denies it. Give, in outline, the various grounds on which it might be (*α*) proved, (*β*) surmised, that he wrote the book, his denial notwithstanding. C. S.
11. State and illustrate the canons of historical evidence.<sup>1</sup>
12. By what logical criterion is it possible to distinguish between history and myth? C. S.

active when their interest is concerned,” and the like.—See Weldon’s translation of Aristotle’s *Rhetoric*, pp. 16–19.

<sup>1</sup> In view of a comprehensive treatment such canons may be distinguished into *general* and *special*; the former relating to *facts* and *witnesses*, the latter, to *oral tradition*, *public monuments*, and *written history*.

## CHAPTER VII

### ON FALLACIES

**Definition** and **Scholas**—“any unsound mode of arguing which appears to **tie Classification**. demand our conviction, and to be decisive of the question in hand, when in fairness it is not.”

An argument formally incorrect, if it leads the person receiving it into error is called a *Paralogism*, and if there is the intention to deceive, a *Sophism*.

Fallacies were grouped by Aristotle into two chief classes, namely, those arising from the language employed, hence said to be, *in dictione*, or *in voce*, and those not arising from the language, and hence said to be, *extra dictiōnem*.

To this grouping Whately objects, as not being grounded on any distinct principle of division; at least, not on any principle consistently adhered to by the older logicians. He himself proposes, instead, the dichotomic grouping, *logical* and *non-logical*<sup>1</sup>; the former including all such that the conclusion does not follow from the premisses—the fault being in the reasoning, and the latter those such that the conclusion does follow from the premisses—the fault being, therefore, in the matter.

**Logical Fallacies.** § 107.—The Logical Fallacies may be divided into the *purely logical*, and the *semi-logical*.

The Purely Logical Fallacies are, those of

(1) Four Terms (*quaternio terminorum*),—involving a breach of Rule (1), § 64. (—See footnote there.)

<sup>1</sup> On which see, however, Poste's *Aristotle on Fallacies*, p. 211.

- (2) Undistributed middle,—a breach of (1), § 64.
- (3) Illicit process,—a breach of (2), § 64.
- (4) Negative premisses,—involving breaches of (3), and (4), § 64.

The Semi-logical Fallacies are, those of

(1) Equivocation,—due to the use of the one term or word in two distinct senses : if this be the middle term of a syllogism there arises what is called *the fallacy of ambiguous middle*.

(2) Amphibology,—due to an ambiguous grammatical structure in a sentence.

(3) Composition,—due to the use of a term distributively in the major premiss, collectively in the minor.

(4) Division,—due to the use of a term collectively in the major premiss, distributively in the minor.

(5) Accent,—due to a misplaced accent or wrong emphasis.

(6) Figure of speech,—due to confusion between one part of speech and another of the same root, differing in a shade of meaning,—that is to say, between conjugate or paronymous words.

The following are examples of the Semi-logical Fallacies :

(1) Repentance is a good thing ; Wicked men abound in repentance ; Therefore, wicked men abound in what is good.

(2) "The Duke yet lives that Henry shall depose" (—Shakespeare's *Henry VI.*) is a destined thing ; A destined thing will surely happen ; Therefore, that a certain living Duke shall depose Henry (or, Henry a certain living Duke?) is a thing that will surely happen.

(3) Three and two are two numbers ; Three and two are five ; Therefore, five is two numbers.

(4) Five is one number ; Three and two are five ; Therefore, three and two are one number.

(5) "Thou shalt not bear false witness against thy neighbour" ; A person residing ten miles away is not your neighbour ; Therefore, you need not be scrupulous as to the character of your testimony against such a person.

(6) Projectors are unfit to be trusted; This man has formed a project; Therefore, this man is unfit to be trusted.

Non-logical or Material Fallacies of Aristotle and the Schoolmen are seven in number, and are as follows<sup>1</sup> :—

(1) The Fallacy of Accident, *Fallacia accidentis*, or, *a dicto simpliciter ad dictum secundum quid*, — the fallacy of proceeding from what is said simply, and without qualification, to what is said in a certain respect, or with a certain qualification. In other words, it consists in employing the middle term in one premiss to signify something considered simply in itself and as to its essence, and in the other premiss so as to imply that its accidents are taken into account with it.

(2) The Converse Fallacy of Accident, *Fallacia a dicto secundum quid ad dictum simpliciter*. This consists in employing the middle term in one premiss taking one of its accidents into account, and in the other employing it simply, that is to say, with the omission of that accident.

(3) The Irrelevant Conclusion, or *Ignoratio Elenchi*<sup>2</sup> (literally, “Ignorance of the proof of the contradictory”) consists in arguing beside the point, and is often employed by such as have to support a weak case. One form of it is the *argumentum ad hominem*, an argument turning not on the merits of the case but on the character or position of those engaged in it; or, it may take the form of pressing such persons with conclusions drawn from their own principles. Another, is the *argumentum ad verecundiam*, that is, an argument from the authority of great names.<sup>3</sup> And of the same character, though not specially referable to the present head

<sup>1</sup> The Semi-logical and the Non-logical, or Material Fallacies, are sometimes found mnemonically grouped together in the treatises of the schools thus :—

Equivocat. Amphi. Componit. Dividit. Acc. Fi.  
Acci. Quid. Ignorans. Non Causa. Con. Petit. Inter.

<sup>2</sup> An *Elenchus* (Ἐλέγχος) is an unassailable argument of our own to prove the contradictory of the conclusion of an opponent's argument.

<sup>3</sup> As Whately remarks, we should by no means universally call these arguments fallacies : they may be referred to this head, however, when they are unfairly used, and so far as they are fallacious.

only, is the *argumentum ad ignorantiam*, which consists in challenging a man to produce a better principle than yours. Contradistinguished from all these is the *argumentum ad rem* or *ad judicium*, which bears directly and absolutely on the real question, and appeals to the judgment.

One form of the fallacy of *Ignoratio Elenchi* may be pointed out as giving an unfair advantage to the respondent, namely, when something is proved not necessary that should have been proved not probable, or improbable that should have been proved impossible.

(4) The *Petitio Principii* (otherwise, *Petitio Quaesiti*), or Begging the Question, consists in taking one of the premisses, whether true or false, as either plainly equivalent to the conclusion, or dependent on the conclusion for its own reception.

Aristotle enumerates five ways in which this fallacy occurs.<sup>1</sup> The first is when the very thing that should be proved is assumed. This cannot easily pass when the terms are the same; but when synonyms are used or a name, or a circumlocution, it may escape detection. Whately observes that the English language offers facilities for obscuring the presence of the fallacy owing to the fact that, from the diversity of its sources, it abounds in synonymous expressions which have no resemblance in sound, and no obvious connection in etymology; so that a sophist may bring forward a proposition expressed in words, say, of Saxon origin, giving as a reason for it the very same proposition, but stated in words, say, of Norman origin. A second way in which the fallacy occurs is, when a particular ought to be proved, and the universal is assumed, or when what ought to be proved alone is assumed in conjunction with other propositions. A third way is, when a universal ought to be proved, and the particular is assumed, or when what should be proved in conjunction with other propositions is assumed alone. A fourth way is, when we divide, or break up, the problem to be proved into distinct parts, and assume the particulars in detail. A fifth way

<sup>1</sup> See Poste's *Aristotle on Fallacies*, p. 181.

is, when two facts are reciprocally involved, and we assume one to prove the other.

The most plausible and important form of the fallacy of *Petitio Principii* is the *circulus in probando*, or, Argument in a Circle, which consists in assuming, in a train of reasoning, or a series of syllogisms constituting a chain of proof, the conclusion of the final syllogism as one of the premisses of the first or fundamental syllogism. Such a procedure is, of course, the less easily perceived, the longer the circuit, or train of reasoning. To expose the fallacy, therefore, the best way is to shorten the circuit by cutting off the intermediate steps, and when the same proposition comes round a second time, to exhibit it in the same words.

“Arguing in a circle” observes Mill, “implies more than the mere passive reception of a premiss by one who does not remember how it is to be proved. It implies an actual attempt to prove two propositions reciprocally from one another, and is seldom resorted to, at least in express terms, by any person in his own speculations, but is committed by those who, being hard pressed by an adversary, are forced into giving reasons for an opinion of which, when they began to argue, they had not sufficiently considered the grounds.”

Closely associated with the fallacy of *Petitio Principii* are, what have been called by Bentham, *Question-begging Appellatives*. These are epithets so selected as to assume the point at issue and, so, to unfairly prejudice the minds of the persons addressed. For instance, in politics, the word Innovation (Mill’s *Logic*, ii. p. 404).

By Dr. Campbell, Dugald Stewart, and others, every syllogism has been held to involve a *Petitio Principii*. Whately and Mansel dissent from this view. Mill very carefully distinguishes. According to his view, there is a *Petitio Principii* inherent in every syllogism if we consider the major premiss to be itself the evidence by which the conclusion is proved, instead of being, what in fact it is, an assertion of the existence of evidence sufficient to prove any conclusion of a given description. In this way, the major premiss may be

divested of the *Petitio Principii*. As Minto observes, *Petitio Principii* is an argumentative artifice, "a conscious or unconscious act of deception, a covert assumption ; and the syllogism, so far from favouring this, is an *expositio principii*, an explicit statement of premisses such that, if they are true, the conclusion is true. The syllogism merely shows the interdependence of premisses and conclusion ; its only tacit assumption is the *Dictum de Omni*" (*Logic*, p. 234).

(5) The Fallacy of the Consequent, *Fallacia consequentis*, or *Non sequitur*. This consists in the assertion of a conclusion which has no real connection with the premisses.

(6) The Fallacy of False Cause, *Non causa pro causa*, or *Post hoc ergo propter hoc*. This is an inductive fallacy, and consists in assuming without sufficient grounds that one thing is the cause of another.

(7) The Fallacy of Many Questions, *Fallacia plurium interrogationum*. This consists in putting more questions than one as one question, so that a snare is laid for the unwary ; since a single answer, if given without distinguishing, being truly applicable to but one question, might be interpreted as intended to meet the other or others.

Aristotle points out the certain way for exposing a fallacy of this kind. "Several questions put as one should at once be distinguished apart. Only a single question admits of a single answer ; so that neither several predicates of one subject, nor one predicate of several subjects, but only one predicate of one subject, ought to be affirmed or denied in a single answer."

Examples of the Non-logical, or Material, Fallacies. § 109.—The following are, respectively, examples, in order, of the non-logical, or material Fallacies. fallacies :—

(1) What you bought yesterday you eat to-day ; You bought raw meat yesterday ; Therefore, you eat raw meat to-day.

(2) Physicians advise some of their patients to take opium ; Opium is a poison ; Therefore, physicians advise some of their patients to take poison.

(3) In a certain law-suit, the attorney for the defendant is said to have handed counsel his brief marked, "No case : abuse the plaintiff's attorney." It must suffice to add that the counsel, thereupon, roundly rated the attorney on the opposite side ; thereby furnishing an excellent illustration of the Ignoratio Elenchi, but one which writers on Logic have some hesitation to reproduce.

(4) To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the state ; for it is highly conducive to the interests of the community that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments.

(5) Episcopacy is of Scripture origin ; the Church of England is the only episcopal Church in England ; Ergo, the Church established is the Church that should be supported.

—*De Morgan.*

(6) Night invariably precedes day ; therefore, it must be the cause of day.

(7) "King Charles II.'s celebrated inquiry of the Royal Society may be referred to this head. He asked the cause why a dead fish does not (though a live fish does) add to the weight of a vessel of water. This implies *two* questions—the first of which many philosophers for a time overlooked : viz., 1st, Is it a fact ? 2ndly, If it be a fact, what can cause it?" —*Whately* (see Fowler's *Inductive Logic*, p. 256).

**Mill's Classification.** — § 110.—Mill's classification of Fallacies<sup>1</sup> is as follows :—

<sup>1</sup> A catalogue of the varieties of apparent evidence which are not real evidence is an enumeration of fallacies (Mill's *Logic*, ii. p. 300). Such an enumeration excludes mere blunders, and restricts itself to the intellectual causes of error. The moral causes, such as indifference, and bias, though, of course, most powerful, are yet indirect and remote, and act only through the intellectual causes. Consequently to guard against these last is to guard against every other cause of error. We have a classified enumeration of the most potent of the moral causes of error, in those *Idola mentis humanae*, or "Phantoms of the Human Mind," of which an account is given by Bacon in the *Novum Organum*. The *Idola* are false notions, or causes of error, that beset the mind of man, and impede us in a successful search for, or a correct estimation of evidence. They are four in number ; namely, (1) the *Idola tribus*, or

(a) Fallacies of Simple Inspection, or Fallacies *à priori*. These consist in receiving a proposition as self-evident, or as an *à priori* truth requiring no evidence, upon, as it were, a simple inspection of it, or the mere comprehension of its meaning.

(β) Fallacies of Inference, which are distinguished into (a) those from evidence distinctly conceived, and (b) those from evidence indistinctly conceived.

The class (a) is subdivided into Inductive Fallacies—those of Observation and Generalization—and Deductive Fallacies, or Fallacies of Ratiocination.

Those from evidence indistinctly conceived are treated in one group as Fallacies of Confusion.

Thus, Mill's table, in accordance with his view and treatment of Logic, covers both Deduction and Induction. We shall consider his ultimate heads briefly in detail.

(1) Fallacies *à priori* relate to the assumptions we make on the border-land of the objective and the subjective—to those fundamental propositions which bear upon the efficient causes and necessary substrata of all phenomena, and which we receive without proof.

(2) Fallacies of Observation relate to the first step—Observation—in the performance of the proving process. They may be either of Non-observation, or Mal-observation. The former may take place either by overlooking instances, or by overlooking some of the circumstances of a given instance—by leaving something unseen. The latter takes place by mistaking for conception what in fact is inference, so that something seen is seen wrong.

(3) Fallacies of Generalization relate to the second step—

Idols of the Tribe,—those false notions which man is liable to from his nature as man ; (2) the *Idola specus*, or Idols of the Den or Cave,—those that beset the mind of each individual man from his individual position ; (3) the *Idola fori*, or Idols of the Market-place,—those that arise from our abuse of language in our ordinary intercourse with mankind ; and (4) the *Idola theatri*, or Idols of the Theatre,—those sources of error which lie in false theories and systems of philosophy, and relate to the poetic, artistic, or ideal cravings of the mind.—*Novum Organum*, Bk. I., Aph. 38-70.

Generalization—in the performance of the proving process. They exhibit a fundamental misconception of the legitimate mode of drawing conclusions from observation and experiment. The field covered by this class is very large, and is one of much debate.

(4) Fallacies of Ratiocination are those which have their seat in the ratiocinative or deductive part of the investigation of truth. They are those to which, in the logic of the schools, the appellation of Fallacy is in general exclusively applied.

(5) Fallacies of Confusion are those in which the source of error is not so much a false estimate of the probative force of known evidence, as an indistinct, indefinite, and fluctuating conception of what the evidence is.

**Illustrations.** § 111.—The following are a few of the illustrations given by Mill under the foregoing heads:—

(1) The same order must obtain among the objects in nature as obtains among our ideas of them.

(2) The faith which the uneducated portion of the agricultural classes in this and other countries continue to repose in the prophecies as to weather supplied by almanac makers.

(3) Inferences from the order of nature existing on the earth, or in the solar system, to that which may exist in remote parts of the universe.

(4) Increase of money is increase of riches, even when produced by means subversive of the conditions under which alone money may be riches.

(5) That common form of argument against Berkeley's theory of the non-existence of matter which consists in knocking a stick against the ground.

**Miscellaneous Questions and Studies on Fallacies.** § 112.—The following questions and studies relate principally to the matter of the preceding sections of the present chapter:—

1. Terms are liable to be indistinct, propositions to be false, and arguments inconclusive (*mendosa collectio*). How does Logic provide against these defects?

2. A certain argument having been shown to involve para-  
logism, inquire into the conditions under which this failure  
does or does not tend to establish the contradictory  
conclusion.
3. Explain and illustrate the fallacies of *Illicit Process*,  
*Post hoc ergo propter hoc*, *Aequiroatio* or *Homonymia*.
4. Is it true that all formal fallacies, logical and semi-  
logical alike, are forms of the fallacy of four terms? Illustrate  
your answer by examples.
5. Examine the following arguments--reducing them  
where necessary to the syllogistic form; and where you find  
them invalid explain the nature of the fallacy:—
  - (a) Virtuous men alone are happy; Socrates, therefore, is  
happy, being a virtuous man.
  - (b) He who says you are a man speaks truly; He who  
says you are a fool says you are a man; Therefore, he who  
says you are a fool speaks truly.
6. Explain briefly the following terms:—*Dilemma*,  
*Fallacia divisionis*, *Real kind*, *Subaltern mood*. C. S.
7. Write a note on the division of fallacies into those *In  
dictione*, and *Extra dictionem*.<sup>1</sup>
8. Why is *Ignoratio Elenchi* so named? Distinguish and  
illustrate arguments *ad hominem*, *ad populum*, and *ad  
verecundiam*, showing when they are, and when they are not  
fallacies.
9. What is the fallacy of *Petitio Principii*? What are its  
various forms?
10. From what points of view can the syllogism be  
regarded, (1) as being, (2) as not being, a *petitio principii*?
11. The fallacy of Accident is reckoned among non-logical  
fallacies; it is said to be really a case of ambiguous middle;

<sup>1</sup> See Whately's *Logic*, p. 104; and for Question 15, p. 129. As to  
the division of fallacies into *In dictione* and *Extra dictionem*, Mansel  
points out that Whately's is not the ancient principle of distinction  
which these terms imply, and which is stated with more or less clear-  
ness by several logicians. Fallacies *in dictione* arise from defects in the  
ordinary signs of thought, and disappear on translation into another  
language. Fallacies *extra dictionem* are in the thought itself, and  
adhere to it in whatever language it may be expressed.

it is also said to be the converse of the fallacy *a dicto secundum quid ad dictum simpliciter*. Examine these several statements, explaining the terms employed.

12. Examine the following :—He who is most hungry eats most ; He who eats least is most hungry. Therefore, he who eats least eats most.

13. What we eat grew in the fields ; Loaves of bread are what we eat ; Therefore, loaves of bread grew in the fields.

14. Whatever body is in motion must move either in the place where it is, or in a place where it is not ; Neither of these is possible ; Therefore, there is no such thing as motion.<sup>1</sup>

15. He who necessarily goes or stays is not a free agent ; You must necessarily go or stay ; Therefore, you are not a free agent.

16. Strong drinks must be a cause of strength.

17. He that is of God heareth the words of God ; Therefore, you hear them not, because you are not of God.

18. There is no great man but his place could be supplied by another as great ; Therefore, all great men could be dispensed with.

19. It is certain that wealth often makes the mind uneasy, for it tends to fill it with care ; and it is equally

<sup>1</sup> For an interesting discussion of this sophism, see Monck's *Logic*, Chapter XIX. The word *Place*, as employed here, involves, as Mr. Monck points out, an ambiguity. It may mean either the precise space which the body occupies at a given instant, supposing it at rest, or a limited portion of the surrounding space. Taken in the latter sense, there is no difficulty, without further refinements in conceiving that a body may move in the place where it is, since the essential idea in motion, namely, *change of position with regard to a fixed point*, is not inconsistent with this use of the word *place*. Taken in the former sense, the *place* of a moving body is but instantaneous, just as the position of a moving point is instantaneous. And just as it may be said to begin to be at any moment in the place where it is, so it may be said to begin to move in such place. Consequently, whichever of the two meanings we attach to the word *Place* in the major premiss of this sophism, the minor appears to be untenable. For this and other sophisms of Zeno see *La Physique d'Aristote*, par J. B. Saint-Hilaire, tome ii., p. 400.

certain that it fills it with care, for wealthy men are seldom at ease in their minds.

20. All vegetables grow most in the increase of the moon; Hair is a vegetable; Therefore, hair grows most in the increase of the moon.

21. Nothing is better than wisdom; Dry bread is better than nothing; Therefore, dry bread is better than wisdom.

22. "Let Achilles run a race against a tortoise; then, if the tortoise has the start, Achilles, though he run ten times as fast as the tortoise, will never overtake him. For, suppose them to be at first separated by an interval of a thousand feet: when Achilles has run these thousand feet, the tortoise will have got on a hundred; when Achilles has run these hundred, the tortoise will have run ten, and so on for ever; therefore, Achilles may run for ever without overtaking the tortoise." Explain this sophism.<sup>1</sup>

23. Write out, with a brief explanatory commentary, a classification of Fallacies.

24. State and illustrate some of the leading forms of the fallacy of Ambiguity (Whately's *Logic*, Bk. III., §§ 8-12).

25. Determine the formal validity of the following inferences, and give the corresponding technical names:—

(a) To be good is to be happy; Angels are happier than men because they are better.

(b) The Cretan, Epimenides, said that all the Cretans were incredible liars. Is he to be believed or not? If we believe him, we must, he being a Cretan, disbelieve him.

26. State and exemplify some fallacies incident to logical division.

27. If, according to the Aristotelians, all correct reasoning rests upon the *Dictum de omni et nullo*, must not all fallacy

<sup>1</sup> Mill, after Hobbes, considers (*Logic*, Bk. V., Chap. VII., § 1) that the gist of this fallacy consists in confounding a time which is infinitely divisible with an infinite time. The "for ever" in the conclusion means for any length of time that can be supposed; but in the premisses, "ever" does not mean any *length* of time; it means any number of subdivisions of time.

consist in a breach of this dictum ? Of course, you will support your answer by reasons.

28. How is a fallacy in argument to be distinguished from an error of fact ? What is the function of special knowledge in detecting fallacies ? C. S.

29. Examine the following argument (*Ignava Ratio*) :—

“If it is fated that you shall recover from the present disease, you will recover, whether you call in a physician or not. If it is fated that you shall not recover, then, with or without a physician, you will not recover.”

But either the one or the other of these two contradictions is fated.

Therefore, it will be of no use to call in a doctor.

## CHAPTER VIII

### ON METHOD

**Nature of Method.** § 113.—“Method,” it has been observed, “is rather a power or spirit of the intellect pervading all that it does than its tangible product.” In so far, however, as this peculiar intellectual power leaves its impression on intellectual work of any kind, a certain definable feature is communicated to the more tangible product, so that we may describe Method as the appropriate *form* in which we must present to the mind the matter of any given discussion or inquiry. “Method,” according to the *Port Royal Logic* (Mr. Baynes’ translation), “may be called, in general, the art of disposing well a series of many thoughts, either for the discovery of truth, when we are ignorant of it, or for proving it to others when already known.” According to this view of the subject, it includes not merely all that relates to choice of sequence and direction in our progress from point to point, but also to the ordering of our thoughts within the sequence, to the determining of what is to be treated and what omitted, and to the due proportion and unification of the parts of a discourse to a definite end.

Many of the older writers treat Method as the fourth part of Logic. But to this arrangement it is objected, that while the other three parts describe, each, a distinct and complete product of thought, no such whole is treated in the doctrine of Method, which may be used, as Dr. Thomson

observes, "for making a whole science, or a whole speech, a system, or a sentence." Method is to be regarded rather as a complete practical Logic than as a part of Logic rationally co-ordinate with those parts that treat, respectively, of concepts, judgments, and syllogisms.

According to Mr. Bain's views, which harmonise to a certain extent with the foregoing, the science of Logic, Pure and Applied, may, as a whole, be regarded as a body of Method auxiliary to the search for truth. There are various admitted uses of Logic that fall under Method. One of these, as expressed by Hamilton, is "the rendering explicit in the statement whatever is implicit in the thought." Another is the arranging of an argument or chain of reasoning into the form that best discloses to the mind its conclusiveness or inconclusiveness, as taught by syllogism and the inductive canons. There are, besides, certain modes, Logic familiarises us with, of presenting to the mind all the known facts and premisses of a subject such as to suggest the conclusions involved, and this is a positive aid to discovery.

The views of these more recent writers appear to find an approach to satisfactory expression in a definition of Method that makes it the collection of the processes, both logical and experimental, that must be employed in the search after, and the scientific demonstration of truth.

In the present chapter, we shall consider but some very general features of Method, and these merely in outline; leaving the outline to be filled up to a certain measure of adequacy by help of the chapters on Applied Logic that immediately follow.

**Rules of Method.** § 114.—For the guidance of the reason towards the discovery of truth, Descartes, in his *Discourse on Method*, gives the following four rules which have been much insisted on by more recent writers:<sup>1</sup>—

(1) Never to accept anything as true, which we do not clearly know to be so.

<sup>1</sup> In particular by Père Malebranche, in his *Recherche de la Vérité*.

(2) To divide each of the difficulties we examine into as many parts as possible, or as may be necessary for resolving it.

(3) To conduct our thoughts in order, commencing with objects the most simple and the most easily known, so as to ascend by degrees to a knowledge of the most complex.

(4) To make, in relation to everything, enumerations so complete and reviews so general, as that we may be sure of having omitted nothing.

**The Analytic Method and the Synthetic.** § 115.—There are two important varieties of the method in general, the Analytic and the Synthetic. The Analytic Method (Greek, *ἀνάλυσις*) is so called “because it traces things back to their elements, and resolves knowledge into its first and original principles.” It is known also as the Method of Resolution, of Invention, and of Discovery. The Synthetic Method (Greek, *σύνθεσις*) is so called because “it gathers together the elements of knowledge, combining them into one whole, or system, in such a manner as that the understanding is enabled distinctly to follow truth in all her different stages and gradations.” The point of departure of this method is some general truth the formulation of which is arrived at by synthesis. It is called also the Method of Doctrine, of Demonstration or Instruction, and of Composition.

Illustrations of the practical handling of both these methods may be had from various fields of thought. The writer of a work on a new subject, for example, pursues the method of discovery in laying down his plan, and determining the details to be filled in. In the setting forth of his matter, however, towards building up the edifice of thought he has designed, he pursues the method of instruction or of synthesis.

As applied to natural phenomena, the method of discovery begins with facts apparent to the senses, and proceeds in search of laws or general principles which serve towards their unification; or from things which are, as it has been said, *nobis notiora*,—better known to us, to things *notiora naturae*, or

better known in nature. The method of instruction, on the other hand, begins with general principles,—things *notiora naturae*, and proceeds to account for the things *nobis notiora*.

Taking an illustration, say, from Astronomy, the method of discovery, as applied to the apparent movements of the heavenly bodies, takes the complex facts, as they are presented to our senses, to begin with, and proceeds, by a comparison of them in all the variety met with, to a determination of the uniformity underlying. The object of the process here is to discover uniformity and law. The process is, in its distinguishing features, inductive. The method of instruction, on the other hand, as applied to external nature, shows how an apparent complexity is due to a real simplicity and uniformity. The process is synthetic and deductive.

As applied to certain branches of mathematics,—for example, to Geometry, the method of discovery or analysis, starting, say, from a theorem which is to be proved, admits for the moment the truth of such theorem, and reasons backward thence, deductively, to some other theorem more obviously deducible from first principles. This furnishes the key to the proof of the theorem to be established, and to the path to be pursued from the more elementary position to reach it by the method of instruction or synthesis. The method of analysis is employed here for the discovery of proof. From point to point, however, the method of instruction or synthesis goes hand in hand, or alternates with it. The latter method, in its various applications, shows how, from initial data consisting of the simplest principles, a given proposition of even a complex character may be proved.

The synthetic method, as thus applied, is deductive; but it is to be observed that the method of discovery, or analytic method, and the method of instruction, or synthetic method, are not to be taken as synonymous with induction and deduction, respectively. There is very often a close correspondence between analysis and induction, and especially between synthesis and deduction; but how far

it exists in any particular case must be determined by reference to the circumstances of such case.

In contrasting these methods, namely of discovery and of instruction, or of analysis and of synthesis, and their special uses, it is to be borne in mind that it is often possible to discover new methods by synthesis, and to teach old ones by analysis. It is, of course, hardly necessary to observe that when the accumulation of knowledge, not merely the elucidation of a single proposition, is the object in view, and when, instead of being bound to follow a particular path, we may work out and combine conclusions at will, registering the results towards after use, the method of instruction combined with such registration becomes, as far as the process of registration proceeds, a method of discovery; though the work has been in other respects in the main synthetic in character.

#### QUESTIONS.

1. Distinguish between abstract and concrete names. To which of the classes belong (a) adjectives,<sup>1</sup> (b) names of states of consciousness? Are any abstract names connotative?<sup>2</sup>

2. The terms intension (or comprehension) and extension, strictly speaking, belong to the conceptualist logic: explain. How is it shown that analysis in extension is the same process as synthesis in intension, and that analysis in intension is synthesis in extension?<sup>3</sup>

<sup>1</sup> Adjectives are concrete general names: they are given to a plurality of things agreeing in a certain respect. Names of states of consciousness are abstracts. See Mill's *Logic*, Bk. I., Chap. II., § 4.

<sup>2</sup> Mill holds that even abstract names, though the names only of attributes, may in some instances be justly considered as connotative: for example, such a word as *fault*, equivalent to *bad* or *hurtful quality*. This word is a name common to many attributes, and connotes *hurtfulness*, an attribute of those various attributes. According to Dr. Fowler's use of the term *connote*, however (see § 21 of the present treatise), abstract terms are connotative, not denotative.

<sup>3</sup> To analyse or divide a *class* of things, that is to say, a collection of objects exactly resembling each other in certain definite qualities, we must add a quality or difference. Thus we divide the class *animal* when we add the quality *rational*, thereby separating the rational from the irrational. Again, to analyse a *notion*, we must subtract a quality

3. What attributes are included in the connotation of a class-name? Are proper names connotative,—for example, Mississippi, the Goodwin Sands, the Wye, the Reeks, Mont Blanc, William Rufus, *The Children of the Abbey, Paradise Lost*?

4. Mention various difficulties that occur in the process of Definition, and show how they are to be met. What kind of words are indefinable, and why?<sup>1</sup> Explain the maxim, “omnis intuitiva notitia est definita.” When do we define by negation, and when by example?<sup>2</sup> Define the terms, Monarchy, Colony, Food, Solid.

5. What is the subject of an impersonal proposition?<sup>3</sup> Give reasons for your answer.

6. Among the heavenly bodies, the Rings of Saturn, and among animals, the Ornithorhyncus, or Australian Duck-bill, are sometimes spoken of as being *sui generis*<sup>4</sup>: explain.

7. What are the Principles of Identity, Contradiction, and Excluded Middle, respectively? Mr. Jevons regards the first as a definition?<sup>5</sup> What caution is required in the use of the third?<sup>6</sup> Why, according to Mr. Monck, are or attribute. Thus, we analyse or divide the notion, rational animal, by subtracting the quality rational, which, in effect, amounts to adding the class of irrational to that of rational animals.

<sup>1</sup> Ultimate, or simple notions, such as coincidence, unity, plurality, mathematical point, line, angle, and arbitrary names used for particular objects: for these, verbal expression necessarily fails. See Bain's *Logic*, ii. p. 168. But, even a singular may be conceived by the mind as a conflux of generals (*ib.*, p. 154); and often the best explanation will be to show or indicate the thing itself.

<sup>2</sup> When we have to correct popular and received definitions based on inaccurate classifications, or when the object belongs to a new group that we have formed by a process of generalization. In such cases we may unfold the character of a generality either by (1) laying out two arrays of representative instances—one for the given notion, the other for its negation—or (2) confining ourselves to the representative instances coming under the given notion alone. We choose the one or the other method according to the ease with which the fields of instances are respectively overtaken. See Bain's *Logic*, ii. pp. 154–165.

<sup>3</sup> When an object is so peculiar and unlike other things that it cannot easily be brought into one class with them, it is said to be *sui generis*, or of its own genus.

<sup>4</sup> He regards it as the best we can give of identity or sameness. *Lessons in Logic*, p. 118.

<sup>5</sup> That it must be understood of a term and its *contradictory*, not of a term and its *opposite*.

these principles an insufficient basis for the doctrine of Syllogism?<sup>1</sup>

8. Examine the following :—"S is P resolves itself into, S is S, P is P, S is not P"; "The categorical proposition, taken as it stands, is a contradictory and self-destructive form of expression"; "In the categorical judgment each constituent can be conceived only as selfsame."—*Lotze*.

9. Explain the following statement from Leibnitz :—"Ce qui est A ne sauroit estre non-A; item, AB ne sauroit estre non-A."

10. Examine the following :—"The major is, relative to the conclusion, an anterior knowledge, and the middle is a knowledge in some sort simultaneous"; "The universal, that addresses the understanding, is in nature anterior to the particular that addresses only the senses"; "The knowledge of principles is an indemonstrable knowledge."—*J. B. Saint-Hilaire*.

11. Every proposition has its contradictory. Write out the contradictories of the following propositions respectively<sup>2</sup>:

- (1) Either no S or all S is P.
- (2) Either some S is not P or all P is Q.
- (3) Some S is not P, and all P is Q.

12. What is meant (1) by the conversion, and (2) by the contraposition of a proposition? Apply these processes as far as possible to the following:

- (a) All invertebrates have cold blood.
- (b) Some cold-blooded animals are not vertebrates.
- (c) No wingless birds are songsters.
- (d) Some winged birds are not songsters.

What can you infer from (a) and (b) jointly, and what from (c) and (d) jointly?

C. S.

<sup>1</sup> Owing to their being, in his view, analytical propositions. The real basis of Logic, according to Mr. Monck, is not the *Dictum*, or the Axioms as such, but the synthetical proposition that all valid reasonings can be brought under them. See Monck's *Logic*, footnote, p. 99.

<sup>2</sup> The contradictory of a proposition formed by the disjunctive affirmation of any two propositions is the proposition formed by the copulative affirmation of their respective contradictories, and *vice versa*.

13. On what logical laws do the different kinds of conversion respectively depend ? Give the converse, the obverse (or permuted, or aequipollent form), and the opposites of

All who have rights have duties.

None but the brave deserve the fair.

All equilateral triangles are equiangular.

A stitch in time saves nine.

P struck Q.

Wherever A and B are present, either C or D is also present.

14. Is obversion correctly described as immediate inference ? "In the case of singulars, the distinction between contraries and contradictories seems to disappear" : trace out the consequences of this doctrine and inquire into its truth.

15. Convert the following propositions :—

Birds of a feather flock together.

A killed C unjustly.

Only a sweet and virtuous soul,

Like seasoned timber never gives.

The exception proves the rule.

Of many people it hath been said

That Tenterden steeple Sandwich haven hath decayed.

All join to guard what each desires to gain.

16. Can we infer from All S is P, or from No S is P, that Some not-S is not P ? Convert the proposition, A is probably B. What inference does this proposition give us concerning B ? C. S.

17. Comment upon :—*Inter contraria non datur medium. Prae contradic. Post contra. Prae-postque subalter.* Universals are in diametrical opposition.<sup>1</sup>

18. "From truth nothing follows but truth ; but from falsehood truth may follow." Show that the correctness of this statement appears from considering the extent of the conditions implied respectively by the truth and falsehood of a proposition. D.

<sup>1</sup> See Bain's *Logic*, i. p. 98.

19. What are the logical opposites of the following :—  
 The good alone are great.  
 Few shall part.  
 Most men will be true to their own private ends.  
 I hope you'll not deny that whatever is, is.  
 Error is talkative.  
 When the wicked bear rule the people shall mourn.

20. "Two straight lines cannot enclose a space." On what grounds do you accept this proposition ? What other views are taken as to the grounds on which it is received ; and why do you prefer your own ?<sup>1</sup> C. S.

21. Examine the following arguments :—  
 As polygamy is in many countries illegal we may infer the variability of the moral standard.  
 If gold is wealth, to export it diminishes the national resources.  
 If all good people are happy, unhappiness is an indication of vice.  
 One may be sure of the benefit of inuring young children to cold, from the strength of all men and women thus treated in their infancy.  
 Bad drainage breeds cholera ; so we are safe, as our drains are good.  
 Good things are dear, and scarce things are dear ; therefore, good things are scarce.  
 Squares are rectangles ; so, conversely, rectangles must be squares.  
 Strong drugs are dangerous ; this drug is not strong, and so, is not dangerous.  
 To err is human ; Euclid was human ; hence, there are mistakes in Geometry.  
 No man is perfect ; all men have an idea of perfection ; hence, all imperfect beings have an idea of perfection.

22. Explain the following facts :—  
 No mood of the third figure has an initial C.  
 No mood of the second figure has an initial D.

<sup>1</sup> See Mill's *Logic*, Bk. II., Chap V., § 4, and Lotze's *Logic*, by Bosanquet, p. 510.

No sorites has an O premiss.

Two singular premisses warrant a conclusion but two particular premisses are worthless.

If O be one premiss of a legitimate syllogism, the mood must be one or other of two.

In ordinary syllogisms, O occurs seldom as a premiss but frequently as a conclusion.

If a conclusion can be drawn from two negative propositions by regarding one or both affirmative, it must be of the form, Some things which are not A are not B.

In a legitimate syllogism, if the conclusion be interchanged with one of the premisses show that the new syllogism cannot be valid.

If from one of the premisses of a legitimate syllogism and the opposite of the conclusion a new conclusion be drawn, it is opposed to the suppressed premiss or to its converse.

23. Prove the following results in the case of the application of *Reductio ad impossibile* to any of the received syllogisms :—

Moods with an O conclusion involve a special difficulty.

If subcontrariety be introduced, it will result.

Introduced subcontrariety may be valid but is always useless.

Introduced contrariety is always invalid.

A syllogism having a particular premiss and a universal conclusion is intractable.<sup>1</sup>

24. "All reasoning is from particulars to particulars." Examine this statement.

25. Write Cesare and Darapti in the notation of Sir William Hamilton. By the aid of his new propositions construct a syllogism in the second figure that shall violate both the special rules of that figure, and yet give a valid conclusion. Reduce your syllogism to one of the old moods if possible.

<sup>1</sup> The *Dictum de omni* proves immediately that such a syllogism as this cannot be valid in the first figure. If we assume it to be so in any other figure, *Reductio ad impossibile* becomes inapplicable.

26. In what figures, if any, are the following Thomsonian moods valid? Where the conclusion is weakened point out the fact:—AUI; YAY; UO $\eta$ ; IU $\eta$ ; UEO.

27. Reduce to the syllogistic form and estimate the following:—

$$a+b=c+d; a=d; \text{ therefore } b=c.$$

A is north of B, and B west of C; ∴ C is south-east of A.

Knowledge is impossible, if, as philosophers have shown, we cannot know either matter or mind.

A is probably C, and C is certainly E, and E is very likely M; ∴ A is not improbably M. C. S.

28. Discuss each of the following:—

What moods are common to all the figures?

The rules of syllogism violated by the moods, AAE, AIO, IAO, and IAE.

The validity of EA and AI carries with it that of OA and IA only in the third figure.

In what figures an I major premiss is inadmissible.

The validity of the moods EAE, OAO, in each figure.

Whether the name of the mood Darapti might not with advantage be changed to Dapmatis.

The mood, or moods, in which the conclusion agrees in quantity with the major premiss, and disagrees in quality with the minor.

The mood and figure when one premiss of a legitimate syllogism is particular, and its extreme universal in the conclusion.

The nature of the conclusion when the middle term is twice universal.

29. State the extensive syllogism, All men are mortals, Caius is a man, therefore Caius is a mortal, in an intensive form.

30. If no X is Y, and some Z is X, and P is the name of those Z's that are X; is it a verbal proposition to say that No P is Y? What is meant by a contradiction in terms?<sup>1</sup>

<sup>1</sup> Attaching to a given subject an attribute which cannot co-exist with any of the essential attributes of the subject; as when we say, for example, "a square circle."

31. Examine the following arguments, throwing them, when necessary, into a formal shape :—

Since no one is responsible for a crime unless he has done it himself, it cannot be right to punish a person who has inherited his cruel qualities from his parents.

As he has disappointed us three times in keeping his engagement, he is sure to keep it next time. It would be a bad thing for every one to wear fine clothes, for then nobody would work.

Those who hold that the insane should not be punished ought in consistency to admit also that they should not be threatened ; for it is clearly unjust to punish any one without threatening him.

32. Draw the conclusions respectively inferable from the following :—

—Bring me to the test,

And I the matter will reward, which madness  
Would gambol from.

All persons who die either of traumatic, or of idiopathic tetanus exhibit a certain course of symptoms : Cook did not exhibit that course of symptoms.

The members of the board were either bondholders or shareholders, but not both ; and the bondholders, as it happened, were all on the board.

He loved all mankind, for fortune prevented him from knowing that there were rascals.

Lias lies above Red Sandstone ; Red Sandstone lies above coral.

The soul has no parts, and that which has no parts cannot perish by the dissolution of its parts.

None but cavalry were lost ; all my friends were in the cavalry ; the losses of the army were heavy.

33. Illustrate the principle that the relations of logical symbols are independent of space relations. C. S.

34. Discuss the possibility of embracing all modes of deductive reasoning under one comprehensive form.

35. Describe Bacon's Idola. The four classes are not mutually exclusive? C. S.

36. Explain the different meanings attached to the pair of terms, Analysis and Synthesis. How would you define these terms?<sup>1</sup>

37. "There are two general ways of reasoning, called arguments *a priori*, and *a posteriori*, or according to what logicians style the synthetical and analytical method." Explain—

- (1) The validity of the distinction indicated above.
- (2) The propriety of the phraseology in which it is expressed.

38. Discuss, or criticise, the following statements :—

Chance is a word expressive of our own ignorance.

—“All chance, direction which thou canst not see.”

Probable evidence admits of degrees; circumstantial does not.

Syllogism has been represented as a snare; but escape from the snare is possible.

Logic deals with discursive truths.

Aristotle's syllogism was an inference in matter necessary; his enthymeme was an inference in matter probable.

If by New Truth is meant something not implied in anything already known, no new truth, as thus defined, can be elicited by any process of reasoning.—*Whately*.

<sup>1</sup> The general idea of analysis is separation, of synthesis, composition, or combination. Analysis, from its derivation, signifies a backward solution, as when, for example, we untie a knot for the purpose of learning the intricacies of its fastening. Analysis is the process of separating a whole into its parts; while synthesis is the combination of parts into a whole. Synthesis has throughout a reference to the deductive and combining processes of science: analysis relates to generalization, or induction, everywhere except in mathematics, in which it is merely the mode of deductive synthesis adapted to the solution of special problems.—See Bain's *Logic*, Appendix F.

A is the most difficult proposition to prove, and the easiest to overthrow; O is the easiest to prove and the most difficult to overthrow.

The fourth figure was not thought of by Aristotle, because it implies no difference of knowledge from that which is implied in the first figure.—

*Moberly.*

Baroko and Bokardo are the *crux logicorum*.

Contradictory opposition is said to be the best, because you can always infer by it.—*Karslake*.

“Murder should be punished with death; This man is a murderer; ∴ This man should be punished with death.” However just in this case the conclusion may be, the syllogism cannot properly be called an argument.—*Munro*.

39. Examine the following sophisms:—

(a) If a man says, I lie, either he is telling the truth, or he is not. But if he tells the truth he is lying, since that is the statement which by hypothesis is true; and if he is telling a lie, he tells the truth, inasmuch as he said that he lies.

(b) A grave case was submitted in the following terms to Sancho Panza, when Governor of Barataria:—

“There is a deep river that divides a certain lord’s estate into two parts, and upon this river is a bridge, and at the end of it a gallows and a kind of court-house, in which is administered the law imposed by the owner of the estate, which runs thus—‘If any one passes by this bridge from one side to the other, he must first swear whither and for what he is going: if he shall answer truly he may be let pass, but if he shall tell a lie let him die for it, hung on the gallows put up there without any remission.’ Now, it fell out that on putting the oath to a certain man; he swore and said that by the oath he had taken he was going to die on that gallows which stood there and nothing else. The judges deliberated on the oath and said, ‘If we let this

man pass free, he lieth in what he swore, and according to the law he should die ; and if we hang him, he swore he went to die on that gallows ; and having sworn truly, by the same law he ought to go free.' 'We ask your worship, Lord Governor,' then said those in charge of the case, 'what shall the judges do with this man ? for they are still in doubt and great perplexity.'"<sup>1</sup>  
 —Watt's *Don Quixote*, vol. v., p. 154.

40. There are two series of propositions,

$$x_1, x_2, x_3, \dots \dots \dots x_n,$$

and,

$$y_1, y_2, y_3, \dots \dots \dots y_n,$$

such that, of the first series, one at least is true, and of the second series, not more than one is true, and, generally, that, if  $x_r$  of the first series is true,  $y_r$  of the second is true ; prove, syllogistically, that, of the first series not more than one is true, and, of the second, one at least is true, and, generally, that, if  $y_r$  of the second series is true,  $x_r$  of the first is true.—A generalization of No. 243, Keynes' *Formal Logic*, 3rd edition, p. 311.

<sup>1</sup> See Fallacy of the Crocodile, in Monck's *Logic*, Chap. XIX.

## BOOK IV.—ON APPLIED LOGIC

### CHAPTER I

#### SCOPE OF APPLIED LOGIC

Truth.      § 116.—Applied Logic is the science of the necessary laws of thought as employed in attaining truth respecting the objects to which thinking is directed. These objects are, Man himself, the Universe of Things other than Man, and the Ultimate Basis on which every species of truth must rest ; in other words, a First and Efficient Cause who is Absolute Being. The kind of truth we are concerned with in Applied Logic is objective, or material truth,—the conformity of thought with its object. This we are said to be in possession of, when our thoughts correspond with facts ; that is, when the views we take of objects are substantially correct. Formal Logic furnishes us with various criteria of formal truth. Some of these, such as the principle of contradiction, the principle of identity, the principle of excluded middle, and the principle of sufficient or determinant reason, have been advocated by logicians as criteria of truth in general ; but there can be no adequate standard for judging of the material and truth of a proposition, or adequate criterion of objective truth, other than the entire science, and the whole of the rules of evidence, of which these logical criteria, indeed, constitute a section, but only a very small section.

Science.      § 117.—We have observed that all objects of all human thought may be included under two or three great heads ; but these take in so vast a range, that, ordinarily, we contemplate them in our speculations only in a very partial way. To these objects, or some of them, we are unconsciously applying the thinking process every instant. In so far as we arrive at truth with regard to them, however fragmentary, such truth may be called knowledge ; and, if such knowledge have a certain completeness in itself, and form a systematic and harmonious whole, we call it science. A science is thus a system or complete structure of truths relating to one well-defined object, or set, or class of objects, or, as it has been already defined, a system of principles and deductions to explain some object-matter. Whatever the object-matter may be, a science must have attained to true statements respecting it so far as the nature of the case, and the present degree of progress of knowledge allow. For this purpose it must employ Induction and Deduction. It must be able to define the object-matter and its subordinate parts with clearness and precision. For this purpose it must employ Definition. It must be able to indicate the extent of the domain the object-matter covers. For this purpose it must employ Division. And it must exhibit its results in a systematic and harmonious shape. For this purpose it must order its contents according to the principles and rules of method.

The vast field of human thought is now in various parts appropriated by sciences each in a more or less advanced state. Every science is separated from the rest in virtue of its having for subject-matter, some one set of facts or laws bound together by one conception of which it can give an account. The sciences which relate to the objective universe form in themselves an important group called the natural sciences. They have for matter the numberless things that nature, considered as an assemblage of objects and a succession of events, furnishes us for study.

The universal procedure of science is an analysis of its

objects followed by a synthesis; the result of the whole forming a more or less complete classification. Truth being the first requisite of every science, our synthesis of the individual objects a science is concerned with must correspond to the combination which exists in nature. Hence such synthesis cannot be arbitrary. And so, in like manner, the classification which is to serve the purpose of science cannot be arbitrary. In the construction of the natural sciences, the first, and perhaps the most difficult questions which we have to answer are, What classes ought to be formed, and, Does this or that object possess the special attributes entitling it to be ranked under a certain class. Thus, one of the most important contributions of Applied Logic to the formal construction of science, so far as relates to co-existent objects, is the theory of classification. We form classes of real things, or natural objects as a groundwork for the science of Natural History. We form classes of qualities, irrespective of the real objects in which they inhere, classes of colours, sounds, shapes, and so forth, a single group of them being sometimes found important enough to be made the basis of a distinct science, as in the case of Acoustics, or Optics. We classify the relations of things, in like manner, irrespective of the other qualities and differences of the things related, thus laying a foundation for Geography and large portions of various other sciences.

We have observed that nature may be regarded as an assemblage of objects and a succession of events. We classify events according to the uniformity of their succession in time. Several departments of Physics are made up chiefly of classifications of this sort; for example, Mechanics and Hydrostatics; and, generally, this basis of classification, as well, will be easily seen to be of far-reaching importance.

The advancement of the science, in various departments, depends to a very considerable extent on the success of the attempts that have been made to improve the classifications adopted in them. Sometimes the ground of division determining a whole group of phenomena has been altered, as in

Botany and Natural History. This, however, may be regarded as an improvement in the method rather than an enlargement, strictly, of the domain of the science. But most of the questions that science encounters in its progress relate to successive improvements and extensions of old classifications, not to the substitution of entirely different ones. And by such improvements chiefly the domain of science is extended.

Then, there is the classification of the sciences among themselves, so as to reduce them into one complete and orderly system of human knowledge. This is a problem to solve which numerous attempts have been made, some of them, such as those of Descartes and Comte, very nearly meeting the requirements of modern progress and intelligent thought, but respecting which discussion has not yet been completely set at rest.

We shall be concerned with classification in a more distinct sense under the head of the subsidiaries of induction.

**§ 118.**—It will be seen as we proceed that the **Belief.** knowledge of material truth which forms the object-matter of science is to a considerable extent arrived at tentatively. Science moves cautiously from point to point, feeling its way for firm ground in passing between dangerous swamps from landmark to landmark, or in its endeavour to attain a glimpse of prospects that lie beyond the horizon, and often compelled to decide provisionally as to direction and foot-hold in advance of that experience which alone could give certitude. As a consequence, the attitude of the mind in pursuit of scientific truth in relation to the various judgments presented to it in the course of scientific inquiry, when not of absolute negation, will be often signified by various degrees of belief ranging along what has been called the scale of modality. These various degrees of belief may, on a broad principle of division, be resolved into three. Some judgments may, at a certain time, be held to be neither subjectively nor objectively true. Such judgments may eventually turn out to be both one and the other. But, in

so far as, at a given stage, they are neither held with entire certainty by the thinker himself, nor are such that they can be shown to be materially true, they constitute but mere Opinion. Judgments of this kind have been called by Aristotle problematical. Such judgments often lead the way to great discoveries. On the other hand, in many subjects, from the nature of the case, our judgments, again, may be such that the inquirer himself is fully persuaded of their truth, but is, at the same time, unable to give such grounds for his belief as shall compel men in general to coincide with him. Such judgments are, to him, subjectively, but not objectively certain; and they constitute to him Belief in the special and more distinctive sense of that term. Aristotle calls judgments of this nature assertorial, or assertive. They commend themselves to the moral nature of the inquirer; and, in so far as other men are of the same disposition, they will accept them likewise. Lastly, a third class of judgments present themselves to us for consideration. They are judgments that range themselves at the top of the modal scale. Not only is the inquirer fully persuaded, in his own mind, of the truth of these judgments, but he is able to give such grounds for them as shall compel others to accept them also. Judgments of this class Aristotle calls necessary, or apodictical. Such judgments are both subjectively and objectively true, and such alone can be accepted as knowledge and as a matter of science. At the same time, even in respect of the subject-matter of these judgments a distinction is drawn. In so far as they relate to abstract science, hypothetical certainty may, in the language of some philosophers, be attached to them; but, in so far as they lie in the domain of the concrete, they cannot be regarded as absolutely certain. "No one," Dr. Venn observes, "acquainted with the complexity of physical and other evidence would seriously maintain that absolute ideal certainty can be attained in any branch of Applied Logic."

**Subjects for Studies.** § 119.—The greater number of the following questions relate to the matter of §§ 116–118:—

1. Define the following terms, *Axiom, Postulate, Theorem, Problem, Thesis, Hypothesis.*

An *Axiom* is an indemonstrable theoretical, or speculative judgment. By its being called indemonstrable is meant that it does not require or admit of proof.

A *Postulate* is an indemonstrable practical judgment.

A *Problem* is a demonstrable theoretical judgment.

A *Thesis* is a judgment proposed for discussion and proof ; or, with Aristotle, an axiom of some special science or disputation.

A *Hypothesis* is a judgment provisionally accepted as an explanation of some group of facts, and, at the same time, liable to be discarded if found inconsistent with them.

2. Discuss the several implications of Pilate's question—“What is Truth ?”

3. Explain the distinctions between Belief and Knowledge, Intuition and Inference, Probability and Certainty.

4. Is a philosophic criterion of truth possible ?

5. “The ultimate criterion of truth is the testimony of consciousness.” Examine this statement.

6. Give the criterion by which a proposition may be decided to be (a) an axiomatic, (b) a necessary truth. C. S.

(a) A proposition, to be an axiomatic truth, must be a real proposition, and *self-evident*, that is to say, independent of any other principle within the science.

(b) A necessary truth, properly speaking, is one the denial of which would be a contradiction in terms. “The same thing cannot be in two places at once,” is in this sense a necessary truth. Such a truth as this is expressed by an analytical proposition. Necessity is, however, very commonly held to mean certainty, on which understanding an inductive truth would be held to be necessary. Necessity is, again, sometimes employed to mean inconceivability of the opposite. Mr. Bain, commenting on this meaning, observes, “Truths conceivable in one age and country, are not only

conceivable under a different state of education, but so thoroughly engrained that their opposites are inconceivable."

7. Examine the distinction between logical, metaphysical, and mathematical necessity. Is inconceivability of the opposite the ultimate test of all beliefs?

8. Examine the following statements:—

(a) "To obtain a criterion of truth we must previously know what truth is; whilst, again, to know truth, we should still require a criterion."—*Bp. Huet.*

(b) "The criterion of truth must be sought for either in our sensuous impressions, or in the reason itself; but, whatever is a judgment of the understanding must have a relation to sensation, and, on the other hand, sensation is but a fleeting, transitory thing; therefore, there can be no rational solution of the difficulty."—*Carneades.*

(c) "Though nothing is absolutely, yet many things are probably true."—*Ib.*

(d) "All reasoning terminates in first principles; all evidence is ultimately intuitive; common sense<sup>1</sup> is the standard of truth to man."—*Beattie.*

(e) "The logic of induction is the criterion of truth inferred from facts, as the logic of deduction is the criterion of truth inferred from necessary principles."—*Whewell.*

(f) "We cannot know,  
For knowledge comes of things we see."

—*Tennyson.*

<sup>1</sup> *Common sense*,—judgment unaided by art or any system of rules. The term originated with Père Bouffier, who explains it to be, "that quality or disposition which nature has placed in all men, or evidently in the far greater number of them, in order to enable them all, when they have arrived at the age and use of reason, to form a common and uniform judgment with respect to objects different from the internal sentiment of their own perception, and which judgment is not the consequence of any anterior principle."

- (g) "Knowledge is the relation of reason to the universe."—*Sir W. Hamilton.*
- (h) "Science is the principle of thought modified by the objects to which it is directed."
- (i) "Physical truths are all contingent."
- (j) "There is no independent science called Logic, but simply one comprehensive precept, Avoid contradictoriness in thought."
- (k) "Evidence means the grounds which make evident."

## CHAPTER II

### INDUCTION AND QUASI-INDUCTION, ANALOGY, AND EXAMPLE

**Induction.** § 120.—We have observed that a science must have attained to true statements respecting its object-matter, so far as the nature of the case, and the present degree of progress of knowledge allow. For this purpose, it must employ Induction and Deduction.

Induction and Deduction are often spoken of as converse processes, since by the former we deduce a general law from a sufficient number of particular cases, and by the latter we prove that some property belongs to a particular case from the consideration that it comes under a general law.

Induction is defined by Mill, as, “that operation of the mind by which we infer that what we know to be true in a particular case, or, in particular cases, will be true in all cases that resemble the former in certain assignable respects, or that what is true of certain individuals of a class is true of the whole class, or that what is true at certain times will be true in similar circumstances at all times.” It is a process by which we step from the known, as it is said, to the unknown ; that is to say, from cases in which actual examination is possible, to cases in which actual examination is not possible to us ; and it, therefore, distinctly takes its position as the logic of *prevision*, an instrument ancillary not alone to the various sciences, but to the practical affairs of everyday life. How much of our knowledge may be said to be

due to induction, and what is the character of the knowledge so acquired, are points that have led to some differences of opinion. As to the first of these questions, some philosophers consider that the greater part, if not the whole of our knowledge is due to inductive reasoning, many of the simpler ideas and truths the mind possesses having been gathered by inductions of an unconscious kind. As to the second, it is maintained that in some sorts of matter, at all events,—notably, when we are dealing with natural phenomena—*inferences* with regard to new instances cannot always be depended on as resulting in certain knowledge. The first of these questions has been incidentally before us in dealing with the principles of demonstration, or first truths ; but our limits preclude our dwelling upon it longer. The second is of a different character ; not resemblances merely, but the *degree* and the *nature* of the resemblances, being, according to the above view, the justification of inductive inference. Jevons, examining this question, maintains, in his *Principles of Science*, that it is impossible to expound the methods of induction in a sound manner without resting them on the theory of Probability. "Perfect knowledge alone can give certainty, and in nature perfect knowledge would be infinite knowledge, a measure of knowledge which is clearly beyond our capacities." To this, Dr. Venn in his chapter on "Induction and its Connection with Probability" (*Logic of Chance*, 3rd ed., pp. 203-234) replies, yes, as to absolute ideal certainty ; but otherwise, except as a needful protest against attaching too great demonstrative force to the conclusions of Inductive Logic, he regards it as decidedly misleading to speak of its resting on probability : "Logic within its own domain knows nothing of hesitation or doubt."

Before entering with some detail into a discussion of the methods of Inductive Logic, it will set the inductive process in a clearer light to begin with, and justify the foregoing explanation as to its character and objects, to illustrate its application in a particular case.

The metal potassium was discovered in 1807 by the action

of the electric current upon hydrate of potash ; and thus were presented to the chemist's mind for further consideration and, possibly, additional fruit, the following two facts :—

Potash is an alkali ;

Potash yields potassium.

This naturally led on to the thought—in which a leap in advance is seen to be made, because of proceeding to a general proposition where only a particular conclusion is warranted :—

Potash yields a metal ;

Potash is an alkali ;

∴ Every alkali will yield a metal.

Here we have a general law, unwarranted by the subjective criteria of truth, but still involving no formal contradiction. Being a statement of a material truth, we must, then, have recourse to criteria of a material kind, and so another alkali is experimented on in accordance with the deduction suggested by the previous result, thus—

All alkalies will yield a metal ;

Soda is an alkali ;

∴ Soda will yield a metal.

—Soda is experimented on, in a similar way to potash, and yields the metal sodium. This greatly strengthened the chemist's belief that a law had been discovered expressible in the form of the general proposition, All alkalies contain a metal ; and further experimentation, in 1855, on the alkaline earths, and the production of the metals, calcium, strontium, and barium, led to a very decided confirmation of the soundness of the view that Every alkali will yield a metal, which thus passed from the stage of opinion to that of belief. Ammonia, owing to its volatile nature, happens to show itself refractory ; but towards an inclusive interpretation of the general proposition, and the law it expresses, the chemist assumes a theoretically possible metal to complete the list ; and thus, what was at an earlier stage but scientific opinion

may be regarded as having now made good its ground as a fact of chemical science.

Before passing away from this illustration, it will be instructive to distinctly exhibit the steps it discovers from experiment to experiment, in our progress towards the establishing of a general proposition—here a chemical law. They are these:—

- (1) An Induction, the expression of which, so far as we have expressed it, is really non-syllogistic.
- (2) A Deduction, based upon the preceding inductive inference as one of the premisses.
- (3) An Experimental Verification of the deductive inference.

Thus, “a correct induction furnishes the premiss for a sound deduction, and a doubtful induction must be verified by deductions from it.” This, however, is not all. We require a distinct statement of the principle that, as here, underlies the belief of the experimentalist, and that his mind leans upon in proceeding to generalize from a single case, or a small number of instances or cases. The ground of his belief is very obviously an assumption with regard to the course of nature; namely, that what happens once will under conditions with a sufficient degree of similarity happen again, and not only once again, but as often as the same circumstances recur; or briefly, that the course of nature is uniform. This principle, Mill and other logicians hold to be, itself, a product of induction; in fact, to be, itself, a great generalization founded upon prior generalizations; while, philosophers of a different school hold it to be a principle which, antecedently to any verification by experience, we are compelled by the constitution of our thinking faculty to assume as true.

With this principle, however, in hand, we can throw the matter of the non-syllogistic expression—

Potash contains a metal;  
Potash is an alkali;  
∴ Every alkali contains a metal—

into the syllogistic form—

- A thing true of an alkali, as such, in this instance, is true of alkalies in all instances;
- That an alkali contains a metal, is a thing true of an alkali in this instance;
- ∴ That an alkali contains a metal, is a statement, or thing true of an alkali in all instances,

—a syllogism, of the truth of the conclusion of which we must regard the major premiss, however obtained, as a necessary condition; and, in the major premiss of which, whatever its origin, lies the very essence of the induction.

**Quasi-Induction, or** may be regarded as inference from the known to **Induction** improperly the unknown. It is to be carefully distinguished so called. from each of the following processes usually classified as inductions improperly so called, which bear some resemblance to it, and have often been confounded with it, namely, (1) *Verbal transformations*, (2) *Inductions*, incorrectly so-called, in Mathematics, (3) *Descriptions in general terms*, or *Colligation of a set of phenomena*.

(1) *Verbal transformations*.—Under this head we must place the so-called Induction of the older logical writers based upon the formula of Aristotle. By Aristotle, Induction (*Ἐπαγωγὴ*) is defined as, *Tὸ διὰ τοῦ ἔτερον θίτερον ἄκρον τῷ μέσῳ συλλογίσασθαι* (*An. Pr.* ii. 23), that is to say, the process of “proving the major term of the middle by means of the minor,” in which the terms *major*, *middle*, and *minor*, are used relatively to their *extension*, to denote, respectively, the attribute proved, the constituted species of which it is proved, and the aggregate of individuals constituting such species. In the form in which this so-called inductive reasoning appears, there is a resemblance to the third figure of the syllogism. Thus:—

X, Y, Z (minor) are B (major);

X, Y, Z are all A (middle);

∴ All A is B.

But the resemblance is apparent only ; the true distinctions being, (1) that, in the minor premiss of the induction, the copula does not represent the subject as *contained under*, but as constituting the predicate, (2) that, in consequence of this distinction, a universal conclusion is logically drawn in this form which is not admissible in the third figure of the syllogism. The form, however, is not, in a true sense, a syllogism : it is merely a process of equivalence.

By Murray, Induction is defined as "an argument in which, from the position of all the parts, the position of the whole is concluded." The proceeding thus from the position of all the parts to the whole is, in the older treatises, regarded as *the* Perfect Induction ; an argument from the position of some of the parts to the whole, being in relation to it called an Imperfect Induction. But perfect induction as thus defined, while useful as a kind of shorthand registration of facts, Mill declares to be no part of the investigation of truth.

By Whately, Induction is explained to mean two different things ; more properly, the process of the investigation, and the collection of facts, and less properly, that of deducing syllogistically an inference from those facts. The former is the original and strict sense of the word (—the Greek term, 'Ἐπαγωγή), at least that most in favour, and may be defined as "the bringing in ( $\tauὸ\;ἐπάγειν$ ) of examples or comparisons," or, "one by one, of instances bearing on the point in question till a sufficient number has been collected."

The following is an example, after Whately, of the so-called *Perfect Induction* :—

The Earth, Mercury, Venus, &c., all move round the sun from West to East ;

These are all the known planets ;

∴ All the known planets move round the sun from West to East.

This argument is a true Perfect Induction ; but, though in form it appears to be a syllogism in the third figure, namely *Darapti*, it is, in reality, no ordinary syllogism.

The following example—to which, of course, the strictures appended to Aristotle's general form apply—is one very commonly given to illustrate Imperfect Induction (for instance by Aldrich):—

This, that, and the other magnet attract iron ;

This, that, and the other magnet are all magnets ;

∴ All magnets attract iron.

As an improvement, Whately proposes that there should be expressed in one of the premisses the assumption that the magnets that have been examined constitute a fair specimen of all magnets, so that the foregoing might be written as a syllogism in *Barbara*, thus:—

That which belongs to, or is an attribute of, this, that, and the other magnet, belongs to, or is an attribute of all ;

Attracting iron belongs to, or is an attribute of, this that, and the other magnet ;

∴ It belongs to, or is an attribute of all.

To explain the grounds of this assumption, it has been urged that what may be described as the major premisses of such inductions are resolvable ultimately into an assertion of the Law of the Uniformity of Nature, or some such proposition. Jevons, commenting on this view, says, that, if there be such a principle, it is liable to exceptions ; and he instances the retrograde motions of the satellites of Uranus, and of the satellite of Neptune, as making against the previously unbroken induction pointing to such a uniformity as that all the satellites in the planetary system revolved in one uniform direction round their planets.

Mr. Bain throws Whately's statement of Imperfect Induction into the following form:—

The magnets that I have observed, *together with those that I have not observed*, attract iron ;

These magnets are all magnets ;

∴ All magnets attract iron.

But he adds, that the major here assumes the very point to be established, and makes the *inductive leap*. “In no

sense," concludes Mr. Bain, " is the inductive syllogism an admissible logical form."

Bearing some resemblance to these forms, but in a certain degree more suggestive, is what has been called *Traduction*. In traduction all the propositions are singulars, and the conclusion asserts merely the identity of singular terms that have been predicated of the respective subjects in the premisses.

For example:—The substance A has, say, the spectroscopic characteristics,  $a, b, c$ ; The substance A has the mechanical, physical, and chemical characteristics,  $a, \beta, \gamma$ ;  $\therefore$  The substance that has the spectroscopic characteristics,  $a, b, c$ , has the mechanical, physical, and chemical characteristics,  $a, \beta, \gamma$ . In this way we associate, in the case of lithium the spectrum of one bright line with the physical characteristic of being the lightest metal known.

(2) *Inductions in Mathematics.* Geometrical reasoning bears a certain resemblance to Induction (that is, to the so-called Imperfect Induction). By it also we conclude that what is true in a particular case will be true also in other cases that resemble this particular case in certain assignable respects. It differs from Induction, however, in the important feature, that we are certain of the complete similarity between the specimen, instance, or case before us, and every other specimen of the cases to which we extend our deduction in all the qualities essential to the proof. Our deduction is thus said to apply to every instance, by *parity of reasoning*. We cannot have this degree of assurance with respect to the cases or instances on which we ground an induction. The difference between the two kinds of reasoning turns on the character of the evidence. In induction, we observe that  $a, b, c, d$ , and so forth, have each a certain property. Our knowledge of any of the individual instances except the specimens before us is, however, not such as to enable us to say that in virtue of its title to a place in the series, it *must* have this property. The evidence is the presence of this property in every instance we have examined, together

with the best proof, or proofs, we can have, though necessarily falling short of demonstration, that the appearance of such property in the individual is connected with the appearance of the individual in the series we are considering, and dependent on the essential principle of the series. On the other hand, in geometrical reasoning, we conclude that,  $a$ ,  $b$ ,  $c$ ,  $d$ , and so forth, have each a certain property, the title of these to their places in a certain series being grounded not anywhere on an inductive leap, but on definition, what is true for one, thus being true for all; and our knowledge of any one individual of the series being such as to enable us to say through a demonstrative proof that it *must* have this property. Thus, the evidence in the one case is *a posteriori*, in the other *a priori*.

In algebra, again, it is not unusual to call certain methods of proof inductions, as, for example, some of those commonly employed in the summation of series. Observation made respecting the form taken by the sum of the series, as we proceed the length of a few terms in the earlier part of it, suggests a general formula of summation: We then proceed to the lemma that, *if* the general expression suggested for the sum, by actual additions of the earlier terms, hold for  $n$  terms it must hold for  $n+1$  terms. This is easily demonstrable. Then we say, *but it does* hold for two terms; therefore, it must hold for three; again, holding for three it must hold for four, and so on; and finally holding for  $n-1$ , it must hold for  $n$  terms. Therefore, that it is a true expression for the sum of the series is demonstrated. Here, again, the broad distinction between such a method and induction is at once apparent. In the illustration chosen, the individual instances are series, of, 1, 2, 3, . . .  $n$  terms. What is established is that a certain law of summation, seen to hold in the cases of 1, 2, 3, . . . terms, holds generally with regard to such series. In induction we are not warranted in determining an order as between individuals which would entitle us to proceed from any one to the next not by a leap but demonstratively, unless on evidence

that must be built up. Here, it is otherwise. The order of the terms, and the relation between two consecutive terms, are *a priori* elements which we employ in the initial step towards establishing the general law of the series in the way explained.

(3) *Descriptions in general terms of a set of observed phenomena.* The descriptive operation which enables us to sum up a number of details with regard to phenomena in a single proposition, and which Dr. Whewell calls the Colligation of Facts, and regards as the type of inductions generally, Mill considers, should be distinguished from Induction, in the sense in which he employs the term, and that in which we employ it here, when it stands alone. The study of facts may be undertaken for three different purposes, namely, towards (1) simple description of them, or (2) their explanation, or (3) their predication—that is to say, the determination of the conditions under which similar facts may be repeated again, that their recurrence may be predictable. According to Mill, Dr. Whewell's observation holds good only in respect of the first of these purposes. Different descriptions may, according to the point of view, be all true; but not different explanations. Compare, for example, the descriptions of the motions of the heavenly bodies from the Ptolemaic and from the Keplerian points of view; and, again, the explanations of these motions based upon, (1) the idea of an inherent virtue in them, (2) impact and vortex whirls, (3) the composition of a centripetal with an original projectile force. Different conceptions may be appropriate to the different descriptions, and yet, not any of them that one which corresponds with the explanation. Of course, whether different explanations may be all true, will depend on our interpretation of the word *different*, that is to say, whether as consistent, or the contrary. Inductions differ from descriptions in relation to proof. Induction is proof. It requires, however, an appropriate test of proof: and to provide that test is a special purpose of Applied or Inductive Logic.

**Analogy.** § 122.—Another process of thought by which, as by Induction, we endeavour to enlarge our knowledge beyond the bounds of experience is Analogy.<sup>1</sup> By induction we conclude from many things to all others of the same species. By analogy we conclude from the known agreement of two things in several qualities that they agree also in some other quality which is not directly known. In our progress from the particular to the general, induction proceeds on the principle that what certainly belongs to many individuals of the same kind also probably belongs to all the other individuals of that kind; the principle of analogy is, that, if two things agree in many respects, they probably also agree in some other respect. Because some one quality exists in many things, therefore, it exists in all of the same kind: this is induction. Because many qualities in this are the same as in that, therefore, one other quality in this resembles that: this is analogy. In other words, induction concludes from one in many to others, by way of extension; analogy, from many in one to the others by way of intension (*Bowen's Logic*, p. 381). The formula of analogical reasoning, considered as inference from resemblance in some points to resemblance in others, is as follows:—

Two things resemble each other in certain respects;

<sup>1</sup> The term Analogy is employed in a great variety of senses. Whately, after several preceding logicians, defines it as "resemblance of relations," and Thomson, as "the similarity of ratios or relations." For example, a mother-country is said to stand to her colonies in the same relation as a parent to children; and, if from this resemblance an inference be drawn, such as, say, that obedience or affection is due from colonies to the mother-country, we call such inference reasoning by analogy. An argument of this kind may amount to nothing, or it may have the force of a rigorous induction. Its value will depend on our being able to show that the circumstance in which the two cases resemble, that is to say, the common circumstance which is the *fundamentum relationis*, is the *material* circumstance: that, on it the rest of the circumstances of the case depend. It is, however, more usual to employ the term, Analogy, to mean, not merely resemblance of relations, but arguments from *any* sort of resemblance, not amounting to a complete induction; so that, generally, to *argue from analogy* signifies to infer from resemblance in some points, not necessarily in relations, resemblance in others.

A certain proposition is true of one of these things ;  
. . . This proposition is true of the other.

Analogy differs from induction in not requiring it to be shown by due comparison of instances that there appears to be an invariable conjunction between the known properties in which the things have an admitted resemblance, and the property which it is sought to decide about. It requires, however, that this latter property should not have been ascertained to be unconnected with the common properties. Consequently, if by deduction from previous knowledge of their laws, or otherwise, it can be concluded that they have nothing to do with it, the argument from analogy fails.

And, in general, where the points of agreement and disagreement between two things are perfectly ascertained, so that there is no element unknown, there is no analogy in the proper sense of the term. This holds with regard to certain figurative resemblances, such as comparing the state to the individual man, and the like, met with frequently, particularly in the ancient philosophy.

If a fair proportion of the properties of two things are known, every additional resemblance in points not known to be unconnected with the undecided property affords grounds for expecting an indefinite number of other resemblances, among which the property in question may perhaps be found. On the other hand, every additional dissimilarity should lead us to expect that the two things differ in an indefinite number of properties, including, perhaps, the property in question. These dissimilarities may even be such as would, in relation to one of the things, imply the absence of that property ; and then every resemblance, in so far as it shows that the two things have a similar nature, strengthens the presumption<sup>1</sup> against the presence of that property.

<sup>1</sup> *Presumption*, in its strictly technical sense, in relation to evidence, means not a preponderance of probability in favour of a supposition, but such a *preoccupation of the ground* as implies that it must stand good till some sufficient reason is adduced against it ; in short, that "the burthen of proof lies on the side of him who would dispute it." — Whately's *Rhetoric*, p. 112.

Hence, the value of an analogical argument depends on the extent of ascertained resemblance, as compared, first, with the amount of ascertained difference, and next, with the extent of the unexplored region of unascertained properties; and, it follows, that where the resemblance is very great, the ascertained difference small, and our knowledge of the subject-matter extensive, the argument from analogy may approach in strength very closely to a valid syllogism.

It hence appears, that the conclusions derived from analogy, as in the extension of derivative laws (see § 125) beyond the limits of observation, are of considerable value only when the case to which we reason is an *adjacent* one; adjacent, however, so far as analogy is concerned, not in place or time, but in *circumstances*. So that, if the conclusion be as to an effect from resemblance of a certain case before us to other cases in which it has been observed, the inference that it will occur in such a case, though deficient in complete resemblance to the observed cases, possesses a high degree of probability.

No competent inquirer into nature will, however, rest satisfied with conclusions derived from analogy when a complete induction is attainable. The great value of analogy, even when faint, is, so far as the field of science is concerned, that it may suggest observations and experiments with a view to the discovery of positive scientific truths, for which, however, the hypotheses based upon analogies cannot be accepted, and must not be mistaken.

**Example.** § 123.—An Example is an argument which

proves something to be true in a particular case from another particular case. It is defined by Aristotle, in the terminology employed by him for Induction, as “proving the major term of the middle by a term resembling the minor,” and, owing to its convenience for illustration, is called by him, “oratorical induction.”

The relation of Example to Analogy is this, that, by it we argue not from one general proposition to another, in virtue of similarities between the classes of things to which they relate, but from one singular or individual case to another

such case. How we reach the one individual case from the other is a matter of difference of opinion among logicians ; some holding that we can proceed from the one to the other directly, others, that we must first ascend from the one case to a common law, and then descend from this law to the other case. Thus, "It may be expected that time will leave Tennyson's fame unimpaired, because it has left Virgil's fame unimpaired," is an example that finds a principle of unification in the general proposition that, "Time leaves the fame of all great poets unimpaired."

Subjects for examination and discussion in the matter of the Studies.      § 124.—The following questions are added for examination and discussion in the matter of the four preceding sections :—

1. Trace the relation to each other of the processes of inductive and deductive reasoning.
2. Define perfect, and also imperfect induction. In what type of syllogism is the former represented ? What was Whately's syllogistic representation of the latter, and to what objection is it liable ?      D.
3. In what sense is the inductive process opposed to the deductive ?
4. "Perfect induction is demonstrative and syllogistic ; imperfect induction is neither." Examine the truth of this assertion.
5. "Induction is always a syllogism." Is this a true analysis of the process of inductive reasoning ?      C. S.
6. What is inductive inference ? What theoretical difficulties attend the process, and how is it attempted to meet them ?
7. Compare Mill's view of induction with any different views that are known to you. Illustrate his position, that induction is "generalization from experience."
8. On what grounds is it sometimes denied that induction belongs to Logic ?
9. Explain and discuss the doctrine that induction is based upon the theory of probability.      C. S.
10. Exemplify inductions improperly so called.

11. What do you understand by the process of mathematical induction? Does it throw any light upon the theory of inductive reasoning?

12. Inquire how far the geometrical reasoning of Euclid is inductive or deductive. Take, for illustration, the fourth and fifth propositions of the First Book.

13. Is it true to say that a universal proposition represents nothing more than a hitherto uncontradicted experience? L.

14. Is it possible for us to form true universal propositions about facts, if we have not actually observed all the individuals designated by the subject of the proposition? If so, how?

15. What is meant by an inductive, and what by a deductive science; and what conditions are necessary that a science may be the one or the other?

16. Explain and illustrate the following observation:—“In the actual conduct of scientific inquiry there is a constant alternation of the processes of induction and deduction.”

17. Illustrate the following statement:—“Logical induction concludes from *each one* to *all*; induction properly so called concerns the matter of thought, and concludes from *some* to *all*.”—*Bowen*.

18. Distinguish between analogy and induction. Is reasoning from analogy syllogistic? State fully the reasons for your answer.

19. Is it true to say that (1) induction is perfect analogy, or (2) analogy is imperfect induction?

20. How does argument from analogy differ from induction, and how from metaphorical argument? C. S.

21. On what does the cogency of argument from analogy depend?

22. Give typical examples of induction and analogy.

23. Explain the fallacy of False Analogy.

The term False Analogy is usually applied only to those cases of analogical inference in which there exists no ground whatever for any analogy.—See Fowler's *Inductive Logic*, Chap. VI., vi.

24. Examine the analogical force of the following statement :—“There are probably inhabitants in the moon, because there are inhabitants on the earth, in the sea, and in the air.”

25. Sir Henry Maine says :—“Analogy, the most valuable of instruments in the maturity of jurisprudence, is the most dangerous of snares in its infancy.” Comment upon this observation.

## CHAPTER III

### LAWS OF NATURE, AND CAUSATION

*Laws of Nature.* § 125.—According to Mill, “if we throw the whole course of any inductive argument into a series of syllogisms, we shall arrive by more or fewer steps at an ultimate syllogism which will have for its major premiss the principle, or axiom, of the uniformity of the course of nature.” “But,” he adds, “the course of nature is not only uniform,” that is to say, within certain limits, “it is also infinitely various,” that is, having regard to the whole field of nature. As our experience widens, we do not always meet phenomena in the same combinations in which we met them at first. Sayings that have long passed as proverbs have to be reconsidered. It is not, for example, for every country, and for all time, a uniformity in the course of nature that “All swans are white,” though it was long thought to be so. The uniformity in question is seen to be, when we look into it, not so properly a single uniformity as the coexistence of uniformities. The general regularity results from the coexistence of partial regularities. The uniformity of the course of nature is, then, to be regarded as a complex fact, compounded of all the separate uniformities that exist with regard to single phenomena. These separate and various uniformities, when ascertained by what can be regarded as a sufficient induction, and reduced to their most simple expression, we usually call, *Laws of Nature*. Among such laws we may, for example, include the following:—(1) Air has weight; (2) Pressure on

a fluid is propagated equally in all directions ; (3) Pressure in a given direction, not opposed by equal pressure in the contrary direction, produces motion, which does not cease until equilibrium is restored. But among them we should not include, for example, a closely related uniformity, the rise of the mercury in the Torricellian tube. This uniformity arises from the three laws just enunciated, and may be deduced from them by reasoning ; or, it may be independently established by experiment ; or, we may verify our reasoning in the case by experiment. Now, in science, it is customary, wherever regularity of any kind can be traced, to call the general proposition which expresses the nature of that regularity a law ; so that complex uniformities such as that observed in the Torricellian experiment may be called *laws* ; but having regard to the phrase we are considering at large, the attribution of certain laws to nature is equivalent to claiming for them as marks the highest *simplicity* and *generality*. Hence, uniformities of the more complex kind, as in the case just mentioned, can scarcely, in the strictness of scientific speech, be called Laws of Nature.<sup>1</sup> We thus arrive at a better understanding of the phrase both intensively and extensively.

<sup>1</sup> It may, however, be convenient to have a nomenclature distinctive of these two classes of laws. For this purpose, the Laws of Nature properly so called may be distinguished as Ultimate, or Primary Laws, and all other uniformities as Secondary Laws. Secondary Laws, again, may be divided into two classes. Some uniformities of this order can be shown to follow Primary Laws, as, for example, that of the Torricellian experiment. Such may be called Derivative Laws. Besides these, there are in the class of Secondary Laws uniformities which cannot as yet be shown to follow from Primary Laws, but which, it is hoped, will ultimately be proved to depend on them. Such uniformities may be called Empirical Laws (from the Greek *εμπειρία*, experience, or trial). They become Derivative Laws as soon as their dependence on Primary Laws is fully established. The following are examples of Empirical Laws : the local laws of the flux and the reflux of the tide in different places ; the succession of certain kinds of weather to certain appearances of sky ; that when different metals are fused together, the alloy is harder than the various elements ; and that gases have a strong tendency to permeate animal membranes. With regard to both these classes of laws, the Derivative, and still more, the Empirical, the caution is to be observed, that they must not be extended beyond narrow limits of time, place, and circumstance. See Bain's *Logic*, ii. pp. 102-113, and Mill's *Logic*, ii. pp. 39-49.

The Laws of Nature, having regard to their number and their essential character, are the fewest and simplest assumptions which being granted the whole existing order of nature would result; or, again, the fewest general propositions from which all the uniformities that exist in the universe might be deductively inferred; and their determination for any given portion of it, so far as the solution of the problem lies within our reach, marks, moreover, an epoch in the progress of science.

Hence the problem of Induction might be summed up in two questions: how to ascertain the laws of nature, and, having ascertained them, how to follow them into their results. Towards this end, a beginning should be made in the form of a survey of the inductions to which mankind have been conducted in unscientific practice, with the object of ascertaining what kinds of uniformities have been found perfectly invariable, that is to say, pervading all nature, and what are those which have been found to vary with difference of time, place, or other changeable circumstances. The rules of experimental inquiry are the practical contrivances for working out the problem to its solution.

As to the criterion of truth in the process, we must make Experience its own test. This being premised, the stronger inductions are the touchstone to which we must always endeavour to bring the weaker. If an induction conflicts with stronger inductions, or with their conclusions warranted on sufficient evidence, such induction, as showing itself the weaker, must give way. But all inductions, whether strong or weak, which can be connected by ratiocination are confirmatory of each other. If on a survey of existing uniformities some appear, to all human intents, quite certain and quite universal, these will serve as tests to bring multitudes of others to a like position of certainty and universality, equally indefeasible within their bounds as the former. Such will, as just explained, be Laws, and, if any of these laws prove to be not a result of other and simpler laws, it will be a Law of Nature.

§ 126.—We may regard nature in another **Causation.** way. Nature is known to us through certain appearances that present themselves to our senses,<sup>1</sup> and which we call Phenomena (plural of phenomenon, from the Greek *φαινόμενον*, that which appears). These phenomena of nature, or physical phenomena, exist in two distinct relations to one another, that of simultaneity, or coexistence, and that of succession. These phenomena do not stand apart. Every phenomenon is related, in a uniform manner, to some that coexist with it, and to some that have preceded, and some that will follow it. Of truths concerning phenomena the most valuable are those that relate to the order of their succession; on a knowledge of such truths being founded every reasonable anticipation<sup>2</sup> we can form with respect to future physical facts.

Among the various uniformities in the succession of phenomena, the one of greatest certainty and universality is the Law of Causation. This is expressed briefly by the proposition that, Every fact that has a beginning has a cause<sup>3</sup>: that is to say, every physical fact within the range of experience has some cause with which experience is competent to make us acquainted. The question is here not of what is by some philosophers called the *vera causa*,<sup>4</sup> the cause which is not

<sup>1</sup> See Dr. Stoney's "Studies in Ontology," *Scientific Proceedings of the Royal Dublin Society*, vol. vi., p. 500.

<sup>2</sup> Anticipation is the presage of a truth of nature before we attempt to establish it by cautious methods, or before the evidence is unfolded.—Thomson's *Laws of Thought*, § 117.

<sup>3</sup> We owe to Aristotle the first attempt at a logical analysis of causation. He distinguishes causes into four kinds: the *material*, the *formal*, the *efficient*, and the *final*. The *material* cause is that of which a thing is constructed; for example, the bronze of a statue. The *formal* cause of a thing is its exemplar, its type in the mind of the artificer or artist; for example, the *design* conceived by the statuary. The *efficient* cause is the original principle of change: the power changing the thing changed; man, or human skill; the maker. The *final* cause, the *τὸν ὅντα ἕπεται*, is the end, that for the sake of which the thing is done; as, health, for the sake of which we take exercise. It is rendered by Cicero, "Causa propter quam quid fit." See Magrath's *Selections from the Organon*, § 96.

<sup>4</sup> The term, *vera causa*, first employed by Newton, is used in three senses; sometimes for the actual cause that produces a phenomenon, sometimes for a cause that we know to be actually existent, sometimes

only followed by but actually produces the effect. We are here concerned with not the essences and inherent constitutions of "things in themselves": we are concerned with physical causes.

If we descend from the general to the particular, from collective sequences to the sequences in the separate parts, we find the Law of Causation everywhere illustrated by the general invariability of those sequences. To certain facts, certain facts always do, and, as we believe, will continue to succeed. Of these, the invariable antecedent is termed the *cause*; the invariable consequent, the *effect*: And the Law of Causation, as applied to the particulars, may be expressed in this way: every one of those particular consequents is invariably connected with some particular antecedent or set of antecedents.

In applying the Law of Causation to the particular phenomenon, we have alluded, for inclusiveness' sake, to a *particular antecedent*. But, if the invariable sequence insisted on is ever found to exist between a consequent and a *single* antecedent, the case is very rare. The sequence is usually between a consequent and the sum of several antecedents; and, although it is usual to single out for some reason one of these and call it the Cause, and the others mere Conditions, as if to imply their being of somewhat less importance, the real cause consists of both the one and the other; that is to say, of the whole of the antecedents. And this is true, whether they be all of the class of antecedents known as *events*, that is, instantaneous changes, or successions of instantaneous changes, or *states*, possessing more or less of permanency. It will happen, sometimes, that in stating the cause, *all* the conditions are not enumerated; some being understood. An accurate analysis, however, requires a complete enumeration of them, the least conspicuous condition no less than the most conspicuous, and antecedent *states* or permanent facts no less than proximate antecedent *events*. And the absence of preventive conditions for a cause analogous to an existent cause. See Fowler's *Inductive Logic*, p. 119.

ditions is a general condition sometimes to be insisted on in accounting for a phenomenon. When we wish thus to survey all the conditions that make for or against it, showing that in certain instances there are counteractions between them, we may indicate opposing conditions by opposite algebraical signs, and say that the cause is the aggregate of the conditions of the phenomenon, positive and negative, or, of the contingencies of every description which being realized the consequent invariably follows. Or, we may say, that the cause is the assemblage of the positive conditions of the phenomenon, if one negative condition be understood, namely, the absence of preventing or counteracting causes.

When we define the cause of anything as, "the antecedent which it invariably follows," this is not to be interpreted as exactly synonymous with, "the antecedent which it invariably *has* followed in our past experience." Thus, we do not call night the cause, or even a condition, of day. Invariable sequence is not synonymous with causation: unless the sequence, besides being invariable, is unconditional, or necessary, one which *must* be, whatever supposition we may make with regard to other things. That which will be followed by a given consequent only when another condition exists cannot, itself, be the cause of that consequent, though it be known to invariably precede it. To prevent misconceptions on this head, we may define, therefore, the cause of a phenomenon to be the antecedent, or the concurrence of antecedents, to which it is invariably and *unconditionally* subsequent, or subsequent, subject to no other than negative conditions.

The appropriateness of the terms *antecedent* and *consequent* to the relations of cause and effect is a matter that demands some attention. The term, antecedent, implies at least some priority; and the questions arise, Must the cause always, at least to some extent, precede the effect? and again, Does the cause exist to an extent concurrently with the effect? To the first of these questions, Mill replies, that even should there be any shade of inappropriateness in these *terms*, it is immaterial, if we do not miss the fact, that the *beginning* of a pheno-

menon is what implies a cause ; causation being the law of the succession of phenomena. To the second question, Mill replies, that the two things, cause and effect, do very generally coexist ; and that there are some appearances, and some common expressions, such as the dogma of the schools, *Cessante causa cessat et effectus*, that seem to imply that causes not only *may*, but *must* be contemporaneous with their effects. But a distinction is to be drawn. The conditions which are necessary for the first production of a phenomenon are occasionally also necessary for its continuance, though, more commonly, its continuance requires no condition except negative ones. Understanding this distinction, we avoid the necessity of admitting that the continuance of the cause is ever required to maintain the effect. We may say, it is not required to maintain, but to reproduce the effect, or else to counteract some force tending to destroy it.

Sometimes the one phenomenal cause seems to be productive of effects, between which, to ordinary minds, there seems to be no relation. In such cases, it is usual to speak of such effects as each produced by a *different property* of the cause ; as, for example, the fall of a heavy body, as due to the attractive force of the earth, and the deflection, or direction, of a magnetic needle as due to its magnetic force. Sometimes, again, the one phenomenal effect, though in the given instance really produced by a certain cause, may also be producible in entire independence of it. It is not true that one effect must be connected with only one cause, or assemblage of conditions, or that each phenomenon can be produced only in one way. There are often several independent modes in which the same phenomenon could have originated ; in other words, there may be a *plurality of causes*. The number of causes of a phenomenon is not indefinite ; and, if we once know them exhaustively, we can be sure that at least some one of them is operative in any given case presented to us for examination.

As to the *joint action*, or *composition* of causes, the result appears in two forms. In an important class of cases, each cause produces its full calculable effect ; one cause never,

strictly speaking, defeating or frustrating another, as, for example, in the *composition of forces* in Mechanics. In another class of cases, however, occurring in other departments of nature, the causes bear no discernible analogy to the effects produced by their conjoint action ; for example, hydrogen, oxygen, and electricity, and the water produced by the passage of the electric spark through the combined gases.

Finally, when we endeavour to ascend from phenomenal causes of a lower, to those of a higher order, we soon find that the inquiry brings into view a number of *Permanent Causes*, or what have been called Permanent Causes, that exist in nature, such as the sun, the earth and other planets, and all distinguishable substances, simple, or compound, of which nature is made up. Now, to some philosophers, the coexistence of such primeval causes seems to be irresolvable. It seems to be a merely fortuitous occurrence ; and, hence, coexistences among the effects of such causes would not be classed as laws of nature, or referred to causation. To a more progressive school, however, these apparent causes, so diverse in character, and seemingly fortuitous in distribution, cover a subtler cause that, in Protean shapes, shows a wonderful unity, and that, as *Conservation of Energy*, seems to fill the whole phenomenal universe. Hence, as dependent in the ultimate on a source of this kind, the effects of such permanent causes would be referred to the Law of Causation.

Some other cases of uniformities of coexistence, seemingly irreducible, or irresolvable, as not known to depend on causation, still remain for solution. These occur among the attributes of natural kinds. Among them may be mentioned the law connecting gravity and resistance to motion ; and, though more ambiguous in character, the law connecting in an inverse proportion the atomic weights of substances and specific heat, and the law connecting the atomic weights with the specific gravities of substances in a gaseous state.

**Examples**      § 127.—The following questions relate to the and Studies. matter of the two preceding sections ; some of them with a view to an extended course of reading :—

1. What are the laws of nature? How are they proved?
2. What is the relation between the law of the uniformity of nature, and the universal law of causation?
3. How may we prove an empirical law? What is the value of a merely empirical law? Why?
4. "Empirical laws can be extended only to *adjacent* cases." Explain the term in *italics*, and state (1) why the extension is thus limited; and (2) by what law even this extension is made. C. S.
5. Statistics show a remarkable coincidence between the rate of mortality and the density of the population. Is this a derivative or an empirical law?
6. What is a cause? In what sense is the word used in (1) the phrase "Plurality of causes," (2) "Every cause has an effect," (3) "The cause is equal to its effect," (4) "The cause is the effect."
7. "Every phenomenon has a cause." What does this mean, and how is it known?
8. What is meant by saying that cause and effect are only "two aspects of one phenomenon"? What is the bearing of this view of causation on the doctrine of the plurality of causes?
9. "As often as the same circumstances are repeated, the same effect will follow, yet when the effect is the same we cannot infer that the cause is the same too." Explain this statement fully, taking especial account of the meaning to be given to the word "same."
10. When two phenomena are causally connected, can you always ascertain which is the cause, and which is the effect? If so, how?
11. Discuss whether an effect is, or is not, simultaneous with its cause; or, whether in some cases it is, and in others, is not.
12. *Cessante causa cessat et effectus.* Discuss this doctrine; and consider whether the cases in which it appears true; and the cases in which it does not, have each some other distinguishing characteristic by which this difference might be explained.

13. Explain what you understand by (a) permanent causes, (b) the composition of causes, (c) plurality of causes, (d) *verae causae*, (e) uniformities of coexistence independent of causation. C. S.

14. Examine the following statement: "To us a cause is not to be distinguished from the group of positive or negative conditions which, with more or less probability, precede an event."—W. S. Jevons. C. S.

15. Write an Essay on the physical causation of phenomena, distinguishing between phenomena that may be regarded as instantaneous, and phenomena of appreciable duration, establishing any variety of terminology you may see necessary, suitable for the purpose.

16. "Here all things always seem the same."—Comment upon this line in view of the relation between time and phenomenal change.

17. Examine the following statement:—"Camphor, olive oil, linseed oil, spirit of turpentine, and the diamond, have high refracting powers; and inflammability, which is true of the first four, may be extended to the adjacent case of the diamond."—See Mill's *Logic*, Bk. III., Chap. XX., § 3.

## CHAPTER IV

### PROCESSES OR OPERATIONS SUBSIDIARY TO INDUCTION

**Observation and Experiment.** § 128.—The chief mental processes or operations subsidiary to Induction properly so called, are Observation and Experiment, Abstraction, Classification, and Naming,—including the construction of Nomenclature and Terminology.

With regard to the first of these processes, Bacon, in the first aphorism of the *Novum Organum*, says, “Man, the Servant and Interpreter of Nature, can do and understand as much as he has observed concerning the order of nature in outward things or in the mind ; more, he can neither know nor do.”

What we learn, respecting the order of nature in outward things or in the mind, by our own observation, we call our individual experience ;—our sensible experience, in so far as it relates to outward things, or the objective world, and is obtained through our senses under the laws of our intelligence ;—our subjective experience, in so far as it relates to the mind, and is due to intuitions of our feelings and volitions, and our thoughts respecting them.

But, “what man has observed respecting the order of nature” is not limited to individual experience. That experience which is the ultimate source of our knowledge of nature and its laws is not the experience of one man only, or of one generation, but the accumulated experience of all mankind in all ages, registered in books, or recorded by tradition.

Experience may be acquired in two ways<sup>1</sup> :—

(1) By simply noting facts as they occur, without any attempt to influence the frequency of their occurrence, or to vary the circumstances under which they occur. This is *Observation*.

(2) By putting in action causes and agents over which we have control, and purposely varying their combinations, and noticing what effects take place. This is *Experiment*.

“To these two sources we must look as the fountains of all natural science. It is not intended, however, by thus distinguishing observation from experiment to place them in any kind of contrast. Essentially they are much alike, and differ rather in degree than in kind ; so that, perhaps, the terms *passive* and *active observation* might better express their distinction ; but it is, nevertheless, highly important to mark the different states of mind in inquiries carried on by their respective aids, as well as their different effects in promoting the progress of science. In the former we sit still and listen to a tale, told us, perhaps obscurely, piecemeal, and at long intervals of time, with our attention more or less awake. It is only by after-rumination that we gather its full import ; and often, when the opportunity is gone by, we have to regret that our attention was not more particularly directed to some point which, at the time, appeared of little moment, but of which we at length appreciate the importance. In the latter, on the other hand, we cross-examine our witness, and by comparing one part of his evidence with the other, while he is yet before us, and reasoning upon it in his presence, are enabled to put pointed and searching questions, the answer to which may at once enable us to make up our minds. Accordingly, it has been found invariably, that in those departments of physics where the phenomena are beyond our control, or into which experimental inquiry, from other causes, has not been carried, the progress of knowledge has been slow, uncertain, and irregular ; while in such as admit

<sup>1</sup> See Sir John Herschel's *Preliminary Discourse on the Study of Natural Philosophy*, § 67, pp. 76-77.

of experiment, and in which mankind have agreed to its adoption, it has been rapid, sure, and steady."—*Sir J. Herschel.*

Our two experiential methods, thus, are not to be distinguished apart by strict lines. Observation is, to a certain extent, preparatory to, and to a certain extent interwoven with experiment. Again, every experiment is made with the view of observing the result. But, as the essence of observation consists in (*passively*) noting the facts, and of experiment in (*actively*) varying artificially the conditions, we may say that, as our study of phenomena inclines more the one way or the other, we chiefly observe or experiment.

In the following table the principal inductive sciences are distinguished according to the degree in which they make use of Observation or Experiment:—

Astronomy is a science chiefly of Observation. In solar eclipses and the like, however, nature herself may be said to perform a vast experiment for us; and the results of the application of the spectroscope to astronomy have a character closely approaching the experimental.

Biology is a science chiefly of Observation. Experiment also is employed in this science, but with limitations due to peculiarities of the living structure.

Botany is a science chiefly of Observation. There is, however, considerable use made of Experiment, namely, in the production of varieties, and in change of climate and soils.

Chemistry is a science both of Observation and Experiment; and so is Mineralogy.

Physiology is a science chiefly of Observation.

Physics is a science of Observation and Experiment.

Politics is a science chiefly of Observation. There are in it, however, certain substitutes for Experiment, as, for example, (1) the introduction of new and extraordinary influences, such as an epidemic like the *Black Death*, an insurrection, a religious

revolution, a famine, such as the Irish potato famine of 1845, new inventions of great importance, such as the steam-engine and the like; (2) the practical operations of government. "The nearest approach to an experiment, in the philosophical sense, which can take place in politics," according to Mill, "is the introduction of a new operative element into national affairs by some special and assignable measure of government, such as the enactment, or repeal, of a particular law."

Psychology is a science of Observation, and, to some extent, of Experiment,—the latter, in the ascertainment of those bodily powers connected with certain mental processes.

Zoology is a science chiefly of Observation. Cross-breeding gives, however, in this science, as in Botany, some room for Experiment; and to this may be added what is called domestication.

*Rules for Observation to Observation and Experiment:*—

*Experiment.* 1. The phenomenon to be studied should be, as far as possible, isolated from sources of disturbance that cannot be made to counterbalance or neutralize each other or be distinctly estimated as to their effect, so that the results shall relate to itself alone. The elimination of disturbing elements may, in some sciences, be effected by multiplying the main effect, so as to render such disturbing effects comparatively trivial.

The experimental study of the effects of the electric current upon a magnetic needle—the magnetic influence of the earth being eliminated—affords a familiar illustration of this rule.

2. In the case of the more obscure phenomena, as many causes of variation as can be conveniently introduced should be tried. This is in accordance with the Baconian rule of "varying the circumstances."

3. In the application to the study of the phenomenon,

of causes of variation, but one should be introduced at a time ; the result in each case being noted.

4. The results should be precise. And, in the case of Observation, in itself, the following faults should be guarded against :—Non-observation, and Mal-observation ; mistaking inference for observation ; and making but observation of *kinds* where precise observation of quantities is required. We must be sure that we do not confound what we actually observe in a given case with what we infer. On this understanding the fact supposed to be observed may be safely received as true. There is, at least, *one* inference in every case, namely from the sensations to the presence of the object ; and hence follows the seeming paradox that a general proposition collected from particulars is often more certainly true than any one of the particular propositions from which by an act of induction it was inferred, the possible erroneousness of the single inference alluded to, in any one instance, being likely to be eliminated in the aggregate of instances as summed up in the general proposition. (Mill's *Logic*, vol. ii., p. 356.)

5. When *quantities* are measured, and there are grounds for regarding the results as deviating more or less from precision, suitable corrections should be applied to them, so as to bring them as close as possible to accuracy.

Observations of a mathematical character are necessarily liable to errors from the imperfections of our senses, and of the instruments we use ; and also from hasty or otherwise incorrect conclusions. These errors may be divided into three classes which may be mixed up with one another ; namely, what are called the *fixed*, the *personal*, and the *casual* errors. Errors of this kind cannot be detected by multiplying similar observations, since the mere repetition of error does not destroy it. There are many modes of detecting them, and of allowing for them ; the only mode, however, of avoiding them is, by taking advantage of the construction of the instrument, to use it for the same purpose

in different circumstances, so that measurements that are too large in one set of results may be as much too small in another; any final error that may appear being reduced to the class of what are called *casual errors*. By a *personal error*, as we have explained in an earlier section, is meant one of the same character as a fixed error, but arising from the temperament or habits of the observer, not from the instrument. Thus, if A should, in noting the time of a celestial phenomenon by the clock, have a tendency to accelerate the moment of its happening, and B, a similar tendency to retard it, the results of the two would differ by the sum of their personal errors. The personal error is found to occur, not once or twice, but nearly always; and in such a manner as to make the average of a set of observations by one observer differ from that by another. Means have, however, been devised recently for making an approach towards an absolute determination of the personal error in the case of any particular observer.<sup>1</sup> As to *casual errors*, by such are meant errors that are absolutely inexplicable, or, of which the cause and tendency are equally unknown.

As to the rule with regard to *varying the circumstances*, we must, in pure observation, take what nature offers, and very often in the precise surroundings in which it is offered: experiment, however, enables us not merely to produce *a much greater variety* in the circumstances than we can have in nature, but, more particularly, to produce *the precise sort of variation* we require for discovering the law of the phenomenon. For the applicability of experiment, there is, at the same time, one indispensable, all-important condition, namely, that the phenomenon is such as it is in our power to produce. When this is so, we can vary the instances at will, and choose what the concomitant circumstances shall be, so as to remove sources of disturbance, and secure precision in our results. When this is not so, we have to seek for instances in which nature produces the phenomenon, to discover what the concomitant circumstances are, and, in all but the simplest

<sup>1</sup> See Jevons's *Principles of Science*, vol. i., p. 402.

cases, it must be added, to find our investigation marked by want of precision and incompleteness with regard to these circumstances.

In proceeding from a given cause to ascertain the effect, or, on the other hand, from a given effect to ascertain the cause, the relative advantages of observation and experiment are put to the test. In the former case, both observation and experiment are open to us; observation,—that is to say, pure observation—if nature offers us an instance of the cause working spontaneously, and experiment, if we prefer to set the given cause in operation ourselves and mark the result. Here all is comparatively plain. But, if we are given the effect, the conditions on which the phenomenon depends being obscure, we are necessarily destitute of the resources of artificial experimentation. We have nothing to do, therefore, but take the effect, where it offers itself spontaneously: and if nature presents us with instances sufficiently varied, and we find among the proximate, or some other order of antecedents, something invariably present when the effect is, and never present when it is not, we may discover by observation alone a real uniformity in nature.

Such a case as we have supposed, of pure observation, appears to compare well with experiment. But it exhibits, also, the inherent imperfection of direct induction not grounded on experimentation. For, assuming that our observations point to what appears to be, and what perhaps is, an invariable antecedent, we have proved it to be *invariable* but within the limits of experience. To prove that it is a cause, or *unconditional* antecedent, we must, somehow, produce it artificially; and, if when we do so, unless the consequent was generated, not by the antecedent, but by the means employed to produce the antecedent, that antecedent is the cause of the consequent so produced.

“Observation,” as Mill points out (*Logic*, Bk. III., Chap. VII.), “without experiment, and supposing no aid from deduction, can ascertain sequences and coexistences, but cannot prove causation.”

**Studies relative to Observation and Experiment.** § 130.—The following questions relate to the matter of the two preceding sections :—

1. Describe the several steps of observation and experiment in the following examples :—
  - (1) The decomposition of water into oxygen and hydrogen by the electric current.
  - (2) The production of carbonic acid snow.
  - (3) The action of light upon chloride of silver.
  - (4) The barometer and thermometer in relation to the weather.
  - (5) The identification of lightning with the electric spark.
2. Describe the function of experiment in scientific investigation. How far, if at all, can we be said to experiment (1) in Astronomy, (2) in Politics ? C. S.
3. State the general rules of chief importance in the conduct of Observation and Experiment.
4. Account for the superiority of Experiment over mere Observation, and mention the leading fallacies incident to the latter process.
5. Under what circumstances has Observation more advantages than Experiment ?
6. Is it always possible to determine where Observation ends and inference begins ? If not, how is Logic to distinguish between the two ?
7. Describe the requisite conditions of a perfect qualitative experiment, that is, an experiment in which no account is taken of the magnitude of the effect.
8. What is meant by the personal error (or personal equation) in observation ? Discuss its importance in different branches of knowledge. C. S.

**Abstraction and Classification.** § 131.---Abstraction, Classification, and Naming are closely-related processes. Abstraction is the *act* of drawing apart, or the separation of the attributes or qualities common to all the individuals of a group, from the several attributes of the individual, that they may form

the distinctive marks of that group or class. It is preliminary to and implied in classification. Classification may be defined as the arrangement of objects in groups according to their resemblances. An arrangement of objects in groups occurs in distinguishing objects by general names. But in such cases the grouping is merely incidental. The giving of the name is the primary object. In classification as one of the subsidiaries of induction, the main objects are arrangement and distribution. For purposes of classification certain marks are to be selected the possession of which is to be the title of admission to the group or class ; and all the objects that possess these marks are to be ascertained towards arrangement in such group or class. Classification, as one of the subsidiaries of induction, is described by Mill as "a contrivance for the best possible ordering of the ideas of objects in our minds ; and for causing the ideas to accompany or succeed one another in such a way as shall give us the greatest command over such knowledge as we have already acquired, and lead most directly to the acquisition of more." He, accordingly, regards the aim of the general problem of classification to be the following : To provide that objects shall be thought of in such groups, and those groups in such an order, as shall best conduce to the ascertainment and the remembrance of their laws. Classifications of this nature refer to real objects, exclusively ; but, at the same time, embrace all really existing objects.

In making a classification of things, the chief rule to observe is that the classification should be appropriate to the object in view. A good classification for scientific ends requires that the objects be formed into groups respecting which a greater number of general propositions can be made, and those propositions more important, than could be made respecting any other groups into which the same objects could be divided. The properties, therefore, according to which objects are classified, should, if possible, be those which are causes of many other properties, or which, at all events, are sure marks of them. In such a classification it is impossible to secure

excellence without an extensive knowledge of the properties of objects. When the properties selected are marks not materially affecting the natures of the respective objects, and belonging in common to many objects differing widely in other, more important, properties, the classification is called artificial. The principle of a natural classification, according to Mill, prescribes that such classification shall recognize and adopt into itself all distinctions of Kind which exist among the objects it professes to classify; all Real Kinds being natural groups. The adoption of such distinctions will, however, not render its groups conventional. The classes are determined by characters, but these are not arbitrary. Kinds, according to Mill, are classes between which there is an impassable barrier; and what we have to seek is, marks whereby we may determine on which side of the barrier an object takes its place. The characters which will do this should be chosen; if they are also important in themselves, so much the better. But though all Kinds must be classes, all classes in a natural classification cannot be Kinds. The distinctions of Kinds are not numerous enough to make up the whole of a classification. Genera and families may be eminently natural though marked out from one another by properties limited in number, provided these properties are important, and the objects in each genus or family resemble each other more than they resemble anything which is excluded from the genus or family. The *Infimae Species*, or lowest classes in a natural classification, will be the lowest Real Kinds. These are to be classed into higher groups, Genera, and so on, according to types of general resemblance or character.

In so far, however, as the distinction between natural and artificial classification depends on Mill's view respecting distinctions of Kind, the modern doctrine of Evolution, in the opinion of many, renders it of less importance (see Darwin's *Origin of Species*, Chap. II.).

With regard to natural classifications, owing to the fact that they have to be arranged under deep and inaccessible affinities, they are not always convenient for consultation.

Accordingly, separate schemes of an artificial nature, and so constructed as to serve as indexes or keys to them, are provided in Chemistry, Botany, and other subjects. Botany, especially, has an elaborate *Index Table* for ready use in the discrimination of plants.

The object of classification into natural groups is to make us think of those objects together which have the greatest number of important common properties, and which therefore, in the course of our inductions generally, we have most frequent occasion thus to recall; but when our object is to facilitate the inquiry into some particular phenomenon, we must arrange the various groups into a *series*, following one another according to the degrees in which they severally exhibit the phenomenon. In an inquiry, for example, into the phenomenon of "Animal life," we must arrange the various kinds of animals in a series, beginning with man, and ending with the most imperfect kind of zoophytes. In other words, the instances from which the law of animal life is to be inductively collected should follow the degrees of the main phenomenon, implying the existence of a type species, the arrangement being adapted to the application of one of the processes explained in the succeeding chapter, the Method of Concomitant Variations; often the only available one in the case of phenomena which we have but limited means to artificially separate.

**§ 132.—Applied Logic in the section of Induction**  
**Nomen-** clature and Termin- ology. **concerns itself with the discovery of causes, and with the determination and record of new laws.** The discovery of causes and the determination of new laws are main questions. The expression and record of new laws is a subsidiary question: it relates to language, and the means language affords us of describing and recording our observations. Language, as an instrument of record, should be precise and adequate. Relative to the study of nature, it should furnish us with names for the various phenomena, names for the great leading classes of objects, names of such a structural character as to facilitate

our remembering the individuals of a class, and all the special terms, besides, necessary for the description of the several objects, their parts, or elements, and of the various phenomena, and of everything relative to them. In other words, language, for the purposes of science, must furnish us with a systematic and regular nomenclature, and a precise and copious terminology. A *nomenclature* is a collection, in the main, of names of groups or of the various divisions and subdivisions that constitute a classification from *summa genera* to *infimae* species: but it includes the names of individual objects, if they require separate record. A *terminology* is a collection of the names or terms we require for describing the various phenomena, or distinguishing the properties or parts of the individual objects any science is concerned with. Botany and chemistry excel in nomenclature; and the language of botany especially is very complete in terminology. In other sciences, as, for example, astronomy and geology, the nomenclature is more or less imperfect, according to the nature, position, or state of progress of these sciences in relation to distinctive subject-matter. (See Fowler's *Deductive Logic*, Chapter II.)

Subjects for Studies, and general matter of the present chapter. They Examples. § 133.—The following questions bear upon the assume, at the same time, a liberal course of supplementary reading:—

1. Examine what is meant by a "Natural or Scientific classification," and discuss the following dictum of Mill: "Kinds are classes between which there is an impassable barrier."
2. Give a brief estimate of the present state of the nomenclature of the following sciences, respectively:—Mineralogy, Chemistry, Astronomy, Geology.
3. Classify the following terms, as whether of nomenclature or terminology:—haematite, granite, ethyl, silurian, lava, stamen, bauhinia, bilobate.
4. State the canons that determine a correct natural

classification. What purposes are subserved by a scientific classification?

5. Discuss the importance of Naming as one of the subsidiaries of induction. (See Herschel's *Discourse*, §§ 129-136.)

6. What does Mr. Bain regard as "the golden rule" of classification? (*Logic*, vol. ii., p. 185.)

7. Explain the object of the device called *double-naming* used in certain sciences, such as Botany, Zoology, and Chemistry.

8. Discuss the propriety of the allegation that "Natural groups are given by *Type* (—a well-selected average member of a class), and not by *Definition*."

## CHAPTER V

### METHODS OF INDUCTION

**Methods of Induction.** § 134.—In the modern history of Induction five names have a distinct prominence, Bacon, Newton, Herschel, Whewell, and Mill.

The importance of Bacon's contribution to Induction consists in his condemnation of rash generalizations, in his insisting on the necessity of a thorough acquaintance with the facts of nature as a foundation for an inquiry into general laws, and in his showing, according to a method of his own, in the *Novum Organum*, how such facts could be conveniently marshalled towards this end. He points out that, in the centuries before him, there had been wanting well-ordered experience, while, at the same time, there prevailed too great a reverence for antiquity and the authority especially of the ancients, forgetful that, considering the age of the world, we ourselves are really the ancients, "Antiquitas saeculi juventus mundi." He sees, however, in the aspects of things, grounds of hope that, as a new continent, a few short generations before, rewarded the courage and perseverance of Columbus, so the instauration of the sciences would be possible to philosophy, diligently and courageously pursuing a suitable course.<sup>1</sup>

Newton, in his own scientific work, set the example of

<sup>1</sup> In the Second Book of the *Novum Organum* there are some rough approximations to Mill's inductive methods. Certain tables of Instances there given, relating to Heat, would, with suitable modifications, furnish examples to illustrate each of them. See Fowler's *Inductive Logic*, pp. 206-209; Bacon, in the *English Philosophers Series*, p. 199; Bain's *Logic*, vol. ii., pp. 403-408; Mill's *Logic*, vol. i., p. 361.

rigorous and cautious inquiry. His *Regulae Philosophandi*, or Rules for Philosophizing, long regarded as maxims of weight in scientific investigation, are as follows :—

(1) (a) Only *verae causae*, or real or actually existing causes, are to be admitted in explanation of phenomena. (b) No more causes are to be admitted than are requisite to explain the phenomena. *Natura nihil agit frustra, et frustra fit per plures quod fieri potest per pauciora.*

(2) The same causes are, as far as possible, to be assigned for the same kind of natural effects.

(3) Such qualities of bodies as can be neither increased nor diminished in intensity, and that are to be found in all bodies within reach of experiment, are to be considered qualities of all bodies whatever.

(4) Induction relating to phenomena are to be held as either exactly or approximately true, until other phenomena occur affording data either to render them more exact, or to prove them liable to exceptions.

Sir John Herschel, in his *Discourse on the Study of Natural Philosophy*, discusses the principles on which physical science relies for its successful prosecution, and the rules in accordance with which a systematic examination of nature should be conducted. The importance of experience as the sole fountain of physical knowledge, the value of accurately recorded observations, the utility of classification, and appropriate nomenclature, are all insisted on in the *Discourse*. He lays down (*Discourse*, pp. 151-157) nine rules for philosophizing, four of which are selected by Mill as canons of the Experimental Methods. Compared with Mill, his aim is Discovery, while Mill's is not merely Discovery but Proof.

Dr. Whewell, in his *Novum Organum Renovatum*, professes from an examination of the progress of actual discovery to revise and improve the methods by which science must grow. He pursues throughout what he conceives to be a fundamental antithesis, namely, that between Ideas, or Conceptions, and Facts ; the grand object of science being, in

his view, to colligate, or bind together, the facts by means of appropriate conceptions. His principal aim is to furnish arts of Discovery ; and he gives some quantitative methods which are a material aid towards this end, and are much employed by observers. Of these, the Method of Curves, the Method of Means, and the Method of Least Squares may be briefly noticed. The *Method of Curves* is a graphical method of exhibiting the variations in two phenomena, one of which depends on the other. It consists in drawing a curve of which the observed quantities are represented by the ordinates, the quantities on which the changes of these quantities depend being represented by the abscissae. Many of the meteorological and other physical instruments in our observatories are provided with fittings for automatically tracing such curves. The *Method of Means* enables us to eliminate with more or less completeness from the effects of a constant cause accidental accompaniments of any kind by taking an average of several observations.<sup>1</sup> The *Method of Least Squares* determines that one, among two or more means to be the most probable, which is such, that the sum of the squares of the deviations of it from each of the actually observed quantities is less than the similar sum formed for the remaining mean or means.

According to Mill, Induction is the operation of discovering and proving general propositions : Dr. Whewell's theory of the Logic of science would, he admits, be very perfect, if it did not pass over altogether the question of proof.

For the discovering and proving of the laws of nature Mill gives the following experimental methods :—

(a) *The Method of Agreement.*

*Canon.*—If two or more instances of the phenomenon under

<sup>1</sup> To take an average is to add the several quantities together, and divide by the number of quantities thus added : the result is a quotient lying among, or in the *middle*, of the several quantities. On the several significations of a *mean result*, see Jevons's *Principles of Science*, vol. i., p. 414.

investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon.

(b) *The Method of Difference.*

*Canon.*—If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former, the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.

(c) *The Joint Method of Agreement and Difference.*

*Canon.*—If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.

(d) *The Method of Residues.*

*Canon.*—Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.

(e) *The Method of Concomitant Variations.*

*Canon.*—Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.

The following are illustrations of these methods:—

(a) *The Method of Agreement.*

(1) Given a cause,  $a$ , to ascertain the effects.

Try  $a$  with  $\beta, \gamma$ , and let the effect be,  $a b c$ .

Again, try  $a$ , with  $\delta, \epsilon$ , and let the effect be,  $a d e$ .

The effect of  $a$ , whatever it was, must have been produced in both instances.

Now,  $a$  is the only part of the effect common to both instances, or what they *agree* in.

And,  $a$  must have been the effect of  $a$ , or  $\beta$ , or  $\gamma$ , or  $\delta$ , or  $\epsilon$ .

But it was produced when  $\delta, \epsilon$ , were not ; also when  $\beta, \gamma$ , were not ;

$\therefore a$  must be an effect of  $a$ .

Suppose, for example,  $a$  to be a combination of an oil and an alkali, and  $a$  a detergents, or saponaceous substance. By trying, thus, the combination, under several varieties of circumstances, it may be proved that such a combination causes the production of a soap.

(2) Given an effect,  $a$ , to ascertain the cause.

Here, since we may exclude possible success due to random trial, or to accident, we are limited to *observation*.

Suppose we observe  $a$  in the combinations,  $a b c$ ,  $a d e$  ;

and let the antecedent circumstances of  $a b c$  be known, or discovered, to be,  $a \beta \gamma$  ;

and the antecedent circumstances of  $a d e$  be  $a \delta \epsilon$ .

Then, as in (1), it can be shown that  $a$  is the antecedent connected with the consequent  $a$  by a law of causation.

Suppose, for example, that  $a$  represents crystallization ; and let  $a$ , in the known instances, be the deposition of a solid matter from a liquid state, either in a state of fusion or of solution. Then, it may be proved that solidification of a substance from a liquid state is an invariable antecedent of

its crystallization. In fact, in this example we may go farther, and say that it is the cause, or if not, at least the proximate event that completes the cause; for from this equivalent of  $a$  we are able to produce  $a$  artificially. If we found ourselves unable to do so, the cause of  $a$  would remain subject to considerable doubt.

A characteristic defect of the Method of Agreement, pointed out by Mill, is that it is liable to be affected by a Plurality of Causes, a defect, however, which affects neither the Method of Difference nor the Joint Method.

*(b) The Method of Difference.*

(1) Given a cause  $a$ , to ascertain the effect.

Find  $a$ , in such a combination of ascertained circumstances as,  $a\beta\gamma$ .

Let the effect of  $a\beta\gamma$  be  $a b c$ , and the effect of  $\beta\gamma$  be  $b c$ .

Then the effect of  $a$  is  $a$ .

For example, let  $a$  be sodium thrown into a flame or other source of heat and light, to ascertain the character of the spectrum thus produced. We find that when sodium is present there occurs a bright double line in the spectrum, and that when it is absent there is no such line there. Hence, this line in the spectrum is due to the presence of sodium in the flame.

(2) Given an effect  $a$ , to obtain the cause.

Take an instance,  $a b c$ , in which the effect occurs.

Let  $a\beta\gamma$  be the antecedents of  $a b c$ .

Find an instance,  $b c$ , in which the effect does not occur.

Let  $\beta\gamma$  be the antecedents of  $b c$ .

Then the cause of  $a$  must be  $a$ , either alone or in conjunction with some of the other circumstances present.

Sir Humphry Davy showed that if two pieces of ice be rubbed together in a vacuum heat is produced, as proved by their melting ; hence, he inferred that friction is the source and cause of the heat so produced.

The Method of Agreement and the Method of Difference are both, as Mill points out, methods of elimination. By *elimination* is here meant the successive exclusions of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those amongst them which can be absent consistently with the existence of the phenomenon. The Method of Agreement is based upon the principle that whatever can be eliminated is not connected with the phenomenon by any law : the Method of Difference, on the principle that whatever cannot be eliminated is connected with the phenomenon by a law. The first of these methods depends more upon experiment ; the second, more upon observation.

(c) *The Joint Method of Agreement and Difference.*

The Method of Difference requires of us to find a positive instance,  $a \beta \gamma$ ,  $a b c$ , and a negative instance,  $\beta \gamma$ ,  $b c$ .

But sometimes neither experiment nor observation of nature readily affords us these.

We then proceed from examined instances, establishing by the Method of Agreement that wherever  $a$  occurs,  $a$  is present to  $a \beta \gamma$ ,  $a b c$ .

And, in the same way, we proceed from examined instances, establishing that where  $a$  does *not* occur,  $a$  is not present to  $\beta \gamma$ ,  $b c$ .

Having reasoned out the instances thus, on a double application of the Method of Agreement, we infer that  $a$  is the cause of  $a$ .

This method is appropriate when the agency by which we can produce the phenomenon is not that of one single antecedent, but a combination of antecedents which we have

no power of separating from each other and exhibiting apart. For example, suppose we are investigating the cause of the double refraction of light. We take, say, Iceland spar, which produces the phenomenon, and is therefore an instance of  $a\beta\gamma$ ,  $a b c$ ; but here we cannot find a substance affording the negative instance,  $\beta\gamma, b c$ . We must, therefore, proceed, by the Method of Agreement, to ascertain a connection between the property in question and some other property in bodies, and this we find to be in crystalline structure. Hence, by the Joint Method, we infer that either crystalline structure or the cause that gives rise to that structure is one of the conditions of double refraction.

The Joint Method consists in reality of a double employment of the Method of Agreement, and proof by it is not equivalent in cogency to proof by the direct Method of Difference. Mill regards it but as the Method of Agreement greatly extended and improved; and hence he does not number it as a distinct experimental method.

*(d) The Method of Residues.*

(1) Given a cause  $a$ , to ascertain the effect.

Suppose that, by virtue of preceding inductions, we know that the effect of  $\beta$  is  $b$ , and of  $\gamma$ ,  $c$ .

And suppose, also, that on like grounds, we know that the antecedents,  $a\beta\gamma$ , are followed by the consequents,  $a b c$ .

Then, subducting the sums of the former causes and effects, respectively, from the latter, we arrive, without fresh experiments, at the result that  $a$  is the effect of  $a$ .

(2) Given an effect  $a$ , to ascertain the cause.

Reasoning here in a similar way, we infer that  $a$  is the cause of  $a$ .

For example, let the cause be the attraction of the sun, it is required to ascertain how much of the spring tides is due

to it. This we can do, if we have previously determined the height of the tide due to the moon, which may be regarded as about the average height of the tides during a lunar month. Subtracting the moon's tide from the whole, the remainder is that part of the spring tide due to the attraction of the sun.

The Method of Residues Mill regards but as a peculiar modification of the Method of Differences. Of the two instances required by the latter method, the one positive, the other negative, the negative one is arrived at, not by observation and experiment, but by deduction.

Of Mill's four experimental methods, the Method of Residues and that of Concomitant Variations are sometimes called *quantitative*, inasmuch as by these methods we measure the *quantity* of an effect. The other methods are *qualitative* merely. The Method of Residues is that which is employed in making allowance for errors of observation, and the corrections to be applied in the readings of instruments. Consider, for example, the readings of an ordinary thermometer. As one of the easiest errors to correct, its zero point may be wrong. We determine, then, the true zero point by inserting the stem into melting snow, being 0° Centigrade, or 32° Fahrenheit; and finding the correction, we apply it, according to its sign, positive or negative, to the observed readings, to determine the true reading in each case.

The Method of Residues has been found of much use and interest in Astronomy; some of the most remarkable discoveries of causal relations in that science being due to the observation of *Residual Phenomena*: for example, the discovery of Neptune by Adams and Le Verrier, owing to the observed residual effects noticed in the motion of the planet Uranus; it being sometimes behind its true place, though all previously known attractions had been taken into account in the calculation. Again, the retardation of the comet Encké, a residual phenomenon yet remaining to explain, may, it is thought, be due to the existence of a resisting medium diffused through space. Thus, such phenomena are fruitful of suggestion.

(e) *The Method of Concomitant Variations.*

If some modification in the antecedent,  $a$ , is always followed by a change in the consequent  $a$ , the other consequents,  $b$  and  $c$ , remaining the same; or, *vice versa*, if every change in  $a$  is found to have been preceded by some modification in  $a$ , none being observable in any of the other antecedents; we may, so far, conclude that  $a$  is wholly, or in part, an effect traceable to  $a$ , or, at least, in some way connected with it through causation.

This method presupposes the following proposition as an axiom: that anything on whose modifications—by a modification being meant a change in it, not amounting to its total removal—anything on whose modifications, modifications of an effect are invariably consequent, must be the cause, or connected with the cause, of that effect. Consider, for example, Joule's experiments in Heat. A careful measurement of the work spent in friction, and of the heat produced, led him to the result—since, slightly modified—that the heat required to raise one pound of water from  $39^{\circ}$  to  $40^{\circ}$  Fahrenheit is equivalent to 772 foot-pounds of work at Manchester. Hirn, also, showed, that when heat is made to do work in a steam-engine, part of the heat disappears, and that the heat which disappears is proportional to the work done. Thus, heat and the work spent in friction are causally connected.

The Method of Concomitant Variations is one of very wide usefulness. Among its more extended applications we may notice the employment of it in arguments grounded on statistics,<sup>1</sup> and what is known as the Historical Method.<sup>2</sup> This latter mode of inquiry, which we can here but barely refer to, deals with the conditions of the progressiveness of man and society, including the growth, and stability,

<sup>1</sup> Statistics is defined by Mr. Bain as, "the observation, registration, and arrangement of such facts as can be given in *numbers*."

<sup>2</sup> See Mill's *Logic*, Bk. VI., Chap. X., and Fowler's *Inductive Logic*, p. 200.

or decline, of institutions, customs, and opinions. It is well illustrated in the works of the late Sir Henry Maine on Ancient Law, and on the History of Institutions ; also in those of Lord Avebury and others.

**Plurality of Causes and Method of Agreement** is rendered uncertain by **Intermixture of Effects.** § 135 —It has been pointed out that the **Plurality of Causes** is possible. This may be illustrated as follows :—

Suppose  $\alpha\beta\gamma$  to be followed by  $\alpha b c$  ; and  $\alpha\delta\epsilon$  to be followed by  $\alpha d e$ .

Then, it would appear that  $\alpha$  is the invariable antecedent of  $\alpha$  ; or, if we could be sure that there is no other antecedent common to the two cases, that  $\alpha$  is the unconditional invariable antecedent of  $\alpha$ .

Suppose, for simplicity's sake, that this last is true.

And, now, admit the possibility of more than one cause for  $\alpha$ .

This upsets the proof by the Method of Agreement ; for, on this hypothesis, the modes may have been, for example,  $\gamma$  and  $\epsilon$ ,  $\alpha$  having no influence in either case.

With the still more peculiar and complex case of Inter-mixture of Effects, and the Interference of Causes with one another, the four only possible methods of directly inductive investigation by observation and experiment are for the most part, as we shall presently see, unable to cope ; and Deduction is the only means we have to unravel the complexities so arising, though we may use the experimental methods to supply premisses for and to verify our deductions. We have, in dealing with the composition of causes, touched briefly on the characteristics of the two well-distinguished classes of phenomena that thence arise. Sometimes, the resultant phenomenon is subject to the old laws of the agents ; sometimes, it is for the most part subject

to new or *heteropathic* laws, the old laws having nearly all given way to the new. The conditions of such a phenomenon may be investigated either deductively, or experimentally. We may regard the law of an effect of this description as a result of the laws of the separate causes on the combination of which it depends, and, therefore, in itself capable of being deduced, *à priori*, from these laws; or, we may regard the whole assemblage of concurrent causes that produced the phenomenon as one single cause, and proceed, *à posteriori*, to ascertain the cause, by a comparison of instances. And, adopting this latter view, we may proceed either by observation, or by experiment: by observation, in collating instances of the effect; by experiment, in trying different combinations of causes in hopes of ultimately hitting the precise combination that will produce it, as given to us to account for.

We have thus open to us three possible modes of investigation, namely, deduction, simple observation, and experiment; and of these, the inapplicability of simple observation to ascertain the conditions of effects depending on many concurring causes will be readily recognized. Turning to the other branch of the *à posteriori* method, or what is called the Empirical Method, namely, trying the effect of causes or agents, and taking notice of their effect, we naturally consider that it has a better chance of proving successful than the former. But, on turning to actual experimentation, we find ourselves embarrassed in the endeavour to work out the problem, owing to our being seldom able to even roughly ascertain the set of circumstances into the midst of which we introduce the agent. Suppose, for example, the circumstances the phenomena of life, and we at once see, that given the effect to be aimed at, namely, the recovery, say, of a patient from a disease, two cases similar in all respects but the one—as to the disease—can seldom if ever occur. Hence, anything like a scientific use of the method of experiment is in such complicated cases out of the question. We can, at most, only discover by a

succession of trials that a certain case is *very often* followed by a certain effect. And, as in Medicine, so in History, Politics, and other sciences. Two, therefore, of those possible methods for the study of such complex phenomena being insufficient, there remains, as the main source of our knowledge of the conditions and laws of such phenomena, the mode of investigation, which is called in its most general expression the Deductive Method.

**§ 136.**—The Deductive Method, or as it is called by Jevons, the Combined or Complete Method, consists in the alternate use of induction and deduction. It may be said to consist of three operations:—

- (1) Direct Induction.
- (2) Deduction or Ratiocination.
- (3) Verification.

The problem being, From the laws of each of a set of concurrent causes to find the law of the effect, we begin by examining the causes; deducing their laws by observation and experiment. Assuming such laws as provisionally true, we proceed to reason to their results in other cases; and comparing the reasoned-out results, at length, with the results of direct observation, whenever it is possible, we go through the process of verifying such laws. We either prove them to be true, or else that something in our investigation is imperfect; unless, indeed, we can show that the result we should have anticipated was frustrated by something we can point out in the process of Verification itself. One of the most satisfactory kinds of Verification is when we arrive deductively at Empirical Laws previously formulated from direct observation and the collation of instances. Of this kind was the verification of Newton's theory of gravitation found in its leading deductively to Kepler's Laws relating to planetary motion.

**Bacon's Improvements on the Induction of the Ancients.** **§ 137.**—The Induction of the ancients is described by Bacon under the name of “*Inductio per enumerationem simplicem, ubi non reperitur instantia contraria.*” This consists in ascribing

the character of general truths to all propositions that are true in every instance within our knowledge, without following any principle of selection. Bacon conceived that the peculiar value of his Logic consisted in the Method of Exclusions or Rejections, which he devised, and towards the working out of which he directs the arrangement of facts in three different tables: (1) the *Tabula Essentiae*, or Table of Essence or Presence, containing instances agreeing in the presence of the phenomenon to be investigated; (2) the *Tabula Declinationis, sive Absentiae in Proximo*, or table of Deviation, or of Absence in Allied Instances, containing instances wanting in the phenomenon to be investigated; but in other respects, very closely approximating to some one or other of the instances in the first table; and (3) the *Tabula Graduum, sive Tabula Comparativa*, containing instances of different degrees of the phenomenon—the actual table, that of Heat—which he takes as an example. Bacon's enunciation of the Method of Exclusion is as follows:—"The first work of true induction, as far as regards the discovery of forms, is the rejection or exclusion of the several natures which are not found in some instance where the given nature is present; or are found in some instance where the given nature is absent; or are found to increase in some instance where the given nature decreases, or to decrease where the given nature increases." What he means by *form* will appear from the following, among many definitions he gives of the term:—"When we speak of *forms* we understand nothing more than those *laws* and *modes* of action which regulate and constitute any simple nature,—such as heat, light, weight, in all kinds of matter susceptible of them; so that the *form* of heat, or the *form* of light, and the *law* of heat, and the *law* of light, are the same thing; nor do we ever lose sight of practice, and things as they are." Bacon's three tables could, by a judicious selection of instances, be utilized towards the application of the three inductive methods, namely of Agreement, Difference, and Concomitant Variations. Bacon, however, made but a very rough approach to the accuracy of

these methods. As subsidiary to his Method of Exclusion, and to the Tables, he enumerates certain remaining helps of the understanding, nine in all, that "promote the interpretation of nature and a true and perfect induction." These he leaves undiscussed, except one, which is that of the *Prerogativaे Instantiarum*, or Prerogatives of Instances. The Prerogative Instances are, says Herschel (*Discourse*, § 190), "characteristic phenomena, selected from the great miscellaneous mass of facts that occur in nature, and which, by their number, indistinctness, and complication, tend rather to confuse than to direct the mind in its search for causes and general heads of induction. Phenomena so selected, on account of some peculiarly forcible way in which they strike the reason, and impress us with a kind of sense of causation, or a particular aptitude for generalization, Bacon considers, and justly, as holding a kind of prerogative dignity, and claiming our first and especial attention in physical inquiries." Of these, the so-called *Solitary Instances* have a certain correspondence with those coming under the Methods of Agreement and Difference, and the *Travelling Instances*, with those coming under the Method of Concomitant Variations. Other instances, the names of which are familiar to readers of Bacon are, the *Glaring* or *Striking Instances*, the *Clandestine Instances*, or *Instances of the Twilight*, the *Collective Instances*, the *Conformable Instances*, and the *Crucial, Oracular, or Commanding Instances*. A *Crucial* or *Guide-Post* Instance, is an instance that decides between two or more contending hypotheses. A chemical test is a familiar example; the application of it being an *experimentum crucis*. Dr. Fowler points out that, in a classification of logical methods, crucial instances should be regarded as applications of the Method of Difference.

§ 138.—The following questions and exercises  
Studies in the Methods will furnish suggestions and materials for studies of Induction, relative to the methods of induction:—

1. "The induction of the ancients has been well described

by Bacon under the name of 'Inductio per enumerationem simplicem, ubi non reperitur instantia contraria.' ”

In what did this consist, and what improvements were devised by Bacon? There are cases in which it may, or even must be employed?<sup>1</sup>

2. It has been said that a complete enumeration of instances furnishes a deductive, not an inductive argument; state your view on this point, with the reasoning on which it is based. R.

3. Explain the Method of Agreement. Why is it called a method of elimination? How does it differ from an Inductio per enumerationem simplicem?

4. What kind of Fallacy is involved in the Inductio per enumerationem simplicem?

5. What is the characteristic imperfection of the Method of Agreement, and by what means can it be counteracted?

6. Explain the Method of Difference, and discuss its value.

7. Explain, “All crucial instances (*instantiae crucis*, as they are called by Bacon) are applications of the Method of Difference.” C. S.

8. State fully, and illustrate, the Method of Concomitant Variations. What are its special advantages? How is it related to the Method of Difference? Is it merely a substitute for that method, or something more? What is the major premiss implied in both methods?

9. What data are required for the Methods of Residues and Concomitant Variations and to what errors are they liable?

10. Describe what is meant by *residual phenomena*, and estimate their value in inductive science. C. S.

11. “In a certain degree, the Method of Residues and the

<sup>1</sup> In inductions relative to moral and social phenomena.—See Fowler's *Inductive Logic*, Chapter VI., “On the Fallacies incident to Induction,” one of the most admirable chapters in an able and admirable standard work.

Method of Means are *opposite* to each other. For the method of extricating laws from their combinations brings them into view in succession, while the Method of Means discovers each law not by bringing the others into view, but by destroying their effect through an accumulation of observations."

Describe the methods here mentioned, and explain what is here said of them.

12. Observation shows that a clear sky and a cold night are commonly associated. Indicate the methods to which you would resort to ascertain whether either, and if so, which, is the cause of the other.

13. Defining *Dew* to be "The spontaneous appearance of moisture on substances exposed in the open air when no rain or visible *wet* is falling," show how the complete, or Deductive method, is to be employed to establish the law of causation of the phenomenon from the annexed table of observations:—

<i>Antecedents.</i>	<i>Consequent.</i>
(1) Cold metal or stone ; Breathing upon.	(1) Deposit of Dew on metal or stone.
(2) Hot weather ; Glass of water drawn from deep well ; Exposure to external air.	(2) Deposit of Dew on glass.
(3) Inner surface of walls of dwellings ; Long continued frost ; Thaw ; Moisture in air.	(3) Deposit of Dew on the walls.
(4) Night-time ; Exposed surface ; Thermometric cold ; Circulating air ; Thermometric heat.	(4) Deposit of Dew on exposed cold surface.

14. How far are the four experimental methods of Mill anticipated by Bacon in his *Novum Organum*?

15. When two phenomena exhibit periodic changes, in what way may we ascertain whether one is, or is not, causally connected with the other?

16. Is an inductive inference safe, if it be obtained by comparing different instances in which the phenomenon occurs ; or, if it be obtained by comparing instances in which the phenomenon does occur with instances in other respects similar in which it does not ?

17. Prove, or disprove, inductively, or deductively, that
  - (a) Taxation and representation should coincide.
  - (b) Ague is cured by quinine.
18. Explain and illustrate the historical method of socio-logical inquiry.
19. What is the logical value of Statistics<sup>1</sup>? C. S.
20. "On comparing the accounts of live cattle and sheep annually sold in Smithfield market for some years past, it appears that there is a large increase in cattle, while the sheep are nearly stationary."—*Sir G. C. Lewis.*  
How is this explained?
21. Amidst a multitude of facts, causes, and effects, equally open to observation, how are causes to be distinguished?
  - (1) The cause (or causes) must be sought among the invariable concomitants of the effect.
  - (2) The cause cannot be anything which is present in other cases in which the given effect is not produced, unless the presence of some counteracting cause shall appear, to account for its non-production.
  - (3) The cause is generally suggested by analogy or resemblance, from cases in which the connection of a cause and an effect is better known.
  - (4) The cause is frequently indicated by a variation of degree corresponding to a variation of the degree of the effect.
  - (5) The cause will be the more likely to appear from considering as many forms of the effect as possible.
  - (6) A suspected cause may be tested by allowing it to operate in circumstances of less complication, to see whether the effect is produced.
  - (7) Where complications exist, the effects must be narrowly estimated, to determine whether the causes

<sup>1</sup> It furnishes the facts and data of reasoning in subjects admitting of statistical treatment. As to its value in Induction, Dr. Venn's view is that it, at most, enables us to say, what are the odds for or against any at present indeterminable fact belonging to this class of subjects.—See Venn's *Empirical Logic*, p. 556.

discovered or suspected account for the whole of them ; if there is any residual effect, even in the form of a modification of the effects of known causes, we must seek for its cause also.

22. How are causes discovered which are less open to observation than the effects ?

— See Thomson's *Laws of Thought*, § 117, on Anticipation.

23. When should an incomplete induction, that is to say, an examination of facts that stops short of complete enumeration, be deemed sufficient to establish a general law ?

— Point out the parts that analysis and synthesis, or induction and deduction respectively play in the process.

## CHAPTER VI

### HYPOTHESIS, EXPLANATION, TENDENCY, THEORY

**Hypothesis.** § 139.—A hypothesis (Greek, *ὑπότι*, under, and *θέσις*, a placing) is, according to Mill, “any supposition which we may make, either without actual evidence, or on evidence avowedly insufficient, in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real; under the idea that, if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be, or, at least, is likely to be true.”

Hypothesis is, from a psychological point of view, a product of the imagination: and so, is limited only by the limits of the imagination. To serve the purposes of science, however, it must be confined within certain limits, or conform to certain rules.<sup>1</sup>—And, first, as to what we

<sup>1</sup> When of such a character, it can proceed, according to Thomson, only from “an active imagination, supplied with materials by a clear understanding carefully disciplined.” It is the product of Anticipation, “a power or combination of powers granted only to a few,” and which is capable of “penetrating into the secrets of nature before the evidence is unfolded.”—*Laws of Thought*, p. 229, On Anticipation. M. Ernest Naville, in his *Logique de l’Hypothèse*, p. 203, takes a similar view of the motive power that leads to successful hypothesis; but for *Anticipation* substitutes *Genius*, defining it, after Kant, as “the talent of inventing,—a talent that can neither be taught, nor learnt, and which,” he adds, “looking to its psychological basis, is of a very composite nature. Genius supposes imagination, and judgment, or taste, but with something distinctly personal superadded. The union of all the intellectual faculties,” concludes M. Naville, “is the general condition of a discovery; genius is the spark that produces it.”

may form hypotheses about. Our object is to account for a certain effect, call it  $E$ , and towards this end there are two things to consider, namely, the cause, let us call it  $x$ , and the law according to which it acts, let us call it  $y$ . Now, to take an extreme case, we may, if we choose, make hypotheses as to both of these. We may take  $x$  to be a cause purely of our own invention, and not a *vera causa*, or cause otherwise probable ; and we may, in like manner, take  $y$  to be a law of action such as is not to be found among known laws. Such a double hypothesis would, however, leave the phenomenon as obscure as before, and would probably be found nowhere employed in the history of science. In hypothesis then, as allied to science, either the phenomenon assigned as the cause is real, but the law according to which it acts merely supposed ; or the cause is fictitious, but is supposed to produce its effects according to laws similar to those of some known class of phenomena.

Of these two kinds of hypothesis, therefore, the matter may be, respectively, represented as follows :—

- I.  $C$ , a *vera causa* ;  
 $y$ , a supposed law ;  
 $E$ , the given effect. Or,
- II.  $x$ , a supposed cause ;  
 $L$ , a law which is among known laws ;  
 $E$ , the given effect.

Now, the Hypothetical Method, and herein it offers itself for comparison with the Complete and Deductive, consists of three steps, namely,

- (1) Hypothesis ;
- (2) Ratiocination ;
- (3) Verification.

In these steps, our hypothesis is with regard to either  $x$  or  $y$  ; and our ratiocination is either from  $C$  and  $y$  or from  $x$  and  $L$  to the given effect, represented by  $E$ . And if no other cause than that we assumed  $x$  to be, or when it is a question of

the *law*, no other *law* than what we assumed *y* to be, can lead to *E*, then, in the one case, the *cause*, in the other, the *law* must be, each in its way, true.

To the first form of the hypothesis, as just outlined, belongs that of Newton, with regard to the law of planetary central force; namely, that the force varies as the inverse square of the distance. This was verified by proving that it led deductively to Kepler's laws. To the second form, belongs that of Descartes, accounting for the planetary motions as arising from the existence of vortices obeying the known laws of rotatory motion. The vortices were, of course, fictitious.

Inspection of the two forms of hypothesis given above leads to the following conclusions:—

(1) It is a condition of a hypothesis offered as a provisional explanation of any phenomenon, or what we may call a legitimate Hypothesis, that it be of such a nature as to be either provable, or disprovable, by comparison with observed facts.

(2) It must not be at least known to be untrue owing to comparisons of this kind already made.

(3) It must be universal; that is, it must serve to explain all the observed effects of the given phenomenon.

The first of these conditions is very important as a safeguard against the abuse of hypothesis. We are not debarred from assigning causes, or supposing laws, but they must be such as, if not known previously, may be capable of being eventually known and put to the test. When Newton said, "Hypothesis non fingo," he did not mean that he deprived himself of those facilities of investigation afforded by assuming in the first instance what he hoped ultimately to be able to prove. "What he meant," says M. Naville, "is that he was unwilling to offer opinions which could have no foundation in observed phenomena, unwilling to frame, in fact, mere conjectures." Without hypotheses, in short, science could never have attained to its present state; they are necessary

steps to something more certain ; and, nearly everything which is now theory was once hypothesis.

As to the third of the above conditions, most thinkers are in agreement that a hypothesis, even though it accounts for all the known phenomena, is not to be received as probably true, therefore ; since this is a condition sometimes tolerably well fulfilled by two conflicting hypotheses, and might be fulfilled by hypotheses not at all before us. But a hypothesis fulfilling this and the other conditions mentioned is entitled to a very favourable reception, if, moreover, it leads to the anticipation and prediction of other facts<sup>1</sup> which experience afterwards verifies. Thus the undulatory theory of light, which receives so much adhesion from men of science, led to the prediction that two luminous rays might meet each other in such a manner as to produce darkness, a prediction which has received experimental confirmation, and which, with others of a like kind, leads us very close to the point of accepting the theory, and the hypothesis it is grounded upon, as true.—Here we have hypothesis, ratiocination, and verification. (But see Bain's *Logic*, vol. ii., p. 131.)

Finally we must not confound Hypothesis with methods that are common in several branches of physical inquiry, and that are really inductive.—For example, in geology, our mode of reasoning from the present to the past over enormous lengths of time. This is strictly inductive, and is of the nature of circumstantial evidence, a process really of an inductive character. So, too, is the Nebular Hypothesis of Laplace inductive. This speculation is that the atmosphere of the sun originally extended to the present limits of the solar system ; from which, however, it has cooled down to its present dimensions, abandoning in the process, and owing to the greater centrifugal force generated by more rapid

<sup>1</sup> Professor Jevons draws attention to what he believes to be an ambiguity in the use of the term *fact*. "Sometimes it means what is certain, and known by the evidence of the senses, as opposed to what is known only probably, by hypothesis and inference ; at other times it is contrasted with a general law, and is equivalent to a particular instance or case."—*Elementary Lessons*, p. 275.

rotation consequent on diminishing volume, successive rings of vaporous matter which are supposed to have condensed by cooling, and to have become the planets. This, however, is inferior to most geological speculations in point of evidence. (See Mill's *Logic*, Bk. III., Chap. XIV.)

**Scientific Explanation.** § 140.—There are some terms which, especially in this concluding Book, we have from time to time employed, and which as we are approaching the close, may be conveniently considered here, each more or less in itself: namely, Fact, Explanation, Tendency, Theory.

The term, Fact, is in common use sometimes employed as equivalent to truth, and, in science, is often employed as equivalent to a particular instance as contrasted with a general law. More usually, however, we have employed the term as equivalent to phenomenon, in what relates to nature. An individual fact is said to be explained, when we have pointed out its cause; that is, when we have stated the law or laws of causation of which its production is an instance. A law or uniformity is said to be explained, when we have pointed out another law, or other laws, of which that law is itself but a result, and from which it may be deductively inferred. This explanation of laws may take place in one of three ways. In the case of the intermixture of laws, the law of the complex effect is explained by resolving it into the separate laws of the causes that contribute to it. Again, when between what seemed to be the cause, and what was supposed to be its effect, further observation detects an intermediate link, we may show that what at first appeared to be the cause was only the remote cause, that is, the cause of the cause. And, lastly, when we *subsume* one law under another; or gather up the several laws into one more general law which includes them all, we are said to explain such law or laws. The harmonizing thus, of fact and law, so as to bring them under one uniform law of causation, is what is called a Scientific Explanation.

**Tendency.**      § 141.—*Tendency* is a term of very frequent use in Science. “All laws of causation,” says Mill, “in consequence of their liability to be counteracted, require to be stated in words affirmative of tendencies only, and not of actual results.” In some sciences, indeed, there are particular words to express particular cases of such counteraction ; for example, in mechanics, a tendency to motion, but counteracted, is expressed by the one word, *pressure*, and so on. A tendency, then, is a cause which will produce an effect, unless there be an opposing or counteracting cause.

**Theory.**      § 142.—The term *Theory* is ambiguous. It is sometimes employed as synonymous with hypothesis ; thus the Atomic Theory, in chemistry, is really a hypothesis to explain the laws of chemical combination discovered by Dalton. At the same time, there underlies the term, Theory, a certain reference to width of field. In many cases, hypotheses are not immediately verifiable. They then give place to a deduction more or less complicated. The collection of consequences constitutes a system. This system must be compared with facts, and, if it be found conformable thereto, it ranks in science as a Theory. A scientific theory is thus the body of laws relating to any science, laid down accurately, and so as to exhibit them in mutual support and harmony. Thus, the theory of gravitation means all those general laws of motion and attraction constituting Newton’s system of the universe. And we employ the term in a similar meaning, when we speak of the theory of sound, the lunar theory, and so on. Sometimes, when a theory is very sweeping in its character, and appears to throw light on many fields of speculation, we call it a *Doctrine*. The modern doctrine of Conservation of Energy (see Prof. Clerk Maxwell’s *Matter and Motion*, p. 59) is unique in this respect. It amounts, in brief, to this, that all the various forms of force, energy, moving power, or work power, that we know of, are mutually convertible at different rates, and are therefore, in this sense, inter-

changeable as cause and effect. Further, if one form of energy is extinguished, another is created; *work* is said to be done in the transformation, and no force is absolutely lost. The transition to this view from the older views in physical science is interesting to consider, as showing how much our ideas have altered. Considering even but one department of physics, namely heat, books may sometimes be seen, published in the present century, that in their use of the term *caloric* show a trace of the old phlogiston theory which regarded heat as a *substance*, and which owing to the degree in which it bound many facts, respecting heat, together, facilitating the description of them, and in some measure unifying them, was looked upon for many a day as a good working hypothesis. On the ruins of this theory was built up that of heat as a *cause* or an effect; and now we have arrived, in the light of the Doctrine of Energy, at such a knowledge of forces as enables us to say that heat and work are interchangeable, and their amounts, as to equivalence, strictly calculable; and that this is true, not merely as regards molar work, but also as regards molecular, such as in chemistry, electricity, and magnetism, which show a mutual convertibility of molar into molecular work so far as our observation extends.

**Closing Words.** § 143.—It was observed in § 116 that the subject-matter of Applied Logic has no narrower limits than all that relates to Man, the Visible Universe, and Absolute Being. This reminder respecting its scope will serve also to impress upon us how very limited a portion of the field has occupied our attention in these pages. At the same time, for a manual such as the present, the restriction is most fitting; seeing that from the nature of the things that we have been treating of, and the kind of evidence we possess respecting them, our knowledge, for the range of our horizon, may have a character for precision which any attempt to navigate, though ever so little, on the “main sea deep” of transcendentalism would seriously endanger.

**Subjects for Studies.** § 144.—The following questions may be regarded partly as exercises on the matter of the present chapter, and partly to outline a course of somewhat more extended reading :—

1. Taking the “Evolution,” or any other proposed hypothesis, how should we proceed (a) to show how it satisfies the conditions of a legitimate hypothesis sufficiently to entitle it to investigation, and (b) to test its acceptance or rejection as a truth in science ?
2. Explain and illustrate the functions of the Imagination in Scientific Discovery. C. S.
3. Explain the function of hypothesis in scientific inquiry ; and give in detail the conditions of its legitimacy ; distinguishing its use from its abuse.
4. What are the requisites of a good hypothesis ?
5. What do you understand by a working hypothesis ?
6. Examine the hypotheses on which the theories of electricity are usually based. What are their respective merits and what are their defects ?
7. Whewell observes, in his *Philosophy of Discovery*, alluding to the vortices of Descartes, that “the free passage of comets through the spaces in which these vortices should have been, convinced men that these vortices did not exist.” In your opinion *should* this have convinced men that these vortices do not exist ?
8. What is an *Experimentum Crucis* ?
9. What should be the guiding principle of hypothesis ?
10. “Substance, Causality, and Reciprocity, mark three stages in the development of science.” Explain and examine this statement.
11. What do you understand by a “scientific explanation” ? State and examine Mill’s Doctrine concerning the limits of explanation.<sup>1</sup>

<sup>1</sup> We are continually diminishing the number of uniformities thought to be ultimate, by showing that they are derivative, or resolvable into

more general laws ; again, what seemed at one time to be isolated facts are taking their places each in some sequence or other ; and it becomes interesting to know whether there are any limits to the process, or whether it may proceed until all the uniform sequences in nature are resolved into one universal law—for this seems to be the ultimatum towards which complete induction, resting on a basis of observation and experiment, is tending.

In Mill's opinion, the ultimate laws of nature cannot possibly be less numerous than the distinguishable sensations, or other feelings of our nature—those distinguishable in quality, not merely in quantity or degree. For example, there must be ultimate laws of colour, not to be explained, say, from laws of heat, or odour alone, or motion alone—intrinsically unlike all others—*sui generis*. For, though colour may be due to some chemical, or mechanical action, this does not explain why a motion, or a chemical action, can produce a sensation of colour. The last link in the chain of sequences that leads to colour must still be a law of colour, not a law of motion, nor of any other phenomenon whatever ; and so even of every particular colour as compared with others,—not to speak of more than this one of our chief sensations.

The ideal limit, therefore, of the explanation of natural laws, a limit to which we are continually tending, would be to show that each distinguishing variety of our sensations, or states of consciousness, has only one sort of cause, the various modes of production relating thereto being ultimately reducible to one. When the modes of production are reduced to one, we can, so far as simplification is concerned, go no farther.



## MISCELLANEOUS QUESTIONS

1. Show that the operations of naming, and of definition and classification properly come within the scope of Logic, considered as concerned mainly with the problem of evidence.<sup>1</sup>

2. Comment on each of the following statements :—

The ultimate criterion of all logical principles is consciousness.

Contradictory opposition is best, because you can always infer by it.—*Karslake*.

The error lies in regarding everything as the *proper* province of Logic to which it is applicable.—*Whately*.

The end aimed at by Formal Logic, and attained by the observance of its precepts, is not truth but consistency.

Observation is either internal or external.

Logic deals only with discursive thought, not with intuitive.

*Neque enim in plano via sita est, sed ascendendo et descendendo ; ascendendo primo ad axiomata, descendendo ad opera.*—*Bacon*.

<sup>1</sup> Naming necessarily does ; for language is an instrument of thought, as well as a means of communicating our thoughts. Definition and classification do, as auxiliary to the process of advancing from known truths to unknown ; for these operations serve not only for preserving in a readily accessible form our evidences and the conclusions from them, but for marshalling the facts under investigation, so as to enable us to perceive more clearly what the evidence is, and to judge if it be sufficient.

In synthesis we proceed from the simple to the complex.

Enthymeme is a syllogism from antecedent presumptions.

Finding the probability of the conclusion from that of the premisses is a self-informative process : finding it from several probable arguments is a self-corroborative process.—*Bentham*.

3. Distinguish between mathematical, physical, and moral impossibility.<sup>1</sup>

4. What are universals, and how do we arrive at our notions of them ?

5. The business of Inductive Logic is said to be, "to lay down rules for (1) the discovery of the causes of all effects, and (2) the effects of all cause in nature."

How does it proceed in each case ? and compare its success in the one case with its success in the other.

6. Examine the following reasoning :—

An occurrence which always takes place when a certain condition is present, but never when that is absent, must be connected with such condition in the way of cause and effect.

The descent of a shilling and a feather through the same space in the same time, when the resistance of the air is withdrawn is such an occurrence.

∴ The withdrawal of the atmospheric resistance, and the descent of a shilling and a feather through the same space in the same time, must be connected in the way of cause and effect.

<sup>1</sup> A mathematical impossibility involves, in the mere expression of it, an absurdity and self-contradiction : it must be implied, though we may not perceive it, in the terms employed : in other words, the expression of it will be found to involve a *contradiction in terms*. It follows, that inability to accomplish anything in this sense impossible implies no limitation of power. A physical impossibility is something at variance with the existing laws of nature, and which, consequently, no being subject to these laws can surmount. A moral impossibility is that high degree of improbability which leaves no room for doubt ; and this, not because the thing implies a contradiction, or is a violation of the laws of nature, but because, from the odds against it we are rationally convinced it will never occur.

7. Give a typical illustration of a crucial experiment.<sup>1</sup>

8. Examine the following arguments :—

No one is free who is enslaved by his appetites ; a sensualist is enslaved by his appetites ; therefore a sensualist is not free.

None but Whites are civilised ; the ancient Germans were Whites ; therefore, they were civilised.

No one is rich who has not enough ; no miser has enough ; therefore no miser is rich.

He who is content with what he has is truly rich ; a covetous man is not content with what he has ; therefore, no covetous man is truly rich.

To call you an animal is to speak the truth ; to call you a lion is to call you an animal ; therefore to call you a lion is to speak the truth.

9. Is imperfect induction a form of reasoning fundamentally different from the syllogism ? If so, explain in virtue of what principle we pass in it from premisses to conclusion. If it is not, explain how it may be expressed in syllogistic form.

10. What objections have been made to defining induction as the method of discovering and proving general propositions ? Are they tenable ? Do we arrive at propositions ? Are they general ? Show that the matter is an appeal to observation, or fact.

<sup>1</sup> That experiment made by Pascal, on the Puy de Dôme in Auvergne, on the 19th of September, 1648, may be mentioned as an instance. It had been observed that neither will water rise in the common pump, nor mercury in the barometer—then, recently invented by Torricelli—above a certain height ; and the question was, which of two explanations of these facts should be accepted ; the one explanation being that nature's hypothetical “ abhorrence of a vacuum ” was incapable of acting beyond a certain limit ; the other, that the water or the mercury answered to the superincumbent column of the atmosphere of equal weight. Pascal decided the matter in favour of the second explanation, that suggested by Torricelli. He saw, that if this were the true one, the height of the barometric column must *ascend* or *descend*, according as we *descend* or *ascend* on the earth's surface. He, therefore, in the presence of a number of persons, made a decisive experiment on the Puy de Dôme ; the barometric column being found to descend continually until the top of the mountain was reached, the reading of the instrument at the top proving to differ considerably from that of a similar instrument at Clermont.

11. Distinguish, after Hamilton, *formal* from *material* induction.

Examine the statement:—"The ancients seem to have been correct more or less in their idea of the *form* of induction; but quite wrong in their idea of its *matter*; while the moderns, with Bacon at their head, are correct in the doctrine of *material* induction, but quite wrong as regards the *form*."

12. The Aristotelian and the Baconian systems of Logic are said to be opposed in their *objects*, their *evidence*, and their *methods*. Explain this statement.<sup>1</sup>

13. Examine the following statement:—"No *matter of fact* can be mathematically demonstrated, though it may be proved in such a manner as to leave no doubt on the mind."

14. What are the principal kinds of error in analogical reasoning?<sup>2</sup>

15. Examine the following analogical argument:—"The planet Mars possesses an atmosphere with clouds and mist like ours. It has also seas and polar regions like the earth. Being so similar to it, then it must be inhabited."

16. Explain what is meant by a *new truth*. How many kinds of new truths are there? What are the principal applications of the term, *Experience*?<sup>3</sup>

<sup>1</sup> As to their *objects*, the one, according to Mansel, relates principally to the subject,—the Ego, the other to the object,—the Non-Ego; as to their *evidence*, the Aristotelian laws are laws of thought as it ought to be, the Baconian laws are laws of nature as it is; as to their *methods*, in the Aristotelian Logic we proceed from the law to the facts, in the Baconian, from the facts to the law. Again, necessity, in the one case denotes what cannot but be thought; in the other, it means, *what invariably is*.

<sup>2</sup> (1) The error of inferring from resemblance in one point resemblance in another point, though there is not only no evidence to connect the two points in the way of causation, but the *evidence tends positively to disconnect them*. This is, more properly, the Fallacy of False Analogy. (2) The error of pressing the analogy too far.

<sup>3</sup> A new truth is something neither expressly, nor virtually asserted before. No new truth, as thus defined, can be elicited by any process of reasoning. Taken in the widest sense, there are two kinds of new truths. First, such as before they were discovered, though we might deem them probable, were not implied in anything we previously knew. We gain such new truths by observation and testimony, and such truths are called *information*. Second, such truths as were implied in what we already know. The communication of truths of this kind is called *instruction*. As to *Experience*, in a strict sense the term is properly applicable but to what has occurred within a person's own

17. Illustrate the tendency towards expansion, or contraction in the denotation of terms by changes which the following have undergone :—experience, science, mind, law, logic. C.S.

18. Comment on the following statements :—

Intension and extension are complementary and inseparable.

The whole of extension is constituted by objects, the whole of comprehension by attributes.

Formal Logic can recognize no difference between true and false propositions.

It is an awkwardness in nomenclature, that the law of contradiction explains but contrary opposition, and that it is the law of excluded middle that lays down the principle of contradictory opposition.

Discovery without proof is conjecture ; an element of proof is needed to constitute inference, and indeed to constitute discovery.

Causation is no mere phenomenal sequence.

Cause and effect are not divided by time, in the sense of duration, or lapse, or interspace : they are separated in time by an ideal line which we draw across the indivisible process.

There are cases in which, with acknowledged propriety, we generalize from a single instance to a multitude of instances.

When we argue from experience in the past to experience in the future, we sometimes in thoughtlessness employ the incomplete *inductio per similem enumerationem*.

knowledge : more frequently, however, it denotes that judgment which is derived from experience—in this, the primary sense—by reasoning from it in combination with other data. In the first sense it relates to the past ; in the second it can be applied to the future. There are, again, two different applications of the word, viz., as meaning, sometimes, *our own* personal experience, sometimes general experience. The phrase, “contrary to experience,” is sometimes employed to mean, “contrary to our own experience,” say, with reference to a thing we have never seen. In a stricter sense, however, that only is *contrary* to our experience which we know by experience not to be true : such a thing cannot be established by testimony. Experience, from a metaphysical point of view, is employed to denote all accidental knowledge, all that is not part or parcel of the thinking act itself. Finally, some writers restrict the term to the domain of science,

A hypothesis is refuted by refuting its consequences, not proved by establishing them, though it grows in probability as its consequences agree with fact. Induction is really the inverse process of deduction.

Necessary truths are derived from our own thoughts ; experimental truths are derived from our observation of things about us.—*Whewell*.

19. Under what circumstances, according to Whately, does the observation hold good as to the difficulty of proving a negative ?<sup>1</sup>

20. What simplifications were introduced into Logic by Whately and Hamilton respectively ?<sup>2</sup>

21. Reduce the following syllogism in extension to a syllogism in comprehension :<sup>3</sup>—

He who has intelligence and free agency is responsible ;

Man has intelligence and free agency.

∴ Man is responsible.

22. Kant according to Mansel, has done more for logical

<sup>1</sup> When the proposition in question is contrasted with one which has really a term the less distributed, or a term of less extensive sense ; e.g., it is easier to prove that a man has proposed wise measures than that he has never proposed an unwise measure. The one would be to prove that some of his measures are wise : the other, that *all* of his measures are wise. But it will often happen that negative propositions can be much more easily established than affirmatives in the same subject ; e.g. “The cause of animal heat is respiration” is more easily established than a statement of what actually *is* the cause of animal heat.

<sup>2</sup> Whately showed that, by means of contraposition, *reductio ad impossibile* might be dispensed with, and all the moods reduced ostensively. Hamilton discarded the fourth figure, got rid of all but simple conversion, of all reductions, ostensive and *per impossibile*, and abrogated all the special rules of syllogism. See Mill's *Logic*, footnote, vol. i., p. 197.

<sup>3</sup> Applying Mill's universal formula of reasoning, viz., Attribute A is a mark of attribute B ; the given object has the mark A ; therefore, the given object has the mark B, we reduce the above syllogism to the following form :—Responsibility is an attribute of all who have intelligence and free agency ; Intelligence and free agency are attributes of man ; ∴ Responsibility is an attribute of man.

science than any philosopher since Aristotle. In what respects?<sup>1</sup>

23. Explain the terms, Empirical Logic,<sup>2</sup> Empirical Law, Empirical Method.

24. Explain the statement that from the philosophy of Kant there spring, "the three most important doctrines of logical theory."<sup>3</sup>

25. Explain the following terms:—Substitution of similars<sup>4</sup>; Principle of sufficient reason; Uniformities of nature; Mixed mode<sup>5</sup>; Specific difference. C. S.

26. What are the characteristics of an empirical generalization, as distinct from a law of nature<sup>6</sup>? C. S.

<sup>1</sup> In securing to it a distinct field of inquiry by his definition of it as, the science of the necessary laws of thought; in banishing to a distinct region—that of Applied Logic—many speculations that do not properly belong to formal thought; and further, by his demonstration that a universal material criterion of truth is not only impossible but self-contradictory. See Mansel's Aldrich (*Artis Logicae Rudimentae*), Introduction, p. xlvi.

<sup>2</sup> This term has been applied by Dr. Venn to Inductive Logic, to emphasize his belief that ultimate objective certainty is unattainable by human reason.

<sup>3</sup> These are (1) that which regards Logic as purely formal in character, or the science of the laws of thought; (2) that which discusses logical first principles in connection with the question of the origin of knowledge; and (3) that which regards Pure Logic and all that relates to the theory of knowledge as making up a whole of metaphysical science.—Article, *Logic*, in *Enc. Britannica*, 9th ed.

<sup>4</sup> This phrase denotes the power of mutual replacement existing between any two objects which are to a sufficient degree like or equivalent. See Jevons' *Principles of Science*, vol. i., p. 21.

<sup>5</sup> Complex Ideas are, according to Locke, made by the mind out of simple ones. They are either (1) modes, (2) substances, or (3) relations. By *modes*, Locke understands such complex ideas as contain not in them the supposition of substances by themselves, but are considered as depending on, or affecting substances; such are the ideas signified by the words, triangle, gratitude, and the like. Of these modes there are two sorts, (1) simple modes, consisting of but variations or combinations of the same simple idea, as a dozen, or, a score; and Mixed Modes, consisting of ideas compounded of simple ideas of several kinds put together to make a complex one; e.g., beauty, and the like complex ideas.

<sup>6</sup> See, Empirical Law, INDEX. Such a law follows as the result of an *inductio per simplicem enumerationem*. It can be received as true only within the limits of the *time*, the *place*, and the *circumstances*, from which it was derived, or in which the observations have been made. See Mill's *Logic*, Bk. III., chap. xvi. As to *adjacent cases*, if an empirical

27. Give an example of verification by means of a residual phenomenon.
28. Show the bearing on causation of supposing the Universe to be a conservative system.
29. Contrast the conception of a working hypothesis as found in other writers with the conception of it defined in the following paragraph relative to Darwin's *Origin of Species* :—

“That which we were looking for, and could not find, was a hypothesis respecting the origin of known organic forms which assumed the operation of no causes but such as could be proved to be actually at work. We wanted not to pin our faith to that or any other speculation, but to get hold of clear and definite conceptions which could be brought face to face with facts and have their validity tested. The *Origin* provided us with the working hypothesis we sought.”

—T. H. Huxley.

law be extended beyond local limits, the cases to which it is thus extended must be such as are presumably within the influence of the same individual agents.

## NOTES A AND B

### NOTE A

#### EQUATIONAL LOGIC AND HIGHER LOGICAL ANALYSIS

How far it might be possible to extend to Logic the symbolical methods of Algebra is a question that must have often occurred to earlier generations of logicians. Only within recent memory, however, has it been taken up by any writers competent to grapple with it. The first to make a classical attempt to exhibit the science of Logic in the form of a Calculus was the late Dr. Boole of Cork, in his two works entitled, respectively, "The Mathematical Analysis of Logic," and "An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities," an excellent account of the scheme embodied in which will be found in Bain's *Logic*, vol. i., pp. 190-207. These two works, the second of which is of a very elaborate character, have acquired just celebrity, and continue, still, to have an interest for many readers.

Of the various developments by more recent writers of Higher Formal Logic, or Logical Analysis, two may be briefly noticed here, the processes in which are distinguished by a particular simplicity and neatness, and such a freedom from an overcrowding of new and arbitrary notation symbols as must prove a great recommendation to persons about to enter on the study of the application to Logic of algebraical or higher analysis. These are the *Equational Logic* of the late Professor Stanley Jevons, and the beautiful treatise on ratio-cinative analytics which, under the title of "A Generalization of Logical Processes in their Application to Complex Propositions," forms Part IV. of Mr. Keynes' *Formal Logic*.

In the *Equational Logic* of Professor Jevons, propositions are expressed in equations, and intricate reasoning is analysed in such a way as to be intelligible at a glance; thus affording facilities for checking and for interpretation which save a considerable amount of mental labour.

The apparatus of symbols and conventions for this purpose makes but a moderate demand on the memory, and can be readily understood. The *capital letters* are used to denote the *qualities* forming the *intension* of terms. The corresponding small denote the negation, or *absence* of

those qualities. The mathematical sign of equality denotes the identity of meaning of the terms it connects. For example  $A=B$ , means that the qualities denoted by A are identical with the qualities denoted by B. When the letters are placed together, thus,  $AB$ , the combinational symbol denotes a union of the qualities A and B. Such a symbol is employed generally to express the predicate of a proposition. For example, *Man* is an animal, if A be put for the intensive meaning of *man*, and B for that of *animal*, would be expressed thus,

$$A = AB.$$

This equation may be literally read, *Man* is a *man-animal*, and so in all like cases. The algebraic sign of division, changed from the horizontal to the vertical position, means the qualities of one or other of the extremes united by it, or of both together. For example,  $A \cdot B$  means the qualities of A, or those of B, or those of both A and B,—of one and of the other.

These symbols combine according to laws of operation which partake of the mathematical in character. The following equations exhibit the laws of combination briefly and clearly :—

$$AB = BA \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

$$AA = A \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

$$A \cdot A = A \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

$$A(B \cdot C) = AB \cdot AC \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (4)$$

$$B \cdot C = C \cdot B \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

The symbol O denotes a combination of qualities that cannot possibly coexist. With this explanation, the Laws of Thought may be written down in the notation here employed, and at once read off. Equation (6) of the following expresses the Law of Identity, (7) that of Excluded Middle, and (8) that of Contradiction.

$$A = A \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

$$A = AB \cdot Ab \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

$$Aa = O \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

Taking as the premisses of a syllogism in Barbara the proposition, All men are mortal, All poets are men; and writing B for the intensive meaning of mortal, A for men, and C for poets, we have the following expressions :—

$$A = AB;$$

$$C = CA.$$

To draw the conclusion from these, we make use of what is called, *The Rule of Substitution*; that is, For any term in one of the premisses we substitute its equivalent as given in the other, and so we have an equation representing the conclusion. Thus, for A in  $C = CA$ , substitute its equivalent  $AB$ , and we have for the conclusion,

$$C = CAB$$

which, if we choose, can, in ordinary language, be read as expressing a relation between C and B alone.

If A be put for the intensive meaning of wearisome, and C for poets, then, No poets are wearisome, may be written

$$C = Ca \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

If we wish to find the contrapositive of this proposition, we proceed as follows :—

$$\begin{aligned}
 \text{By equation (7), } A &= AC \cdot Ac; \\
 \text{But } C &= Ca. \\
 \therefore A &= ACa \cdot Ac. \\
 \text{But, } ACa &= AAC; \\
 \text{And, by (8), } Aa &= 0, \\
 \therefore A &= Ac \quad \dots \quad (10)
 \end{aligned}$$

The relations (9) and (10) often facilitate the drawing of a conclusion. Take, for example, Cesare, and let the premisses, E and A be represented respectively by,  $C=CB$ , and  $A=Ab$ . To draw the conclusion, E, take the contrapositive of  $A=Ab$ , by (9) and (10), and we have,  $B=Ba$ ; and by substitution in the equation,  $C=CB$ , we have,  $C=CBa$ .

We have been dealing, so far, with but universal affirmative or universal negative propositions. When particulars are concerned, that is to say, the forms I and O, a difficulty connected with the ambiguity of the word *Some* presents itself, which, however, in cases not very complicated may be easily got the better of. Some A is B, and, Some A is not B, propositions exemplifying these forms may be, respectively, thus expressed :—

$$\begin{aligned}
 SA &= SAB; \\
 SA &= SAb.
 \end{aligned}$$

Some of the moods involving I and O can be expressed equationally without the use of the symbol S. For example, Darapti, of which the major premiss may be written,  $A=AB$ , and the minor premiss,  $A=AC$ ; whence,  $AB=AC$ , or,  $BA=CA$ . When a particular, however, occurs in the premisses, the S must be employed : for example, in Bokardo. Here let the major premiss be  $SA=SAb$ , and the minor,  $A=AC$ , and we have for conclusion,  $SAC=SACb$ .

For combinations of propositions in disjunctives, and such, we require an extension of the Law of Contradiction. The principle employed is easy to understand.  $AB$  negatives the form  $Ab$  and also,  $aB$ . So, in like manner,  $A=ABC$ , negatives  $AaC$ ,  $ABC$ , and  $Abc$ . But,  $Bc=aBc$ , negatives only  $Abc$ . In reversing the process, then, any one negative combination negatives but one corresponding affirmative : for example,  $ABC$  negatives but  $ABC$ . Moreover, such a combination as  $AB$  leaves uncontradicted,  $ABC$ ,  $abc$ , and so on.

We are now in a position to express equationally such a disjunctive syllogism as the following :—Neither A nor B nor C is X; All Y is either A or B or C; Therefore, Y is not X. Thus :—

$$\begin{aligned}
 abc \ X &= X; \\
 Y &= AY \cdot BY \cdot CY; \\
 \therefore YX &= abc \ XAY \cdot abc \ XBY \cdot abc \ XCY; \\
 \text{or, by (8), } YX &= 0 \cdot 0 \cdot 0 = \bar{Y}y = Xx.
 \end{aligned}$$

As an example of a more complex kind, let us take, next, De Morgan's problem, Every A is one only of the two, B or C; D is both B and C, except when B is E, and then it is neither;  $\therefore$  No A is D. Expressing the premisses in the form employed by Professor Croom Robertson (Jevons' *Studies in Logic*, p. 203), we have,

$$\begin{aligned}
 A &= ABC \cdot\cdot AbC; \\
 D &= DeBC \cdot\cdot DEbc. \\
 \therefore AD &= DeBC (ABC \cdot\cdot AbC) \cdot\cdot DEbc (ABC \cdot\cdot AbC), \\
 &= DEbc (ABr \cdot\cdot AbC), \\
 &= DEbc (AbC), \\
 &= 0 = Ad = aD;
 \end{aligned}$$

the quantities at the left side being cancelled by successive applications of the principle of Contradiction.

An extension of this method may, by a slightly modified use of the symbols, be made to quantitative statements; groups of qualities in brackets being employed to express the number of objects possessing those qualities, and + and - being employed to unite them.

The sign  $\cdot\cdot$  employed in the preceding illustrations, and meaning both *or* and *and*, Jevons calls *the sign of unexclusive alternation*.

Such are the fundamental principles and the elementary processes of *Equational Logic*. The student for whom the subject has an interest will find himself able to pursue it further, and to an adequate measure of completeness, in Jevons' *Studies in Deductive Logic* and in other recent works.

An exposition of Mr. Keynes' Method, which differs from that of Jevons in not being of an equational character, and therefore which, while affording algebraical means for concentrated analytical expression, keeps closer to the ordinary understandings in Logic with regard both to the intension and the extension of terms, will be found in his *Formal Logic*, 3rd edition, pp. 378 to 460.

## NOTE B

### HISTORY OF LOGIC

ARGUMENTATION is found among all peoples, rude or civilized, its development being chiefly a question of difference of degree. The state, however, of Greece in the fifth century B.C., the peculiar character of its governments and institutions, and the talents of its people, conspired to give birth to, besides other branches of knowledge, the subject that has occupied us so much in these pages: Greece was the cradle of logical science.

More than two thousand years have passed away since the days of Zeno of Elea (fl. 460-440 B.C.), a pupil of Parmenides, and the first philosopher who, according to Aristotle, taught in Greece the regular dialectical science under which were included the principles of Logic. Many questions were then agitated by the Pythagoreans that still occupy the philosophical world, the nature of things in themselves, the principle of change, and its source, the principle of harmony as seen in man and nature. Zeno wrote a treatise, and delivered lectures on these recondite matters; and his appearance in Greece was epoch-making as an expounder of logical method, illustrating his theory by his practice of basing his reasonings upon certain general principles.

The next really important name in the history of Logic is that of Socrates. Socrates obtained his elementary knowledge of Logic in the school of Parmenides and Zeno, and from these philosophers learnt the importance, if we would get at the truth, of viewing facts both positively and negatively, and of hearing, as it is said, both sides of the argument,—one among many of those general principles on which the entire fabric of the Socratic method, his peculiar mode of reasoning, was based. According to Xenophon, Socrates was one of the first to distinguish things into genera or families, so as to teach their nature with the more ease; and Aristotle claims for him the inductive method of proof and the general definition of ideas.

Briefly noticing, of the writings of Plato (429-347 B.C.), his inquiry into the origin of our knowledge, and the nature of truth, and his disquisition on the functions of language as an instrument of thought, we consign his name to the history of philosophy, in order, the sooner to reach Aristotle.

And in Aristotle (380 B.C.) we come to a name almost synonymous with logical science, one of the most surprising men for talent, reputation, and influence, that the world has seen. Aristotle can be truly

known but in his own pages ; and, even after all these centuries, he has to the modern reader an attractive freshness about him on many topics, and a general fertility and penetrativeness of thought that is very remarkable. What we are going to say about his logical writings, therefore, must be understood as merely by way of suggestion to make acquaintance with himself, whether in the language in which he wrote or in a translation. The works of Aristotle treating of Logic are the following :—The Book of the Categories ; the Book of Interpretation ; the First Analytics ; the Last Analytics ; the Topics ; and the Book of Sophisms. It has been usual to publish the whole of these treatises as one collection under the name of Aristotle's *Organon*. A complete translation of them in French has been executed with an unexcelled grasp of his author by J. B. Saint-Hilaire.

The Book of the Categories is preceded by some general and explanatory remarks called by the schoolmen, the *Ante-prædicamenta*, touching on words and on predication, including the theory of inhesion. Passing over the Categories, which elsewhere in the present work have received sufficient attention, and the *Post-prædicamenta*, namely, the four kinds of terms expressive of opposition,—relative, privative, of contrariety, and of contradiction, we come to the treatise termed, Interpretation. This treatise relates to language, to words as parts of speech, to words as signs of mental things, and to writing as consisting of the signs of words. Both the signs of ideas, and the signs of words are variable among mankind, but the mind in itself, and its belongings, remain the same.

The two treatises called the Analytics develop the doctrines and the principles of the Syllogism. The Categories, the Predicables—treated in the Topics—and the Interpretation, are not claimed by Aristotle as original in matter ; but he claims the syllogistic system as of his own entire invention and development. The First, or Prior Analytics embrace four leading topics, the conversion of propositions, the structure of syllogisms in the different figures and moods, the invention of the middle term, and the resolution of syllogisms. The Last, or Posterior Analytics treat of the matter of syllogisms—*true, false, probable, or improbable*. The Book of Sophisms deals with Fallacies, thirteen kinds of them, six relating to diction and language, and seven to matter. Such are the leading features of Aristotle's writings.

The Peripatetic School, represented by Theophrastus of Lesbos, took up Logic where Aristotle left it, deviating from him, however, in certain respects, and treating the matter of logical research as though it were independent of Logic as a science. The distinct recognition of the hypothetical and disjunctive proposition and syllogism belong to this school.

Next in order, follow the Sceptics, the Epicureans, and the Stoics. We pass on to the latter. These occupied themselves chiefly with speculations on the foundation of truth or science. They sought to frame categories of their own of a more accurate and comprehensive character than those of Aristotle—that which forms the groundwork of things, that which has qualities or attributes, that which has a general relation, and that which has a particular relation to some other thing. They conceived that no investigations into logical forms could prove of utility other than those based upon correct notions of the fundamental principles of scientific truth. Among the later Stoics, Carneades distinguished himself as an expounder of Logic. He called in question the

criterion of truth, but insisted on our power to lay hold on the truth of things by a system of probabilities. Philo, of the same school, wrote largely of the nature of truth and the rules of evidence.

From the commencement of the Christian Era till the time of Charlemagne (b., A.D. 742, d., A.D. 814), there appear a series of writers that may be ranged in three divisions—The Fathers of the Church; the Alexandrian School, or Later Platonists; and Miscellaneous Authors. Of the Alexandrian School, the logical writings of Plotinus (A.D. 206), St. Clement (A.D. 218) and Porphyry (A.D. 223) are remarkable. The writings of the first of these are mystical in character; those of the second treat largely of the logical connections subsisting between faith and science; and among those of the third—one of the most subtle logicians of his age—is a work on the *Predicables* of Aristotle. Among the Miscellaneous Authors, Sextus Empiricus is a voluminous writer, of sceptical views; and Boetius—a celebrated name—translated the *Categories* of Aristotle into Latin.

From the eleventh to the thirteenth century several distinguished Arabian and Jewish writers on Logic appear. Among the Arabians, Alfarabi is one of the most famous. He has left a tract upon the sciences which forms a sort of dictionary, or methodical classification of various branches of human knowledge. Another is Avicenna, who follows Aristotle, though not slavishly. And last to be mentioned is Averroes of Cordova, who flourished in the twelfth century.

The Scholastics stretch from the ninth to the middle of the fourteenth century. The idea on which the Logic of the schoolmen rests is an idea of theological unity. This is its most striking feature. Another remarkable feature of it is the controversy on the nature of universal and particular notions or ideas. Hence the distinction of logical doctrinaires into Nominalists, Realists, and those who attempted to steer a middle course, or Conceptualists. A few of the more important names are:—John Scotus Erigena (A.D. 900), Lanfranc (A.D. 1036), Roscellinus (A.D. 1089), Lombard (A.D. 1170), John of Salisbury (A.D. 1180), St. Thomas Aquinas (A.D. 1274), Duns Scotus (A.D. 1308), William of Occam (A.D. 1320), and John de Gerson (A.D. 1363).

The period from the middle of the fourteenth century to the date of the publication of Bacon's *Novum Organum*, 1620, is one of transition, distinguished by various impulses tending to widen the boundaries of knowledge,—impulses which the invention of printing diffused and strengthened. Peter Ramus (1515–1572) is perhaps one of the most conspicuous of the philosophical logicians of this period.

The *Novum Organum* is epoch-making for the impetus given by it to Inductive Logic. As the work itself is easily accessible, either in Bacon's collected writings, or in Dr. Fowler's excellent edition, it is unnecessary to say more about its contents than we have had occasion to do already. The keynote of it is struck in the opening, or first aphorism—"Man, the Servant and Interpreter of Nature, can do and understand as much as he has observed concerning the order of nature in outward things or in the mind; more, he can neither know nor do." Among the processes included by Bacon under the art of discovery syllogism is not included.

Descartes (1596–1650) must be regarded as only not forgotten, in the line or two we can devote to him. His logical method is well-known; and his *Rules for the Direction of the Understanding* is a treatise much

commended by Hallam. Acquaintance must be made with him in his own writings. The *Port-Royal Logic*, published in 1662, and written, it is considered, by Arnauld and Nicole, forms one of the most important modern contributions to logical science. It is, in the main, an exposition of Descartes' views.

The next name of note is John Locke (1632-1704). Locke's influence has operated in two directions. It may be safely affirmed that his *Essay on the Human Understanding* (1690) has given birth to a more diversified series of logical systems and speculations than any other work since the days of Aristotle. Of syllogism Locke thought little. His logical method is really the inductive method that Bacon applied to the study of physical science. His fundamental position is that the philosophy of mind is a science of facts revealed to us by consciousness. With Locke as with Mill, Logic is the basis of psychological empiricism.

And after Locke, Kant (1724-1804). Kant's *Criticism of Pure Reason* appeared in 1781, and has imparted to the logical philosophy of Germany a peculiar and interesting character. His views on Logic, however, are intimately connected with his views on metaphysical science. On the question of the origin of knowledge, he maintains that experience is not the only source of knowledge. We have certain notions altogether independent of the senses, and which are the product of the understanding itself. Of the many systems which have sprung up in Germany through the teachings of Kant, Logic, as united with a more general theory of knowledge, is represented by Lotze. Hegel, in his *Wissenschaft der Logik* (1816), denies that Logic is expressive merely of the *forms* of thought: thought constitutes its very essence. "There is thought in its immediate existence: thought is communicated; and thought is forming a full and complete conception of its own self." From this point of view Logic is identifiable with metaphysics.

This brings us down to the period of Whately and John Stuart Mill, and their contemporaries: and here we must conclude.

We have proceeded far enough to show that the *history* of Logic is of very great interest. The interest, however, as will be seen from our survey, will be found to centre in a comparatively small number of justly celebrated names. Of the seven or eight hundred authors the titles of whose treatises have been laboriously sought out by the late Professor Blakey of Belfast, and which occupy thirty-two pages of his *Historical Sketch of Logic*, only a small number are known to any but the curious. But this small number, though we of a later day may not be always wholly in accord with them, are secure in unending fame. These are they who with such light as was vouchsafed to them have laboured in severity and justness of thought to reach *The True*.

## **INDEX, REGISTER, AND GLOSSARY**



## INDEX, REGISTER, AND GLOSSARY

*The numbers refer to the pages.*

### A

ABELARD, PETER, b. 1079 at Palais, near Nantes in Brittany, d. 1142. To him are due the earliest scholastic writings. His principal work is entitled, *Dialectica*, 26.  
*Abscissio infiniti*, 138.  
Abstraction, 25.  
Academy, The New, was instituted by Archesilaus, b. at Pitani, B.C. 316.  
Accent, fallacy of, 204.  
Accident, 205.  
Accidental definition, 85.  
Added determinants, 125.  
*A dicto secundum quid ad dictum simpliciter*, 205.  
Adjacent cases, cases in which, presumably, the necessary collocation of causes exists, 251.  
Aequivalence, 120.  
Affections of propositions, 30.  
Affirmative propositions, 29.  
Agreement, method of, 280.  
ALBERTUS MAGNUS, a Dominican Friar of Cologne, b. in Suabia, 1205, d. 1280. He and his pupil Aquinas have been called the "Plato and Aristotle of Scholasticism." Albertus wrote commentaries on the *Organon*, and treatises on Universals, and on Division.  
Alexander of Aphrodisius, the earliest Greek commentator on Aristotle.

Ambiguity of terms, 204.  
Amphibology, 204.  
Analogy, 249.  
Analogy and imperfect induction are classed by Kant as syllogisms of the judgment.  
Analogy, fallacy of false, 253.  
Analysis, method of, 218.  
Analytic, the nearest approach to the term, Logic, in the writings of Aristotle.  
Analytical proposition, 72.  
Analytic, the New, 42.  
Anselm, St., b. at Aost, 1033, d. at Canterbury, 1109, was a most distinguished logician. His method is exemplified in "The Monologium," in which he attempts to show how an ignorant man by the power of thought alone, might construct for himself a system of true and rational cognition.  
Antecedent, 164.  
Anticipation, 297.  
Antinomy (Greek *ἀντί*, against, and *νόμος*, a law), a term employed by Kant to denote a conflict of laws that presents itself to speculative reason, in its endeavour to conceive nature as a complex whole. Kant sets forth each antinomy in the shape of a thesis and an antithesis. The theses, four in number, are:—The world is limited in space and time; it consists of parts always com-

posite; it includes no causality but that of natural law; it implies the existence of no necessary being. Opposed to each thesis is an antithesis which is no less dogmatic, made, or suggested, by empirical thinking. Kant intends by these to show that we act erroneously in seeking for a speculative knowledge which can be given in no experience. The antinomies indicate that reason has here gone beyond its powers. They are, as it were, placed as sentinels, to warn the philosophic inquirer that such modes of proceeding as give rise to them are erroneous.

Antisthenes, b. about 422 b.c., was a pupil of the sophist Gorgias, and was afterwards a student under Socrates. He was founder of the sect of the Cynics, and pointed out the extreme importance of attending to the Art of Definition.

*A posteriori*, 14.

Applied Logic, 3.—The term is objected to by Kant.

Apprehension, simple, 26.

*A priori*, 14.

AQUINAS, ST. THOMAS, called "The Angelic Doctor," was born at Rocca Sicea, near Naples, in 1227, and died on his journey to the General Council of Lyons, in 1274. He has left commentaries on various parts of Aristotle, including the *Hermeneia*, and the *Posterior Analytics*, also, *opuscula on Demonstration, on Modals, on Accident, and on the nature of Syllogism*.

Archytas, the *Categories* attributed to, are now considered forgeries of a later period.

Aristotle, 9, 50, 243.

Arnauld, Antoine, *le Grand*, of Port-Royal-des-Champs, b. 1589, d. 1674.

Art, 3.

Arbor Porphyriana, 95.

Argument, 129.

Aristotelian and Baconian Logic, 310.

Ases' Bridge in Logic, according to the schoolmen, the discovery of the middle term, or medium of comparison.

Attributes, 45.

Averages, 280.

Averroes, an Arabian commentator on the works of Aristotle. The distinction he draws between first and second *intuitions*, was probably derived from the Arabian, Avicenna, 321.

Axiom, 236.

Axiom, in Bacon, a law of nature arrived at by induction.

## B

Bacon, Francis, Viscount Verulam, b. 1561, d. 1626, 278, 290.

Bacon, Roger, b. at Ilchester, 1214, d. 1294. He observed that the logical categories contained no real knowledge of objects, and that the art of observation necessitated experimental processes. His chief work is entitled the "Opus Majus."

Bain, Professor Alexander, of Aberdeen University, b. 1818, d. 1903.—Writer on Metaphysics and on Logic, 13.

Begging the question, 206.

Belief, 118.

Bentham, George, 69.

Blemmidas, Nicephoras, wrote in the 13th century an *Epitome Logica*, which contains the earliest specimens of logical mnemonics.

Boetius, b. 470, d. 526, called "The Last of the Ancients," wrote two commentaries on the *Isagoge* of Porphyry, one on the *Categories*, two on interpretation, and translated other parts of the *Organon*. He wrote also original treatises on the *Categories*, on *Hypothetical Syllogisms*, on *Division*, on *Definition*, and on *Topical Differences*.

J/

Boole, Dr., 315.

Brown, Dr. Thomas, his theory of the syllogism, 180.

Buffier, Père, b. 1661, d. 1737. Jesuit. Wrote the *Traité des Vérités Premières*. He settled the question as to the mental substrata on which all thought must rest, as being those first truths that result from common sense, 237.

C

Canons, or Rules of Syllogism, 130.

Canons of Induction, 280.

Categories of Aristotle, 50.

Categories of the Understanding (Kant). These, namely, Quantity, Quality, Relation. Modality, are not the results of experience: they are solely the universal and necessary laws of the understanding.

Cause, 259.

Certainty, 186, 234.

Certitude, 186, 234.

Champeaux, William of, d. 1120. Chance, "cause unseen to human understanding," 187.

*Circulus in probando*, 207.

Citium, Zeno of, fl. b.c. 362, was the founder of the Stoic sect.

Class, 273.

Classification, 272.

Coexistence, 54.

Cognition, or notion, 2, a clear presentation. *Clear* is here contrasted with obscure, 328.

Colligation of facts, 248, 280.

Compiègne, Roscellinus of, flourished about 1080.

Common sense, judgment unaided by art or any system of rules.—*Reid*.

Complex propositions, 63, syllogisms, 162.

Composition of causes, 261.

Composition, fallacy of, 204.

Comprehension of terms, 47.

Comprehension, syllogism in, 312.

Concept, 25.

Conceptualism, 25.

Concomitant variations, method of, 287.

Concrete terms, 43.

Conditional propositions, 63.

Connotation of terms, 45.

Consciousness, the immediate knowledge the mind has of its own feelings.—*Bain*. Sir W. Hamilton has put forward "the testimony of consciousness" as the ultimate and infallible criterion of certainty.

Consequence, ground of sequence.

Consequent, in hypotheticals, 164.

Consequent, in induction, effect.

Consequent, fallacy of the, 208.

Conservation of energy, 302.

Consistency, or implication, 116.

Contingent, that which may or may not happen, 118.

Continuity, law of; *natura non agit per saltum*, or nothing can pass from one state to another, without passing through all the intermediate states.

Contradiction, law of, 112.

Contradiction in terms, or *contradiccio in adiecto* applied to a proposition, implies one that predicates of a subject some attribute contradicted by the connotation of that subject—See Venn's *Empirical Logic*, p. 297.

Contradictory propositions, 112; imperfect negation of so-called, 223.

Contraposition, 121.

Contrary propositions, 116.

Conversion, 118.

Copula, 61.

Criterion of truth, 231.

Cross divisions, 92.

Crucial experiment, 309.

Cumulative argument; see Evidence, circumstantial, 196.

Curves, Method of, Graphical method, a method of determining averages, 280.

D

DAMASCUS, JOHN OF, in the early part of the 8th century, made

a brief analysis of the *Isagogie* of Porphyry, and of the *Categories*, and was one of the first who applied Logic to Theology. *Deduction*, 104.

Deductive method, 290.

Definition, 85.

De Morgan, Professor, b. 1806, at Madura in Madras, d. 1871, 182.

Denotation, 45.

Derivative laws, 256.

Descartes, René, b. 1596, d. 1650, 217.

Desynonymization of terms, a process so called by Coleridge, and called *differentiation*, by Herbert Spencer, consisting in specializing one of a pair of synonymous terms, one to one meaning and one to another. *Determination*, 94.

Development of a term, the, consists in proceeding by a dichotomy to an exhaustive division of a class, 93.

Dianoetic, 10.

Diagrams, syllogistic, 157.

Dialectic (διαλεκτική τέχνη), the original name of Logic, 9. The word was probably invented by Plato; though it was afterwards applied to the works of earlier philosophers, notably to those of Zeno the Eleatic, to whom, according to Aristotle, the first of the science itself is due.

Dichotomy, 93.

*Dictum de omni et nullo*, 130.

Differentia, 80.

Difference, Method of, 283.

Differentiation of terms, desynonymization.

Discourse, reasoning, 126.

Distinct knowledge. Knowledge is, according to Leibnitz, either *clear* or *obscure*; clear knowledge is either *distinct* or *confused*. Knowledge is clear when we are able to recognize the thing again, and distinguish it from other things. Clear knowledge is *distinct*, when we are able to recognize the parts and qualities of the thing known, 1.

Distribution of terms, 31.

Division, 91.

**E**

Eclectic, the Neo-Platonic, or Alexandrian School of Philosophy A.D. 205 to A.D. 483. The aim of this school was to unite the Greek philosophy and the oriental dogmas into one grand whole, 321.

Efficient cause, 258.

Elenchus, 205.

Empirical Logic, 313.

Empirical laws, 256, 313.

Empirical method, 289.

Enthymeme, 152.

Episylllogism, 153.

Error in an argument, may be met by assailing one of the premisses with a contrary instance, by pointing out a formal defect, by proving by an elenchus the contradictory of the conclusion, or by a reduction to an impossibility.

Eristic, or sophistic method, arose from the pre-Socratic schools, and at a later period was adopted to a considerable extent by Euclid of Megara, and his successors.

Essence. Locke distinguishes two kinds, the *real* and the *nominal*. The real is the constitution of an individual; that which makes it to be what it is. The nominal essence is the constitution of a species; being the combination of attributes which is signified by the name of the species.

Euclides, the Megarean, B.C. 519, first gave to the *reductio ad absurdum* its formal structure, and exemplified and defended its use.

Eudemus and the Stoics developed the hypothetical syllogism, invented, it is said, by Theophrastus.

Euler's diagrams, 159.

Excluded middle, 112.  
 Exclusion, 291.  
*Exposita*, 118.  
 Experience, 265.  
 Experiment, 266.  
*Experimentum crucis*, 292.  
 Explanation, 301.  
 Evidence, 195. That which enables the mind to see truth; or the grounds which make evident, and afford a foundation for belief.  
 Evolution, Hypothesis of; Origin of species. According to Darwin's views, plants and animals become modified in the course of generations. Some forms perish, while owing to what he calls *natural selection*, the *fittest* always survive.

## F

Fact, 300, 301.  
 Fallacies, 35, 203.  
 Figures, 34, 134.  
 Final causes, 258.  
 Form, 4. Bacon explains *forms* to mean nothing else than those laws and manifestations of the pure act which order and constitute any simple nature, as heat, light, etc.; and again, that the *form* of a thing is the very thing itself, the *ipsissima res*, and the thing no otherwise differs from the form, than as the apparent differs from the existent, the outward from the inward.—*Nov. Org. Bk. ii.*  
 Formal Logic, 3.  
*Fundamentum divisionis*, 91.

## G

Galen, 135.  
 Gasendi (1592–1655) in his *Excitationes paradoxicae adversus Aristoteleos*, and in the first two books of his *Syntagma Philosophicum*, attacks the Aristotelic Logic.  
 General names, or terms, 41.  
 General notions, 27.

Generalization, 25.  
 Generic property, 82.  
 Genetic definition, 85.  
 Genus, 80.  
 Geometrical reasoning, 246.  
 Glaring instance, 292.  
 Grammar, 17.  
 Gravitation, theory of, 290.

## H

Hamilton, Sir W., b. 1788, d. 1856, 68, 81, 136, 151, 160.  
 Hegel, German philosopher, b. 1770, d. 1831, 322.  
 Herschel, Sir J., b. 1792, d. 1871, 266, 278.  
 Hispanus, Peter, afterwards Pope John XXI., d. 1277, in his *Summulae logicales*, first introduced a *perfect* set of mnemonic verses.  
 Historical evidence, 198.  
 Hobbes, Thomas, b. 1588, d. 1679, 40, 62.  
 Homology, the relation of certain parts of plants and animals to each other, as for example, the hand, in man, with the foot of the dog, the wing of the bat, and the flipper of the seal.  
 Homonymia, equivocation.  
 Hypothesis, a judgment provisionally accepted to explain a group of facts, and liable to be discarded if it be found inconsistent with them, 297.  
 Hypothetical syllogism, said to be due to Theophrastus, but afterwards more fully developed by Eudemus and the Stoics.

## I

Idea (Gr. *ἰδέα*, *εἶδος*), that which is the object of the understanding when a man thinks.—*Locke*. Or, that which is the immediate object of the mind in thinking. For *idea*, as contrasted with *sensation*, see Whewell's *Philosophy of the Inductive Sciences*, vol. i., p. 26.

Identity, law or principle of, 111.  
Idola, Bacon's, 209.

*Ignava ratio*, or *Sophisma pigrum*, ground of uselessness, the master fallacy of fatalism: regarding one's own agency as powerless to avert what fate has decreed. "Either you will recover from your present disease, or you will not recover; one of the two is fated. . . . It can be of no use to call in a doctor."

*Ignoratio elenchi*, 205.  
Illative, or valid conversion, 119.  
Illicit process, 35, 132, 204.  
Immediate inference, 104.  
Import of propositions, 70.  
Impossible matter, 118.  
Impossibility, 308.  
Inconsistent attributes, 308.  
Inconceivability of the opposite, 236.

Indefinite, or indesignate propositions, 30.

*Index tables*, 275.

Indirect demonstration, *reductio ad impossibile*, 146.

Individual, 94.

Induction, 239.

Inferences, 103.

Infima species, 81.

Information, 310; truths of, *ib.*

Inhesion, 71.

Instruction, 310; method of, 218; truths of, 310.

Intension of terms, 47.

Intentions, first and second, 41.

Intuition, 2.

Inversion, 122.

Irrelevant conclusion, fallacy of, 205.

### J

Jevons, W. S., 315.  
Joint Method of Agreement and Difference, 284.  
Judgment, 28.

### K

Kant, Immanuel, b. 1724, d. 1804, 53, 322  
Knowledge, Origin of, 1.

### L

Lambert, 87.  
Lanfranc, b. 1005, d. 1089, logical teacher of St. Anselm.  
Language, 21.  
Law, 5.  
Laws of co-existence, 261.  
Laws of nature, 255.  
Laws of reason and consequent, 164.  
Laws of thought, 111.  
Leibnitz, b. 1646, d. 1716, 16.  
Lemma (Gr. *λαμβάνω*, to take or assure), one of the subordinate elements of science. It is a judgment which does not properly belong to the science in which it appears, but is taken from another.  
Liberal arts, the seven ancient, the *trivium* and the *quadrivium*.  
Limitation, conversion by, 119.  
Locke, John, b. 1632, d. 1704, 18.  
Locke's categories, substances, modes, and relations.  
Logic, 1. The first use of the term as the name of a science is due to Zeno, the Stoic. He divides Philosophy into Logic, Physics, and Ethics.  
Logic as a metaphysic: all formal developments of logical rules are merely general expressions of metaphysical principles.—*Schelling*.  
Lombard, Peter, fl. 1160. "The Master of Sentences," collected and abridged the writings of the early fathers upon theology and philosophy, 321.

### M

Major term, 33; premiss, 128.  
Malebranche, Père, b. 1638, d. 1715, 217, called "The Christian Plato," wrote the *Recherche de la Vérité*.  
Mansel, H. L., b. 1820, d. 1871, 12.  
Many questions, fallacy of, 208.  
Material fallacies, 205.

Material Logic, 3.  
 Matter, or Body, according to Mill, that substance which is "the external cause to which we ascribe our sensations."—*Logic*, vol. i., p. 62.  
 Means, method of, 280.  
 Mediate inference, 104.  
*Mendoza collectio*, erroneous reasoning.  
 Metaphysical division, 86.  
 Metaphysics (*μετὰ τὰ φυσικά*), so-called from those of Aristotle's works that followed, or were intended to be studied after the Physics, including his First Philosophy and Ontology.  
 Method, 36, 216.  
 Methods of induction, 278.  
 Middle term, 33, 128.  
 Mill, J. S., b. 1806, d. 1873, 278.  
 Minor term, 33; premiss, 128.  
 Mnemonic verses, 35, 143  
 Modals, 75.  
*Modus ponens*, 164; *tollens*, 164; *ponendo tollens*, 165: *tollendo ponens*, 165.  
 Moods, 34, 139.  
 Moral sources of fallacy, see Bain's *Logic*, ii., pp. 376-385, and Bacon's *Idola*.

**N**

Names, 40.  
 Natural classification, 274.  
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 Necessity, 236.  
 Negation, conversion by, 120.  
 Negative terms, 43: propositions, 29.  
 Negative premisses, fallacy of, 204.  
 Nicole, Pierre, b. 1625, d. 1695, Port-Royalist.  
 Nomenclatures, according to Mill, "the collection of names of all the Kinds with which any branch of knowledge is conversant."—*Logic*, Bk. IV. chap. vi., § 4.  
 Nominal definitions, 85.  
 Nominalists, 26.  
*Non causa pro causa*, 208.

*Non sequitur*, 208.  
 Notion, concept, or idea, 2.  
*Novum Organum*, 321.  
*Novum Organum Renovatum*, 279.  
 Numerically definite syllogism, 157.

**O**

Objective, 9.  
 Observation, 266.  
 Obversion, 116, 120.  
 Occam, William of, d. 1328, 321.  
 Ontology, the science of Being as Being, and of the attributes that belong to it as such, 16.  
 Opinion, 235.  
 Opposite terms, see Contrary terms, 221.  
 Opposition of propositions, 116. In the *square* of opposition, Aristotle makes use of the diagonal to indicate the full contraries, A and E: the contraditories, namely, A and O, I and E, being marked on the upper and lower pairs of corners respectively.  
 Organon, the, 320.

**P**

Paradox (Gr. *παρά*, against, and *δόξα*, an opinion), an assertion contrary to received opinion.  
 Paralogism, 203.  
 Parcimony, or parsimony, law of, 131. It is always best to assume as little as possible, provided that this will account for the fact.—*Dr. Venn*.  
 Parity of reasoning, extending what is proved, say, from the single diagram before us, to all similar instances; e.g., in the demonstrations of Euclid. To be distinguished from Induction.  
 Paronymous words, of same root.  
 Particular propositions, 30.  
 Particular Logic, in which particular objects are thought of, as contrasted with universal.—*Kant*.

Pascal, Blaise, b. 1623, d. 1662. His eight rules of demonstration are :—

1. To define nothing which cannot be expressed in clearer terms than those in which it is already expressed.
2. To leave no obscure or equivocal term undefined.
3. To employ in the definition no terms not already known.
4. To omit nothing in the principles from which we argue unless we are sure it is granted.
5. To lay down no axiom which is not perfectly self-evident.
6. To demonstrate nothing which is as clear already as it can be made.
7. To prove everything in the least doubtful, by means of self-evident axioms, or propositions already demonstrated.
8. To substitute, mentally, the definition instead of the thing defined.

*Per Accidens*, 119.

Perfect figure of the syllogism, 142. Personal error or equation, 269.

*Petitio principii*, 203.

Philosophical language, 275. Its two principal requisites, according to Mill are, first, precision, or definiteness; and secondly, completeness.

Photius, patriarch of Constantinople, 9th century, wrote on the Categories.

Physical definition, 85.

Plato, b. about 429 B.C. To Plato is due the methods of *συνάγωγή* and *διαίρεσις*, the collection of scattered objects with a view to definition, and a gradual dichotomy by means of contrary or contradictory members, so as to ascertain the subordinate species under each genus. He also analysed the proposition; and the three primary laws of thought are indicated, though not expressly enunciated, by him.

Plurality of causes, 288.

Pluritive propositions, 67.

Porphyry, in the 3rd century, wrote the *εἰσαγωγή*, or an introduction to the Categories. The work is the only source as to the ancient classification of the predicables.

Porphyry, tree of, 95.

Port Royal Logic, 322.

*Post hoc ergo propter hoc*, fallacy of, 208.

Positive terms, 43.

Postulate, 236.

Predicables, 78.

Predicate, 61.

Premiss, 33, 127.

Primary laws of thought, 111.

Principle, a fundamental truth; a truth admitted either without proof, or considered as having been before proved; the major premiss of a syllogism.

Principles of Classification of the Sciences. The Sciences have been classified according to, (1) the purpose of each science, (2) the faculties, chiefly, through which the knowledge is derived to us, (3) the subject-matter, and (4) the principle laid down by Descartes that sound knowledge should advance from the simpler to the more complex phenomena.

Classified on this latter principle, the principal sciences, theoretical, historical and applied, would stand in the following order:—Mathematics, Astronomy, Physics, Chemistry, Physiology, Anthropology, Social Science, and Religious Philosophy.—*Thomson*.

Privative terms, 43.

Probability, 186.

Problem (Gr. *πρόβλημα*, anything put forward), 236.

Proof, 106.

Propædeutic of the understanding and of the reason, Logic a general (*ταῦτα*, to bring up, educate, rear, teach).—*Kant*.

Propositions, 28, 61.

Proprium, 83.  
 Pro-syllogism, 153.  
 Province of Logic, 1.  
 Proximate genus, 81, 93.  
 Psychology, 16.

## Q

Quadrivium, the ancient, consisted of music, arithmetic, astronomy and Geometry.  
 Quality of propositions, 29.  
 Quantification of the predicate, 68.  
 Quantity of propositions, 30.  
*Quaternio terminorum*, four terms, 131, 203.

## R

Ramus, Peter, b. 1515, d. 1572, the first who proposed to make Method a part of the science of Logic.  
 Ratiocination, syllogism, deduction, 103, 126.  
 Realists, 26.  
 Reason, syllogism of the, formal syllogism (—Kant), 103.  
 Reasoning, 32, 126.  
*Reductio ad absurdum*, or *ad impossible*, indirect demonstration, 146.  
 Reduction of the figures, 142.  
 Relation, 155.  
 Residual phenomena, 286.  
 Residues, Method of, 285.  
 Rhetoric, 17.  
 Roscellinus of Compiègne, fl. about 1080.  
 Rules or Canons of the syllogism, 114, 130.

## S

Salisbury, John of, b. 1120, d. 1180.  
 Scholastic views, 26.  
 Scholium, or Scholion, a proposition illustrative of a science, or of that particular proposition in a science where it appears, without being an essential part of it.  
 Schoolmen, a term later employed in an extended sense, was at

first given to the teachers of the cathedral and conventional schools established by Charlemagne and his successors. Of the "Five Great Schoolmen," Italy sent forth Thomas Aquinas and Bonaventura, Germany, Albert the Great, and the British Isles, Duns Scotus and William of Occam.

Science, 3, 232.  
 Scotus, Duns, b. 1265, d. 1308.  
 Second intention, 41.  
*Secundi adjacentis* (incorrect Latin), a proposition containing only the subject and the verb, without a distinct copula.

Semi-logical fallacies, 204.  
 Separable accident, 81.

Similars, substitution of, 313, 316.  
 Singular terms, 42.

Socrates or Sokrates, d. about 400 B.C. We owe to him, according to Aristotle, induction and definition.

*Sophisma heterozeteseos*, irrelevant question.

*Sophisma polyzeteseos* (Gr. ζήτησις, ζήτησεως, a seeking for), otherwise, *calvus*, or the bald, the drawing of a respondent beyond the range of meaning of a word of quantity, e.g. baldness, heap, calf, &c., by asking, for example, does the removal of one hair constitute baldness, does that of two, and so on, until at length he is asked to name the difference between the last case admitted and that not admitted.

Sorites, 154.  
 Specialization of a name, the restriction of it to a narrower class, or the limitation of its extension, e.g., surgeon, originally included barbers, but is now limited to medical men who heal, or alleviate diseases, or injuries of the body, with the employment of manual operation.

Species, 80.  
 Squares, method of least, 280.  
 This is a mode of finding the

most probable result in those cases in which the arithmetical mean is not an applicable expedient. It proceeds upon the assumption that all errors are *not* equally probable, but that small errors are more probable than large. It follows as a corollary from this assumption that the most probable conclusion can be obtained by making, not the errors themselves, but the sum of the squares of these errors, of the smallest possible amount.

Subcontrary propositions, 116.

Subject of a proposition, 61.

Subjective, 9.

Subsidiaries to induction, 265.

Substances, those self-existent underlying things which we can know by phenomena or attributes. Substances are distinguished by metaphysicians into Bodies and Minds.

Substitutive proposition, 151.

Sumption (Hamilton), the minor premiss of a syllogism, 136.

Sufficient reason, principle of, 113. *Sui generis*, 221.

*Summum genus*, 82.

Sumption of a law, Mill's third mode of explaining a law, namely, by resolving it into a law more general than itself.—*Logic*, Bk. III., chap. xii., § 5.

Sumption (Hamilton), the major premiss of a syllogism, 136.

Syllogism, 33, 126.

Symbolical Logic, 155, 160.

Synecategorematic words, 39.

Synthesis, method of, 218.

Synthetical syllogism, one in which the conclusion stands last; analytical, in which it stands first.

System (*σύστημα*, from *συντηνει*, to place or set together), the orderly combination of the related parts of a body of knowledge into one relatively complete whole.—*Ueberweg*.

## T

Tendency, 302.

Term, 27, 39.

*Tertii adjacenter* (incorrect Latin), a proposition in which the subject, the predicate, and the copula are distinctly stated.

Thaumatrope fallacy, the incompatibility of two objects, either of which may be separately attained, is disguised by a rapid and frequent transition from the one to the other.

Theory, 302.

Thesis, 236.

Thomsonian moods, those (*Laws of Thought*, p. 179) based on Thomson's six forms of judgments, (*Ib.* p. 135), namely, A, E, I, O, U, and Y.

Thought, laws of, 111.

*Totum divisum*, 92, 94.

Traduction, 246.

Truth, 231.

Truths, new, 310.

## U

Ultimate laws of nature, 305.

Ultra-total distribution, 156.

Understanding, syllogism of; immediate inference.—*Kant*.

Universe, a whole, the parts of which we are considering.—*De Morgan*.

Universal logic, *logica docens* (Bowen), 13; logic in which abstraction is made of the contents of the cognition (Kant).

Universal propositions, 30.

Universals, 25.

Universal terms, 41.

## V

Variations, method of concomitant, 281, 286.

Verbal propositions, 65.

Verification, 290.

*Verae ratiæ*, 258. An expression occurring in the first of Newton's *Regulae philosophandi*.

Regula I. *Causas rerum naturalium non plures admitti*

debere, quam quae et verae sunt et earum phenomenis explicandis sufficient.

Dicunt utique philosophi: natura nihil agit frustra, et frustra fit per plura quod fieri potest per pauciora.

— Newton's *Principia*, ed. by the PP. Sieur and Jacquier, vol. iii., p. 2.

Verities, two produce a third, in syllogism.

W

Weakened conclusion, 140.  
Whately, Dr. Richard, b. 1787, d. 1863, 12, 244, 312.

Whewell, William, b. 1794, d. 1866, 278.

Working hypothesis, a hypothesis "entertained with good reason, and for useful purposes." It must tally with a large number of facts, and not be incompatible with any, 303, 314.

Z

Zeno, the Eleatic, b. about 590 B.C., is mentioned as the inventor of Dialectic, and as, "skilled to argue on both sides of any question," 319.

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